Problem Set 9 – Due Friday, May 30, 2014

Problem 1. As you did last week, classify each of the following languages as recursive, r.e. but not decidable, co-r.e. but not decidable, or neither r.e. nor co-r.e. Giving reductions where appropriate, prove your results. This problem will count as 30 points, triple a conventional problem.

A $A = \{ \langle M, k \rangle : M$ is a TM that accepts at least one string of length $k \}$.
B $B = \{ \langle M, k \rangle : M$ is a TM that runs forever on at least one string of length $k \}$.
C $C = \{ \langle M, k \rangle : M$ is a TM that accepts a string of length $k$ and diverges on a string of length $k \}$. Assume that the underlying alphabet has at least two characters.
D $D = \{ \langle M \rangle : M$ is a TM that accepts some palindrome \}$.
E $E = \{ \langle G_1, G_2 \rangle : G_1$ and $G_2$ are CFGs and $L(G_1) \oplus L(G_2) = \emptyset \}$. You may assume that $L = \{ \langle G \rangle : G$ is a CFG and $L(G) = \Sigma^* \}$ is undecidable.
F $F = \{ \langle M \rangle : M$ is a TM and $L(M)$ is recursive \}$.

Problem 2 Prove or disprove each of the following claims.

A. $A \leq_m A$.
B. If $A \leq_m B$ and $B \leq_m C$, then $A \leq_m C$.
C. If $A \leq_m B$ then $\overline{A} \leq_m \overline{B}$.
D. If $A$ is r.e. and $A \leq_m \overline{A}$ then $A$ is recursive.
E. If $A$ is recursive, then $A \leq_m a^*b^*$.
F. If $A \leq_m B$ then $B \leq_m A$.
G. If $A \leq_m B$ and $B \leq_m A$ then $A = B$. 