Midterm Exam

Instructions: The exam has six pages, not including this cover page. Read everything carefully. Write everything carefully, too; you will be graded on clarity and correctness. Please write neatly as well.

Relax. Breathe. The exam is not too long or too hard.
— Phil Rogaway

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1 Short Answer

(1.1) Complete the definition: A language over an alphabet $\Sigma$ is

(1.2) List the first five strings, in lexicographic order, of the language

$$L = \{x \in \{a, b, c\}^* : x \text{ contains at least one } a \text{ and at least one } b\}.$$  
Assume, as usual, that characters are ordered $a < b < c$.

(1.3) Explain what the product construction is and what we used it for.
(1.4) Using the procedure given in class and in your text, convert the following NFA $M$ into a regular expression $\alpha$ such that $L(M) = L(\alpha)$. Do not “simplify” anything as you go.

(1.5) You are given the regular expression $\alpha = (00 \cup 11)^*$. *Composing the constructions given in class and in your text* (do not “simplify” anything), imagine converting $\alpha$ into a DFA $M$ for which $L(M) = L(\alpha)$. How many states will $M$ have? Justify your answer.

(1.6) Complete the definition: A CFL $L$ is **inherently ambiguous** if: (be precise with your quantifiers)
(1.7) Complete the definition: A regular grammar is a CFG $G = (V, \Sigma, R, S)$ where:

(1.8) Carefully explain what it means if one says “the CFLs are closed under union.” Then prove that this statement is true.

(1.9) Carefully state the pumping lemma for context free languages. (Don’t use the word “pumps” without defining it.)
2 Justified True or False

Put an X through the correct box. When it says “Explain” provide a brief (but convincing) justification. Where appropriate, make this justification a counterexample. Choose the simplest counterexample you can find.

2.1. For every number $n$, the language $L_n = \{0^n1^n\}$ is regular.

Explain:  

True  False

2.2. If $L^*$ is regular then $L$ is regular.

Explain:  

True  False

2.3. Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA and let $M' = (Q, \Sigma, \delta, q_0, F')$, where $F' = Q \setminus F$ is the complement of $F$ relative to $Q$. Then $L(M) = \overline{L(M')}$.

Explain:  

True  False

2.4. Let $L = \{a^n b^n : n \geq 0\}$. Then $\overline{L}$ is regular.

Explain:  

True  False
2.5. Let noPrefix(L) = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}. If L is regular, then so is noPrefix(L).

Explain: True False

2.6. The pumping lemma is a useful technique to show that a language is regular.

Explain True False

2.7. Language L = \{w \in \{0, 1\}^* : \text{w has an equal number of 01's and 10's}\} is regular.

Explain True False

2.8. If an NFA M = (Q, \Sigma, \delta, q_0, F) has only accepting states (i.e., F = Q), then L(M) = \Sigma^*.

Explain: True False
3 Simple Proofs

(3.1) Let $L = \{x \in \{a, b\}^* : |x| < 5\}$. Prove that there is a 6-state DFA that accepts $L$.

(3.2) Let $L = \{x \in \{a, b\}^* : |x| < 5\}$. Prove that there is no 5-state DFA that accepts $L$.

(3.3) Let $L = \{www : w \in \{a, b\}^*\}$. Prove that $L$ is not regular.