Problem Set 2 – Due Tuesday, April 13, 2010, at 4:15 pm

Instructions: Write up your solutions as clearly and succinctly as you can. Typeset solutions, particularly in *LATEX*, are always appreciated. Don’t forget to acknowledge anyone with whom you discussed problems. Beginning with this homework, homeworks are to be due at **4:15 pm** (no longer 4:40 pm).

**Problem 1.** Let $\text{canExtend}(L) = \{ x \in L : \text{there exists a } y \in \Sigma^+ \text{ for which } xy \in L \}$.

**Part A.** What is $\text{canExtend}(\{0,1\}^*)$? What is $\text{canExtend}(\{\varepsilon,0,1,00,01,111,110,1111\})$?

**Part B.** Prove that if $L$ is DFA-acceptable then $\text{canExtend}(L)$ is too.

A *prefix* of a string $y$ is a string $x$ such that $y = xx'$ for some $x'$. A prefix is *proper* if it is not the empty string. For any language $L$, let $\text{noPrefix}(L) = \{ w \in L \text{ no proper prefix of } w \text{ is in } L \}$.

**Part C.** What is $\text{noPrefix}(\{0,1\}^*)$? What is $\text{noPrefix}(\{\varepsilon,00,01,110,0100,0110,1110,1111\})$?

**Part D.** Prove that if $L$ is DFA-acceptable then so is $\text{noPrefix}(L)$.

**Problem 2.** Using the procedure shown in class, convert the following NFA into a DFA for the same language.

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0, 1
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A ----> 1 ----> B ----> 2 ----> C
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**Problem 3.** let $L = \{1^i : 0 \leq i < 10\}$ (recall that $1^0 = \varepsilon$). Prove that there is a DFA $M$ having 10 accepting states that accepts $L$. Then prove that $L$ cannot be accepted by any DFA having fewer accepting states.

**Problem 5.** Consider applying the product construction to NFAs $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ in order to show that the NFA-acceptable languages are closed under intersection.

**Part A.** Formally specify the product machine $M_3 = (Q, \Sigma, \delta, q_0, F)$.

**Part B.** Does the construction work—that is, is $L(M_3) = L(M_1) \cap L(M_2)$? Informally argue your conclusion.

**Problem 5.** Prove that the DFA-acceptable languages are closed under reversal.

**Problem 6.** Consider trying to show that the NFA-acceptable languages are closed under $*$ (Kleene closure) by way of the following construction: *add $\varepsilon$-arrows from every final state to the start state; then finalize the start state, too.* Show, by finding a small counterexample, that the proposed construction does not work.