Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all 3 problems. Do your best to communicate your ideas clearly and succinctly. Good luck. —Phil Rogaway

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1 Short Answer

1.1 Draw a DFA $M$ for the language

$$L = \{ x \in \{a, b, c\}^*: x \text{ contains exactly one } a \text{ and exactly one } b \}.$$  

Make your DFA have as few states as possible.

1.2 List the first five strings of this language (Problem 1.1) in lexicographic order. Assume $a < b < c$.

1.3 Write a regular expression for this language (Problem 1.1). Make it as short as possible.
1.4 Give a CFG for \( L = (ab \cup aaa)^* baa \). Make your grammar use as few rules as possible.

1.5 Let \( M = (Q, \Sigma, \delta, q_0, F) \) be an NFA with no \( \epsilon \)-arrows. We can convert \( M \) into a DFA \( M' = (Q', \Sigma, \delta', \{q_0\}, F') \) whose language is \( L(M) \) by setting

\[
Q' = \text{and } \delta'(S, a) = \text{and } F' = \{T \subseteq Q : T \cap F \neq \emptyset\}.
\]

1.6 Using the procedure shown in class, convert the following NFA into a regular expression for the same language.

![NFA Diagram]
2 Justified True or False

Put an X through the correct box. Then provide a brief justification. Where appropriate, make the justification a counter-example.

2.1 Every regular language can be accepted by an NFA with only a single final state.

Justification: True False

2.2 The complement of a regular language is context free.

Justification: True False

2.3 Let $h : \Sigma \rightarrow \Sigma^*$ be a function and define $h(a_1 \cdots a_n) = h(a_1) \cdots h(a_n)$ and $h(L) = \{h(x) : x \in L\}$. Suppose $h(L)$ is not regular. Then $L$ is not regular.

Justification: True False

2.4 There is a language $L$ for which $L = L^*$.

Justification: True False

2.5 Every nonempty regular language $L$ is generated by some ambiguous CFG.

Justification: True False
3 Classify

3.1. Let $L = \{ww : w \in \{0,1\}^*\}$. Is $L$ regular? Prove your answer.

3.2. Let $L = \{w \in \{0,1\}^* : w$ contains an equal number of $01$’s and $10$’s$\}$. Is $L$ regular? Prove your answer.