Problem Set 8

Problem 1.

Part A. Suppose that $L$ is r.e. Show that $L^*$ is r.e.

Part B. Suppose that $L$ is recursive. Show that $L^*$ is recursive.

Problem 2. Classify each of the following problems as either decidable—I see how to decide this language; r.e.—I don’t see how to decide this language, but I can see a procedure to accept this language; co-r.e.—I don’t see how to decide this language, but I can see a procedure to accept the complement of the language; neither: I don’t see how to accept this language nor its complement.

Part A. $\{\langle M \rangle : M$ is a TM that accepts some string of prime length$\}$.

Part B. $\{\langle M \rangle : M$ is a C-program that halts on $\langle M \rangle$.$\}$.

Part C. $\{\langle G \rangle : G$ is a CFG and $G$ accepts an odd-length string$\}$.

Part D. $\{\langle M \rangle : M$ is a TM and $M$ has 150 states$\}$.

Part E. $\{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*$.$\}$.

Part F. $\{\langle M \rangle : M$ is a TM and $L(M) = \emptyset$.$\}$.

Part G. $\{\langle M \rangle : M$ is a TM and $L(M)$ is r.e. $\}$.

Part H. $\{\langle G_1, G_2 \rangle : G_1$ and $G_2$ are CFGs and $L(G_1) = L(G_2)$.$\}$.

Part I. $\{\langle M \rangle : M$ is a TM and $M$ will visit state $q_{25}$ when run on some input $x$.$\}$.

Part J. $\{\langle M \rangle : M$ is a TM and $M$ makes use of at most 50 tape cells when run on blank tape$\}$.

Problem 3. Recall that $L = \{ww : w \in \{a, b\}^*\}$ is not context free. Exhibit an unrestricted grammar for it. An unrestricted grammar is like a CFG except that rules can look like $aBCb \rightarrow accbDe$, for example: left-hand sides can contain any string of terminal and nonterminals with at least one nonterminal.