Problem Set 5 – Due Friday, May 2, 2014

Problem 1. Are the following languages regular? Prove your answers. You don’t need to know anything about the number \( \pi \in \mathbb{R} \) except that it’s irrational (meaning: it’s not the ratio of two integers).

(a) \( L_a = \) the set of all decimal digits \( d \) such that \( d \) occurs infinitely often in the decimal representation of the number \( \pi = 3.14159 \cdots \).

(b) \( L_b = \{3, 31, 3141, 31415, 314159, \ldots\} \), the set of all nonempty prefixes of the decimal expansion of \( \pi \), ignoring the decimal point.

Problem 2. In an well-written essay of about a page (200-300 words), discuss the following question.
We have defined various computational models in our class. Some aspects of each definition were essential—the definition has to look basically that way to capture the desired notion—while other aspects were inessential—maybe as insignificant as arbitrary conventions just needed to make things concrete. Select a computational model covered in class (DFAs, NFAs, or regular expressions) and explore definitional alternatives that do and that do not change the essence of the object defined.

Problem 3. Specify a CFG for the language

\[ L = \{x \in \{a, b, c\}^* : x \text{ contains an equal number of two different characters}\} \]

Make your CFG as simple as possible. (If it isn’t obviously right to the TA, it isn’t right.)

Problem 4. Specify a CFG for \( L = \{x \neq y : x, y \in \{0, 1\}^+ \text{ and } x \neq y\} \).\(^1\) With diagrams or clear English, explain how your grammar works.

Problem 5. Prove that every regular language is context free. Do this by by converting a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) into a CFG \( G = (V, \Sigma, R, S) \) for the same language.

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\(^1\) \( L \subseteq \{0, 1, \neq\}^* \); the first “\( \neq \)” is the definition of \( L \) is just a formal symbol.