Problem Set 5 – Due Friday, May 3, 2013

Problem 1. Given an NFA \( M = (Q, \Sigma, \delta, q_0, F) \), define \( \Lambda(M) = \{ x \in \Sigma^* : \delta^*(q_0, x) \subseteq F \} \). In clear English, explain what is \( \Lambda(M) \). Then prove that \( L \) is regular iff there is a machine \( M \) such that \( L = \Lambda(M) \).

Problem 2. Specify a CFG for the language
\[
L = \{ x \in \{a, b, c\}^* : x \text{ contains an equal number of two different characters} \}.
\]
Make your CFG as simple as possible. (If it isn’t obviously right to the TA, it isn’t right.)

Problem 3. Specify a CFG for \( L = \{ x \neq y : x, y \in \{0, 1\}^+ \text{ and } x \neq y \} \). With diagrams or clear English, explain how your grammar works.


Problem 5. Consider the following CFG \( G = (V, \Sigma, R, \text{STMT}) \):

\[
\begin{align*}
\text{STMT} &\rightarrow \text{ASSIGN} \mid \text{IFTHEN} \mid \text{IFTHENELSE} \\
\text{IFTHEN} &\rightarrow \text{if condition then STMT} \\
\text{IFTHENELSE} &\rightarrow \text{if condition then STMT else STMT} \\
\text{ASSIGN} &\rightarrow a:=1
\end{align*}
\]

with \( V \) being the variables in CAPS and \( \Sigma \) being the tokens in \textbf{bold}. We explained in class why \( G \) (or something just like it) is ambiguous. Provide an unambiguous CFG \( G' \), the simplest you can find, where \( L(G') = L(G) \). Explain why \( G' \) is unambiguous.

Problem 6.

Part A. Prove that every regular language is context free. Do this by by converting a DFA \( M = (Q, \Sigma, \Delta, q_0, F) \) into a CFG \( G = (V, \Sigma, R, S) \) for the same language.

Part B. Prove that every regular language is generated by an unambiguous CFG.

Part C. Prove that every nonempty CFL is generated by an ambiguous CFG.

\[^1L \subseteq \{0, 1, \neq\}^*; \text{ the first “\( \neq \)” is the definition of } L \text{ is just a formal symbol.}\]