Problem Set 9 — Due Thursday, March 7, 2002

Problem 1 Prove that $L$ is decidable iff $L$ is listable in lexicographic order. (A language is listable in lexicographic order if some program outputs $x_1 \# x_2 \# x_3 \# \cdots$, $L = \{x_1, x_2, x_3, \ldots\}$, and $x_1 < x_2 < x_3 < \cdots$ where “$<$” denotes the usual lexicographic ordering on strings.)

Problem 2 (Counts as 20 points, same as 2 ordinary problems.)

Part A. Let $L = \{\langle M \rangle : M$ is a TM that accepts some string of prime length$\}$. Prove that $L$ is not decidable.

Part B. Let $L = \{\langle G \rangle : G$ is a CFG and $G$ accepts an odd-length string$\}$. Prove that $L$ is not decidable.

Part C. Let $L = \{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*$\}. Prove that $L$ is not r.e.

Part D. Let $L = \{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*$\}. Prove that $L$ is not r.e.

Part E. Let $L = \{\langle M \rangle : M$ is a TM and $L(M) = L(M)^*$\}. Prove that $L$ is not co-r.e.

Part F. Let $L = \{\langle G, G' \rangle : G_1$ and $G_2$ are CFGs and $L(G_1) = L(G_2)$\}. Prove that $L$ is not decidable. You may use the fact that $A = \{\langle G \rangle : G$ is a CFG and $L(G) = \Sigma^*\}$ is undecidable.