Quiz 3

Your name (neatly):

Notation: TM = Turing machine. [Turing] acceptable = [Turing] recognizable = recursively enumerable = r.e. [Turing] decidable = recursive. $L$ is co-r.e. iff $\overline{L}$ is r.e.

1. We defined a Turing Machine (TM) as a seven-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R)$ where, among other requirements, $\delta$ was a function with domain $\Gamma^* \times Q$ and range $\Gamma^*$. We defined the configuration of $M$ as an element of $\Gamma^* \times Q \times \Gamma^*$, writing $\alpha q \beta$ for $(\alpha, q, \beta)$. We let $\vdash$ be the moves-in-one-step relation on configurations, defined, e.g., by asserting that $\alpha q_0 b \beta \vdash$ if $\delta(q, b) = (r, c, L)$. We let $\vdash^*$ be the reflexive-transitive closure of $\vdash$ and said that machine $M$ accepts a string $w$ if $q_0 w \vdash^*$ (follow conventions of class and your book).

2. Darken (fill in) the correct answer. No justification is required.

(a) True False If a TM $M$ decides a language $L$ then $M$ halts on every input $x$.
(b) True False A TM must accept or reject every string; otherwise, it is invalid.
(c) True False A language $L$ that is not r.e. is co-r.e.
(d) True False The language $A_{TM} = \{\langle M, w \rangle : M$ accepts $w\}$ is r.e.
(e) True False Every context-free language is Turing-decidable.
(f) True False The Church-Turing Thesis was proven by Alonzo Church and Alan Turing.
(g) True False A TM $M$ can be provided its own description, $\langle M \rangle$, as its input.
(h) True False The language $\{\langle M \rangle : L(M)$ is finite\} is r.e.
(i) True False There is a language $L$ that can be recognized by a 2-tape TM but that cannot be recognized by any 1-tape TM.
(j) True False If $A \leq_m B$ and $A$ is r.e. then $B$ is r.e.
(k) True False To show $L$ undecidable, it is enough to show that $A_{TM} \leq_m L$.
(l) True False To show $L$ undecidable, it is enough to show that $L \leq_m A_{TM}$.

3. Give a clear and self-contained proof for the following:

If $L$ is recognizable and $\overline{L}$ is recognizable then $L$ is decidable.