Problem Set 5—Due Friday, March 2. 3:15 hardcopy, 11PM electronic

(25) Problem 1. 11-7

(20) Problem 2 10-3

(30) Problem 3. Vertex covers in Bipartite Graphs We will show that we can use network flow/matching to find minimum cardinality vertex covers in bipartite graphs.

a) Argue that for any matching $M$ in an undirected graph $G$ (whether bipartite or not), the minimum cover of $G$ must be at least of size $|M| = k$.

b) Suppose we have a bipartite graph $G = (L, R, E)$, where $L$ is the left vertex set and $R$ the right vertex set. We can form the normal flow network (as discussed in class and in the book) to find a maximum flow $f$ and its associated matching $M^*$ in $G$. Let $S, T$ be the minimum-$s,t$-cut associated with $f$. Prove that $C = (L \cap T) \cup (R \cap S)$ is a minimum cover of $G$ by showing that:

i) for every edge $(u, v) \in M^*$ exactly one of $u$ and $v$ is in $C$.

ii) for every edge $(u, v) \in E$, At least one of $u$ and $v$ is in $C$. (Thus $C$ is a cover).

iii) $|C| = |M^*|$ (hint: it may be useful to show that if a vertex $x$ is unmatched, then it is not in $C$).

(25) Problem 4. In 10.2 we showed that we can find the maximum weight independent set in a tree in linear time using dynamic programming. Now consider a variation of the independent set problem, a 1-independent set, which is a set $S$ of vertices such at at most one pair of vertices in $S$ is connected by an edge (thus we allow one pair of vertices to be connected by an edge, in contrast to normal IS when we allow zero). We consider the 1-IS problem on trees.

a For the unweighted case, is it still always OK to add a leaf to the solution and obtain a maximum size 1-IS? Justify your answer.

b For the weighted case, modify the DP formulation of 10.2 to efficiently find a maximum weight 1-IS when the input graph is a tree. Justify your answer and give its run time.

(0) Study Problem: not to be turned in 11-10