Problem Set 7 – Due Monday, May 19, 2009, 3:15

1. For each definition below is it a function?, and give its range.
   a) \( f(x) = x^2 \), the domain and co-domain of \( f \) are the reals.
   b) \( g(x) = \lceil x \rceil \), the domain and co-domain of \( g \) is the reals.
   c) \( h(x) = x^{-5} \), the domain and co-domain of \( h \) are the reals.
   d) \( f \circ g (x) \)

2. Prove that a function \( f : A \leftarrow B \) is invertible iff \( f \) is 1-1 and onto.

3. Suppose \( f(x) = \Theta(n^3) \) and \( g(x) = O(n^2) \) What can we say about the following in terms of big O and \( \Theta \) terms?
   a) \( f(x) \times g(x) \)
   b) \( f(x) + g(x) \)
   c) \( f \circ g (x) \)

4. Show that the function \( \text{Fibo}(n) \), which returns the \( nth \) fibonacci number, is in \( O(2^n) \). Hint, use induction.

5. We proved that the positive integers were equinumerous with all integers (and with the rationals). Now let \( A \) be the odd positive integers. Show that \( A \) is equinumerous with \( \mathbb{N} \), and thus that \( A \) is equinumerous with \( \mathbb{Q} \).