Problem 1. Let $G = (V, E)$ be a graph (the “usual” sort, being nonempty, finite, undirected, having no-self loops and no multiple edges). Prove (by giving a convincing argument) or disprove (by giving a smallest counter-example) that the following are equivalence relations for any graph $G$.

Part A. Let $x, y \in V$. Say that $x R_G y$ if there is a path in $G$ from $x$ to $y$ (that is, a sequence of vertices $x_1, \ldots, x_n \in V$ ($n \geq 1$) where each $\{x_i, x_{i+1}\} \in E$ and $x = x_1$ and $y = x_n$).

(Note: the equivalence classes of this equivalence relation are called the “components” of $G$.)

Part B. Let $x, y \in V$. Say that $x R_G y$ if $x$ is adjacent to $y$ (that is, $\{x, y\} \in E$).

Part C. Let $x, y \in V$. Say that $x R_G y$ if $x = y$ or $\{x, y\} \in E$ or there are two vertex-disjoint paths from $x$ to $y$ (paths $x_1, \ldots, x_m$ and $x_1', \ldots, x_{m'}'$, where $x_1 = x_1' = x$ and $x_m = x_{m'}' = y$ and $\{x_2, \ldots, x_{m-1}\} \cap \{x_2', \ldots, x_{m'-1}'\} = \emptyset$).

Part D. Let $x, y \in V$. Say that $x R_G y$ if there is a path from $x$ to $y$ and this remains so even if one removes any edge $e \in E$.

Problem 2. State whether the following propositions are true or false, explaining each answer.

Part A. $\emptyset$ is a language.

Part B. $\emptyset$ is a string.

Part C. $\epsilon$ is a language.

Part D. $\epsilon$ is a string.

Part E. Every language is infinite or has an infinite complement.

Part F. Some language is infinite and has an infinite complement.

Part G. The set of real numbers is a language.

Part H. There is a language that is a subset of every language.

Part I. The Kleene-star (Kleene closure) of a language is always infinite.

Part J. The concatenation of an infinite language and a finite language is always infinite.

Part K. There is an infinite language $L$ containing the emptystring and such that $L^i$ is a proper subset of $L^*$ for all $i \geq 0$.

Problem 3 Friday is a holiday; don’t come to class. Instead, find something to read on the web on César Chavéz. Or join me on Friday from 1:15 pm to 3:15 pm in room 1065 Kemper, when I’ll screen the well-regarded documentary The Fight in the Fields (1997). But double-check our course web page Thursday night after 11 pm or Friday morning to confirm that I was actually able to get hold of the film.\footnote{How annoying is it to have class begin on a Wednesday when the following Friday is a holiday? They couldn’t postpone the start of the term until May 2?}

\footnote{Problem 3 will not be graded and the film is an activity unrelated to your work in this particular class.}