Problem Set 9 — Due Wednesday, May 30, at 3:30 pm

Problem 1. Prove whether each of the following languages is **RECURSIVE**, **RE** but not recursive, **CO-RE** but not recursive, or **NEITHER** r.e. nor co-r.e.

**Part A.** \(L = \{\langle M, w \rangle : M \text{ is a TM that uses at most 17 tape squares when run on } w \}\).  

**Part B.** \(L = \{\langle M, k \rangle : M \text{ is a TM that accepts at least one string of length } k \}\).  

**Part C.** \(L = \{\langle M, k \rangle : M \text{ is a TM that diverges on at least one string of length } k \}\).  

**Part D.** \(L = \{\langle M, k \rangle : M \text{ is a TM that accepts a string of length } k \text{ and diverges on a string of length } k \}\). Assume that the underlying alphabet has at least two characters.  

**Part E.** \(L = \{\langle M \rangle : M \text{ is a TM that accepts some palindrome} \}\).  

**Part F.** \(L = \{(G_1, G_2) : G_1 \text{ and } G_2 \text{ are CFGs which generate the same CFL} \}\).  

Problem 2.

**Part A.** Give two languages \(L_1\) and \(L_2\), each r.e. but not recursive, with empty intersection.  

**Part B.** Give two languages \(L_1\) and \(L_2\), each r.e. but not recursive, with union \(\Sigma^*\).  

**Part C.** Are there languages \(L_1\) and \(L_2\) meeting conditions (A) and (B) simultaneously? Why or why not?

Problem 3  Prove that \(L\) is decidable iff \(L\) is listable in lexicographic order. (A language is listable in lexicographic order if some program outputs \(x_1\#x_2\#x_3\#\cdots, L = \{x_1, x_2, x_3, \ldots\}\), and \(x_1 < x_2 < x_3 < \cdots\) where “<” denotes the usual lexicographic ordering on strings.)