1. You are given a first DFA $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ with $|Q_1| = 10$ states, $|F_1| = 4$ of them final. You are given a second DFA $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ with $|Q_2| = 10$ states, $|F_2| = 4$ of them final. Suppose you use the product construction to make a DFA $M = (Q, \Sigma, \delta, s, F)$ for $L(M_1) \oplus L(M_2)$, the symmetric difference of $L(M_1)$ and $L(M_2)$. It will have $|Q| = \square$ states and $|F| = \square$ of them will be final. Answers are numbers.

2. Recall that, for an NFA $M = (Q, \Sigma, \delta, q_0, F)$, we defined $\delta^*(q, x)$ so as to be the set of all states in $Q$ reachable from $q_0$ by an $x$-labeled path. Given this, we could say that $M$ accepts $x$ if \text{[mathematically rigorous statement involving $\delta^*$ and the components of $M$.]}$

3. In stating the Myhill-Nerode theorem, we associated to any regular language $L \subseteq \Sigma^*$ a binary relation $\sim$ by saying that $x \sim x'$ if \text{[Make sure to include all needed quantifiers.]} As an example, language $L = (0110)^*$ induces a relation $\sim$ where $0 \not\sim 1$, because \text{[Relation demonstrating non-equivalence.]} The Myhill-Nerode theorem tells us that, if $L$ is a language and $\sim$ is the relation associated to it, then $L$ is regular iff \text{[Mathematically rigorous statement involving $\sim$.]}

4. There is a $2^{n+1}$-state DFA $M = (Q, \Sigma, \delta, q_0, F)$ for $L = \{0,1\}^n \{1\} \{0,1\}^n$: namely, let $Q = \{0,1\}^{n+1}$, $q_0 = 0^{n+1}$, $F = \{1\} \{0,1\}^n$, and $\delta(a_0a_1 \cdots a_n, a) = \square$.

5. Draw the smallest NFA you can for the language $L = \{0,1\}^* \{1\} \{0,1\}^3$.

6. State the pumping lemma for regular languages. Be careful and explicit with all quantifiers.
7. Darken the correct box. No justification is required. If you’re not sure, guess.

(a) **True**  **False** Every language is infinite or has an infinite complement.
(b) **True**  **False** The set of real numbers is a language.
(c) **True**  **False** The Kleene closure (the star) of a language is always infinite.
(d) **True**  **False** \( L = \{ w \in \{0,1\}^* : \text{w has an equal number of 01's and 10's} \} \) is regular.
(e) **True**  **False** If \( L \subseteq \Sigma^* \) is not regular and \( h : \Sigma \rightarrow \Sigma^* \) is a homomorphism, then \( h(L) \) is not regular.
(f) **True**  **False** If \( A \) and \( B \) are regular and \( h \) is a homomorphism then \( h(A \cup B) = h(A) \cup h(B) \).
(g) **True**  **False** If \( M = (Q, \Sigma, \delta, q_0, F) \) is an NFA and \( F = Q \), then \( L(M) = \Sigma^* \).
(h) **True**  **False** An NFA \( M = (Q, \Sigma, \delta, q_0, F) \) rejects a string \( x \) if there is a path from \( q_0 \) to a nonfinal state \( q \) of \( M \) and where the concatenation of the arc-labels along this path is \( x \).
(i) **True**  **False** If \( \alpha \) is a regular expression then there is a regular expression \( \beta \) the language of which \( L(\alpha) \).
(j) **True**  **False** For all \( n \geq 0 \), \( L_n = \{ a^i b^j : i \leq n \} \) is regular.
(k) **True**  **False** If \( L \) is regular, the even-length strings of \( L \) are regular.
(l) **True**  **False** If the even-length strings \( L \) are regular and the odd-length strings of \( L \) are regular then \( L \) is regular.

8. If \( L \subseteq \Sigma^* \) is a language, let \( \text{Prefix}(L) = \{ x \in \Sigma^* : xy \in L \text{ for some } y \in \Sigma^* \} \).

(a) Is \( \text{Prefix}(L) \subseteq L? \)

Is \( \text{Prefix}(L) \supseteq L? \)

(b) Given a DFA \( M = (Q, \Sigma, \delta, q_0, F) \), explain how to construct a DFA \( M' = (Q', \Sigma, \delta', q'_0, F') \) for \( \text{Prefix}(L) \). Explain the construction in a sentence or two of simple English.