Problem Set 1 — Due Thursday, January 10, 2002

Instructions: Write up your solutions as clearly and succinctly as you can. Don’t forget to acknowledge anyone with whom you discussed problems. Recall that homework is due at 10:00 am in the box in Eng. II, room #0086.

Problem 1. The following question is to remind you about inductive definitions and their use.

A. Give an inductive definition for a decimal number. A decimal number is a string over the alphabet \{0, 1, 2, \ldots, 9\}. Examples are 4, 120, 007.

B. Give an inductive definition for the value of a decimal number. This is a map \(\nu\) from decimal numbers to nonnegative integers.

Problem 2. For each of the following, give an example language \(L\) to prove existence, or explain why no such language exists. Assume an underlying alphabet of \{0, 1\}.

A. An infinite language with an infinite complement.

B. A language closed under concatenation and containing no even-length strings.

C. An infinite unary language \(L\) such that if \(x \in L\) and \(y \in L\) then there is no string in \(L\) of length \(|x| + |y|\). (A unary language means that the underlying alphabet has just one character, say 1.)

D. A finite language having a longest string \(x\) that is longer than a longest string of any other finite language.