Problem Set 10 — Due Thursday, June 5, at 3:30 pm

Problem 1. State whether the following claims are true or false, briefly explaining your answer.

a. $A \leq_P A$.

b. If $A \leq_P B$ and $B \leq_P C$, then $A \leq_P C$.

c. If $A \leq_P B$ then $\overline{A} \leq_P \overline{B}$.

d. If $A$ is r.e. and $A \leq_P \overline{A}$ then $A$ is recursive.

e. If $A$ is recursive, then $A \leq_P a^*b^*$.

f. If $A$ is r.e., then $A \leq_P A_{TM}$.

Problem 2. Suppose you are given a polynomial time algorithm $D$ that, on input of a Boolean formula $\phi$, decides if $\phi$ is satisfiable. Describe an efficient procedure $S$ that finds a satisfying assignment for $\phi$. How many calls to $D$ do you make?

Problem 3. Let $MULT-SAT = \{\langle \phi \rangle \mid \phi$ has at least ten satisfying assignments$\}$. Show that $MULT-SAT$ is NP-complete.

Problem 4. A graph $G = (V, E)$ is said to be $k$-colorable if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When $k$ is a number, by $kCOLOR$ we denote the language of (encodings of) $k$-colorable graphs. The language $3COLOR$ is NP-Complete. (You can assume this.) Use this to prove that the language $4COLOR$ is NP-Complete, too.

Problem 5. Let

$$D = \{\langle p \rangle : p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root.}\}$$

Prove that $D$ is NP-hard. Is it NP-complete?