Problem Set 4 – Due Thursday, October 25, 2012

Problem 1. Let \( \alpha \) be a regular expression of length \( n \).

(a) Using procedures shown in class, if we convert \( \alpha \) into a regular expression \( \beta \) such that \( L(\beta) = \overline{L(\alpha)} \), how long might \( \beta \) be? Give a reasonably tight upperbound.

(b)* Can you define an infinite family of regular expressions \( \{\alpha_n\} \), \( |\alpha_n| \in O(n) \), but where the shortest regular expression for \( \overline{L(\alpha_n)} \) will have length \( \Omega(2^n) \)?

Problem 2. Using the pumping lemma, show that the following languages are not regular.

(a) \( L = \{www : w \in \{a,b\}^*\} \).

(b) \( L = \{a^{2^n} : n \geq 0\} \).

(c) \( L = \{0^m1^m0^m : m,n \geq 0\} \).

Problem 3. Define \( A = \{x \in \{a,b,\#\}^* : x \text{ contains an equal number of } a\text{'s and } b\text{'s or } x \text{ contains consecutive } \#\text{s or consecutive letters}\} \).

(a) Can you use the pumping lemma to prove that \( A \) is not regular? Explain.

(b) Prove that \( A \) is not regular.

Problem 4. Are the following statements true or false? Either prove the statement or give a simple counter-example.

(a) If \( L \cup L' \) is regular then \( L \) and \( L' \) are regular.

(b) If \( L^* \) is regular then \( L \) is regular.

(c) If \( LL' \) is regular then \( L \) and \( L' \) are regular.

(d) If \( L \) and \( L' \) agree on all but a finite number of strings, then one is regular iff the other is regular.

(e) If \( R \) is regular, \( L \) is not regular, and \( L \) and \( R \) are disjoint, then \( L \cup R \) is not regular.

(f) If \( L \) differs from a non-regular language \( A \) by a finite number of strings \( F \), then \( L \) itself is not regular.

Problem 5. Carefully describe an algorithm to answer the following question: given a regular expression \( \alpha \), is \( L(\alpha) = (L(\alpha))^R \)? What is the asymptotic running time of your algorithm?

Problem 6. For any language \( L \) let
\[ F(L) = \{w \in L \mid \text{no proper prefix of } w \text{ is a member of } L\} \]
Prove or disprove: the regular languages are closed under \( F \).

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2This problem is intended for, at most, the top students. If you can find an elementary solution without consulting the literature, please give it directly to Prof. Rogaway.