Problem Set 3 – Due Friday, April 18, 2014

Problem 1. Using the procedure shown in class, convert the following NFA into a DFA for the same language. Show all work.

Problem 2. Using the procedure shown in class, eliminate all ε-arrows from the following NFA.

Problem 3. Let \( L_1, L_2, L_3 \subseteq \Sigma^* \) be languages and let \( \text{maj}(L_1, L_2, L_3) \) be the set of all \( x \in \Sigma^* \) that are in at least two of \( L_1, L_2, L_3 \). Prove: if \( L_1, L_2, \) and \( L_3 \) are DFA-acceptable then so is \( \text{maj}(L_1, L_2, L_3) \).

Problem 4 Let \( Z(L) = \{ a_1a_2\cdots a_n : a_1a_2\cdots a_n \in L \} \). Prove that the DFA-acceptable languages are closed under \( Z \). Having proved it once: can you think of another, different proof?

Problem 5. How many states are in the smallest possible DFA for \( \{0,1\}^*\{1^0\} \)? Prove your result.

Problem 6 Let \( L_n \) (for \( n \geq 1 \)) be \( \{0,1\}^*\{1\}\{0,1\}^n \). Prove that there is an NFA for \( L_n \) having \( n + 2 \) states, but that there is no DFA for \( L_n \) having \( 2^n - 1 \) or fewer states. In a well written English sentence or two, give a high-level interpretation of your result.