Problem Set 8—Due May 27, 2004

Problem 1 (Counts as 20 points, same as 2 ordinary problems.)

Part A. Let \( L = \{ \langle M \rangle : M \text{ is a TM that accepts some string of prime length} \} \). Prove that \( L \) is not decidable.

Part B. Let \( L = \{ \langle M \rangle : M \text{ is a TM and } L(M) = L(M)^* \} \). Prove that \( L \) is not r.e.

Part C. Let \( L = \{ \langle M \rangle : M \text{ is a TM and } L(M) = L(M)^* \} \). Prove that \( L \) is not co-r.e.

Part D. Let \( L = \{ \langle G_1, G_2 \rangle : G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) = L(G_2) \} \). Prove that \( L \) is not decidable. You may use the fact that \( A = \{ \langle G \rangle : G \text{ is a CFG and } L(G) = \Sigma^* \} \) is undecidable.

Problem 2 A phrase-structure grammar is like a CFG except that the left-hand side of every rule is a nonempty string of variables: rules look like \( \alpha \to \beta \) where \( \alpha \in V^+ \) and \( \beta \in (V \cup \Sigma)^* \). Consider the membership question for phrase-structure grammars: \( L = \{ \langle G, w \rangle : G \text{ is a phrase structure grammar and } w \in L(G) \} \). Is \( L \) decidable? Is it r.e. or co-r.e.??