Quiz — Section 1

Instructions: Please answer the questions succinctly and thoughtfully. Good luck.
— Phil Rogaway

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1 Short Answer [30 points]

(1) Draw a DFA that accepts \( L = \{ x \in \{1, 2\}^* : x \text{ has exactly two } 2\text{'s} \} \).

(2) List the lexicographically-first six strings in the set \( \{0, 1\}^* \). (Lexicographic order of \( \{0, 1\}^* \) is \( \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, \ldots \} \).)

(3) Give a smallest DFA that accepts \( \{\varepsilon\} \). The alphabet is \( \Sigma = \{a\} \).
(4) Using the procedure we have seen in class (“follow the character and then chase down ε-arrows”), convert the following NFA into a DFA that accepts the same language.

(5) Define what is a language over an alphabet Σ.

(6) Recall that, for $L \subseteq \{0, 1\}^*$, $\text{PAL}(L) = \{x \in \{0, 1\}^* : xx^R \in L\}$. Write a regular expression for $\text{PAL}(\{0, 1\}^*)$. 
2 Justified True or False

Put an X through the correct box. Where it says “Explain” provide a brief (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, make your justification a counter-example. Throughout, we use \( L \) to denote a language (maybe regular, maybe not).

1. \( \emptyset^* = \emptyset \)

   Explain:

   - True
   - False

2. Every subset of a DFA-acceptable language is DFA-acceptable.

   Explain:

   - True
   - False

3. If \( M = (Q, \Sigma, \delta, q_0, F) \) is a DFA and \( F = Q \) then \( L(M) = \Sigma^* \).

   Explain:

   - True
   - False

4. If \( L \) is accepted by an \( n \)-state NFA then \( L \) is accepted by some \( 3^n \)-state DFA.

   Explain:

   - True
   - False
5. If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $M' = (Q, \Sigma, \delta, q_0, F')$, where $F' = Q - F$, then $L(M') = L(M)$.  
True  False

Explain:

6. If $L$ is a not-regular language and $F$ is a finite language then $L \cap F$ is a regular language.
True  False

Explain:

7. $(L^*)^* = L^*$.  
True  False

Explain:

8. For $\alpha$ a regular expression, there is an algorithm to decide if $x \in L(\alpha)$ that is efficient enough to run in a reasonable amount of time on reasonable length $x$, $\alpha$.  
True  False

Explain:
3 A Closure Property of Regular Languages [20 points]

If $L$ is a language over an alphabet $\Sigma$ let

$$\text{NoPrefix}(L) = \{ x \in L : \text{no proper prefix of } x \text{ is in } L \}.$$  

Prove that if $L$ is regular then so is $\text{NoPrefix}(L)$. (Describe any construction you use both in clear English and by a formal definition.)