ECS 120 Final – Fall 1995

Instructions: *Check that your exam has all 7 problems (pages 2–9). You'll also find 2 blank pages at the end of the exam. You can use these as scratch paper.*

Answer all the questions. Don't use notes, books, or neighbors. If you don't understand something, ask. Please make your writing logical, succinct, and legible.

Your final exam score, and your grade in the course, will be posted to the newsgroup as soon as they're ready. Happy holidays! —*Phil Rogaway*

Name:

Signature:

On problem	you got	out of
1		30
2		20
3		20
4		20
5		20
6		20
7		20
Σ		150

1 Recall . . . [30 points]

A. *Complete the following definition:*

Let $A, B \subseteq \{0,1\}^*$. We say that A polynomial-time mapping reduces to B, written $A \leq_{\mathrm{p}} B$, if . . .

B. *Complete the following definition:*

A language L is NP-Complete if . . .

C. *State the Cook/Levin Theorem:*

Theorem [Cook/Levin]:

D. *Complete the following statement of the pumping lemma for context free languages:*

> *Theorem.* If a language L is context free then there exists a number K such that ...

E. *In a sentence or two, state the "Church-Turing Thesis:"*

F. *State Rice's Theorem (which says that a certain class of languages is undecidable).*

> **Theorem [Rice]:**

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2 True/False [20 points]

Mark the correct box by putting an "X" through it. No justification required.

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1. For every i, the language $L_i = \{a^i b^i c^i\}$ is context free. **True** **False**

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2. Assume L is a regular language and let L_{1101} be the subset of L which contains the strings that end in a 1101. Then L_{1101} is regular. **True** **False**

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3. If L^* is decidable then L is decidable. **True** **False**

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4. Every enumerable language can be accepted by a TM whose head only moves to the right. **True** **False**

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5. For any language L, the language L^* is infinite. **True** **False**

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6. Let $\langle M \rangle$ be the encoding of a Turing machine M. Let $P(\langle M \rangle) = 0$ if M on ε halts in an even number of steps, 1 otherwise. Then P satisfies the condition of Rice's theorem: it is a nontrivial property of enumerable languages. **True** **False**

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7. The language $L = \{\langle M \rangle : L(M) \in \mathrm{NP}\} \in \mathrm{NP}$. **True** **False**

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8. Suppose $\mathit{3SAT} \leq_{\mathrm{p}} L$ and $L \in \mathrm{P}$. Then $\mathrm{P} = \mathrm{NP}$. **True** **False**

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9. A_{TM} is NP-complete. **True** **False**

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10. If $L_1 \leq_{\mathrm{p}} L_2$ and $L_2 \leq_{\mathrm{p}} L_1$, then $L_1 = L_2$. **True** **False**

3 Language Classification. [20 points]

Classify as:

- **decidable** — decidable
- **enumerable** — enumerable but not decidable
- **co-enumerable** — co-enumerable but not decidable
- **neither** — neither enumerable nor co-enumerable

No explanation is required.

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1. $\{\langle M\rangle : M$ is a TM which accepts some palindrome$\}$[1]

2. $\{\langle M\rangle : M$ is a TM which accepts some string of length $\geq |\langle M\rangle|\}$

3. $\{d :$ the digit d appears infinitely often in the decimal expansion of $\pi = 3.14159\cdots\}$

4. $\{\langle G\rangle : G$ is a regular grammar and $L(G)$ contains an even-length string$\}$

5. $\{\langle G\rangle : G$ is a CFG and $L(G) = \Sigma^*\}$

6. $\{\langle G, k\rangle : G$ is a graph containing no clique of size $k\}$.

7. A language L for which $L_{\Sigma^*} = \{\langle M\rangle : L(M) = \Sigma^*\} \leq_{\mathrm{m}} L$.

8. $\{\langle t\rangle : t$ is an instance of the tiling problem for which you can tile the plane using 10 or fewer tile types.$\}$.

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[1] A palindrome is a string w for which $w = w^R$.

4 A Tight Bound on DFA Size [20 points]

Let $L_5 = \{111\}$ be the language over $\{0, 1\}$ which contains only the string 111.

(A) Show that L_5 can be recognized by an 5-state DFA.

(B) Prove that L_5 can not be recognized by a 4-state DFA.

5 A Decision Procedure [20 points]

If α is a regular expression, we write α^2 for the regular expression $\alpha\alpha$. Show that the following language is decidable (ie., exhibit a decision procedure for this language):

$$L = \{\langle a, b, c\rangle : a,\ b \text{ and } c \text{ are regular expressions and } a^2 \cup b^2 = c^2.\}$$

6 Mapping Reductions [20 points]

Recall that, if $w = a_1 \cdots a_n \in \Sigma^n$ is a string, $w^R = a_n \cdots a_1$ is the "reversal" of w. If $L \subseteq \Sigma^*$ is a language, $L^R = \{w^R : w \in L\}$. Let

$$\begin{aligned} A_{\mathrm{TM}} &= \{\langle M, w\rangle : M \text{ accepts } w\} \\ A_R &= \{\langle M\rangle : L(M) = (L(M))^R\} \end{aligned}$$

A. Show that $A_{\mathrm{TM}} \leq_{\mathrm{m}} A_R$.

B. Show that $\overline{A_{\mathrm{TM}}} \leq_{\mathrm{m}} A_R$.

7 NP-Completeness [20 points]

Let $G = (V, E)$ be a graph. We say that $G' = (V', E')$ is a *vertex-induced subgraph* of G if $V' \subseteq V$ and E' is all the edges of G both endpoints of which are in V'. Now suppose each edge $e \in E$ as an integer *weight*, $w(e)$. Then the weight of the subgraph G' is just $\sum_{e' \in E'} w(e')$.

Show that the following language of graphs with heavy vertex-induced subgraphs is NP-Complete.

Hint: Use *CLIQUE*, and don't forget to argue the correctness of your reduction.

$$\begin{aligned} \mathit{HVIS} \;=\; \{\langle G, w, B\rangle :\; & G = (V, E) \text{ is a graph, } w : E \to \mathsf{Z} \text{ specifies an integer weight,} \\ & w(e)\text{, for each } e \in E\text{, and } B \in \mathsf{Z} \text{ is an integer; and } G \text{ has some} \\ & \text{vertex-induced subgraph of weight at least } B.\} \end{aligned}$$

Example: