Midterm Exam — Spring 2014

Instructions:

• The exam is closed-book, closed-notes, closed-devices, closed-neighbors. Anyone violating these rules will be fed to the saltwater crocodiles that live in the basement of Kemper.
• Write neatly. If the person grading can’t easily read what you wrote, it’s wrong.
• Where answers depend on conventions, use those of Sipser or the lectures.
• An “efficient” algorithm: one that runs in polynomial, not exponential, time.
• Good luck!

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**Problem Z**: For this question you should specify a range (ie, a [lowerbound, upperbound] pair) for which you have 98% confidence—not more and not less—that the correct answer lies within the range you specify.

The average distance between the earth to the moon is between $oxed{\text{lowerbound}}$ and $oxed{\text{upperbound}}$ times the circumference of the earth.
1 Fill in the blank

Fill in the boxes. All answers are numbers.

1. A DFA $M = (Q, \Sigma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 5$ characters. Then there are [_________] points in the domain of $\delta$ and [_________] points in the range of $\delta$.

An NFA $M = (Q, \Sigma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 5$ characters. Then there are [_________] points in the domain of $\delta$ and [_________] points in the range of $\delta$.

A PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ has $|Q| = 10$ states and $|\Sigma| = 2$ characters in the input alphabet and $|\Gamma| = 4$ characters in the tape alphabet. Then there are [_________] points in the domain of $\delta$ and [_________] points in the range of $\delta$.

2. You are given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ with $|Q| = 100$ states, operating on $|\Sigma| = 2$ characters, and employing $|F| = 10$ final states.

If we construct from $M$ an NFA for $(L(M))^R$ it will have [_________] states.

Alternatively, if we construct from $M$ an NFA for $\overline{L(M)}$ by combining procedures seen in class, it will have [_________] states.

3. A CNF (Chomsky-Normal Form) CFG $G = (V, \Sigma, R, S)$ has 7 variables, 6 terminals, 5 rules of the form $A \rightarrow BC$, and 8 rules of the form $A \rightarrow a$. Using the procedure shown in class and in your book, convert $G$ to a PDA $M$ that allows one to push an arbitrary number of symbols onto the stack in one transition. Then $M$ will have $|Q| = [_________]$ states. If, instead, we allow $M$ to push only one symbol onto the stack at a time (which was our original convention), then the construction will need to have $|Q| = [_________]$ states.

4. As per PS5, problem 5, there’s a natural conversion of a DFA $M = (Q, \Sigma, \delta, q_0, F)$ into a context-free grammar $G = (V, \Sigma, R, S)$ for $L(M)$. Suppose that $M$ has $|Q| = 10$ states, $|\Sigma| = 3$ characters, and $|F| = 5$ final states. Then the corresponding CFG $G$ will have $|V| = [_________]$ variables and $|R| = [_________]$ rules.
2. Short answer

1. Use the **pumping lemma** (and anything else that you might need) to prove that $L = \{x \in \{a,b\}^* : x \text{ is not a palindrome} \}$ is **not** regular.

2. In the Myhill-Nerode theorem an equivalence relation $\sim$ is associated to any language $L$. We did this by defining, for a given $L$, that $x \sim x'$ if $\text{(complete the sentence)}$

Let $L = \{x \in \{a,b\}^* : x \text{ is a palindrome} \}$. Then $x = aa$ and $x' = aaa$ **are** / **aren’t** (please circle the correct choice) $\sim$ related to one another because $\text{(complete the sentence)}$

3. Give a **CFG** for the language $L = \{xx^Ryy^R : x, y \in \{a,b\}^* \}$. Your CFG must be as **simple** as possible.
4. Draw a **picture** that helps explain the main idea behind the **pumping lemma** for **CFLs**. You picture should include labels $S$, $A$, and $u,v,x,y,z$. Then, with no more than a couple of sentences, **explain the proof idea** that your picture aims to convey.

5. Carefully explain what it means if I say: “the CFLs are closed under intersection. Don’t indicate if the statement is true or false—just provide a precise mathematical translation of the meaning of the claim.

Now **prove** or **disprove** (circle one) the claim: **the CFLs are closed under intersection**.

6. Let $L$ be the set of all decimal digits $d$ such that $d$ occurs **infinitely often** in the decimal representation of the number $\pi = 3.14159\cdots$.

Is $L$ **regular**? Circle either: **yes** or **no**.

Now **prove your answer**.
3 True or False

Darken (completely fill in) the correct answer. If you don’t know an answer, please guess.

1. True False  The concatenation of finite languages $A$ and $B$ is finite.
2. True False  If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F \neq \emptyset$ then $L(M) \neq \emptyset$.
3. True False  Every regular language can be accepted by an NFA with only one final state.
4. True False  If $L^*$ is infinite the $L$ is infinite.
5. True False  Regular expressions are strings.
6. True False  If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA then $x \in L(M)$ iff $\delta^*(q_0, x) \cap F \neq \emptyset$.
7. True False  The pumping lemma can be used to show that languages are regular.
8. True False  Let $L = \{a^n b^n : n \geq 1\}$. Then $L^*$ is regular.
9. True False  If $A \subseteq L \subseteq B$ and $A$ and $B$ are regular then $L$ is regular.
10. True False  If $L \cup L'$ is regular then $L$ and $L'$ are regular.
11. True False  If $L \ominus L'$ is finite then $L$ regular iff $L'$ is regular.
12. True False  The image $h(L)$ of a context-free language $L$ under a homomorphism $h$ is context free.
13. True False  $L_\pi = \{3, 31, 314, 3141, 31415, 314159, \ldots\}$ is regular.
14. True False  Complementing the final state set of an NFA $M$ gives an NFA for $\overline{L(M)}$.
15. True False  For a DFA $M = (Q, \Sigma, \delta, q_0, F)$, we defined $\delta^*$ as $\delta = \bigcup_{i \geq 0} \delta^i$.
16. True False  $L = \{w \in \{0, 1\}^* : w$ contains an equal number of 01’s and 10’s$\}$ is regular.
17. True False  The intersection of a CFL and a regular language is context free.
18. True False  An efficient algorithm is known to decide if two NFAs accept the same language.
19. True False  An efficient algorithm is known to decide if $w \in L(\alpha)$ for a regular expression $\alpha$.
20. True False  An efficient algorithm is known to decide if $w \in L(G)$ for a CNF CFG $G$. 