Problem Set 8 – Due Tuesday, May 25, 2010, at 4:15 pm

Problem 1. Classify each of the following languages as either (a) **recursive**—I see how to decide this language; (b) **r.e.**—I don’t see how to decide this language, but I can see a procedure to accept this language; (c) **co-r.e.**—I don’t see how to decide this language, but I can see a procedure to accept the complement of the language; or (d) **neither**: I don’t see how to accept this language nor its complement. No justification is needed for your answers.

**Part A.** \{\langle M \rangle \mid M \text{ is a TM that accepts some string of prime length}\}.

**Part B.** \{\langle M \rangle \mid M \text{ is a C-program that halts on } \langle M \rangle\}.

**Part C.** \{\langle G \rangle \mid G \text{ is a CFG and } G \text{ accepts an odd-length string}\}.

**Part D.** \{\langle M \rangle \mid M \text{ is a TM and } M \text{ has } 150 \text{ states}\}.

**Part E.** \{\langle M \rangle \mid M \text{ is a TM and } L(M) = L(M)^\ast\}.

**Part F.** \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset\}.

**Part G.** \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is r.e. }\}.

**Part H.** \{\langle G_1, G_2 \rangle \mid G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) = L(G_2)\}.

**Part I.** \{\langle M \rangle \mid M \text{ is a TM and } M \text{ will visit state } q_{25} \text{ when run on some input } x\}.

**Part J.** \{\langle M \rangle \mid M \text{ is a TM and } M \text{ that uses at most } 17 \text{ tape cells when run on blank tape}\}.

Problem 2. Prove whether each of the following languages is **recursive**, **r.e.** but not recursive, **co-r.e.** but not recursive, or **neither** r.e. nor co-r.e.

**Part A.** \(L = \{\langle M, w \rangle \mid M \text{ is a TM that uses at most } 17 \text{ tape squares when run on } w\}\).

**Part B.** \(L = \{\langle M, k \rangle \mid M \text{ is a TM that accepts at least one string of length } k\}\).

**Part C.** \(L = \{\langle M, k \rangle \mid M \text{ is a TM that diverges (loops) on at least one string of length } k\}\).

**Part D.** \(L = \{\langle M, k \rangle \mid M \text{ is a TM that accepts a string of length } k \text{ and diverges on a string of length } k\}\). Assume that the underlying alphabet has at least two characters.

**Part E.** \(L = \{\langle M \rangle \mid M \text{ is a TM that accepts some palindrome}\}\).

Problem 3.

**Part A.** Give two languages \(L_1\) and \(L_2\), each r.e. but not recursive, with empty intersection.

**Part B.** Give two languages \(L_1\) and \(L_2\), each r.e. but not recursive, with union \(\Sigma^*\).

**Part C.** Are there languages \(L_1\) and \(L_2\) meeting conditions (A) and (B) simultaneously? Why or why not?