Quiz — Section 2

Instructions: Please answer the questions succinctly and thoughtfully. Good luck.
— Phil Rogaway

Name:

Signature:

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1 Short Answer [30 points]

(1) Draw a DFA that accepts $L = \{ x \in \{1, 2\}^* : x$ has at least two 2’s$\}$. 

(2) List the lexicographically-first six strings in the complement of \{b, aa, ab, aaa\}. (Lexicographic order of $\{a, b\}^*$ is \{ε, a, b, aa, ab, ba, bb, aaa, . . . \}.)

(3) Give a smallest NFA that accepts \{ε\}. The alphabet is $\Sigma = \{a\}$. 
(4) Using the procedure we have seen in class, convert the following NFA into a regular expression for the same language.

(5) Define what is an alphabet.

(6) Recall that, for $L \subseteq \{0, 1\}^*$, $\text{PAL}(L) = \{x \in \{0, 1\}^* : xx^R \in L \}$. Write a regular expression for $\text{PAL}(0^*1^*)$. 
2 Justified True or False [40 points]

Put an X through the correct box. Where it says “Explain” provide a brief (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, make your justification a counter-example. Throughout, we use $L$ to denote a language (maybe regular, maybe not).

1. Every DFA-acceptable language can be accepted by a DFA with more accepting states than non-accepting states.  
   Explain:
   \[
   \begin{array}{ll}
   \text{True} & \quad \text{False} \\
   \end{array}
   \]

2. $L^*$ is infinite.  
   Explain:
   \[
   \begin{array}{ll}
   \text{True} & \quad \text{False} \\
   \end{array}
   \]

3. If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $F = Q$ then $L(M) = \Sigma^*$  
   Explain:
   \[
   \begin{array}{ll}
   \text{True} & \quad \text{False} \\
   \end{array}
   \]

4. If $L$ is finite and $\sum_{x \in L} |x| = N$ then $L$ can be accepted by an NFA having $N + 1$ states.  
   Explain:
   \[
   \begin{array}{ll}
   \text{True} & \quad \text{False} \\
   \end{array}
   \]
5. Every co-finite language can be accepted by a DFA.  
   Explain:  
   \[ \text{True} \quad \text{False} \]

6. There exists a language \( L \) such that \( L \) is nonempty, \( L \) is closed under concatenation, and \( L \) contains no string of even length.  
   Explain:  
   \[ \text{True} \quad \text{False} \]

7. \( L^+ \) (that is, \( LL^* \)) does not contain the emptystring.  
   Explain:  
   \[ \text{True} \quad \text{False} \]

8. Let \( M = (Q, \{0, 1\}, \delta, q_0, F) \) be a DFA and let \( L = L(M) \). Suppose \( x01001 \in L \) and \( y01001 \notin L \). Then it is possible that \( \delta^*(q_0, x) = \delta^*(q_0, y) \).  
   Explain:  
   \[ \text{True} \quad \text{False} \]
3 A Closure Property of Regular Languages [20 points]

If \( L \) is a language over an alphabet \( \Sigma \) let

\[
\text{Late}(L) = \{ x \in \Sigma^* : \text{for some } a \in \Sigma, \text{ string } ax \in L \}.
\]

Prove that if \( L \) is regular then so is Late(\( L \)). *(Describe any construction you use both in clear English and by a formal definition.)*