Problem Set 8 — Due March 8, 2005

Problem 8.1. Prove that a language is decidable iff it can be enumerated in lexicographic order. (A language $L$ can be enumerated in lexicographic order if there is a TM that, on input of the empty string, produces an output of $\#x_1\#x_2\#x_3\#\cdots$ where $L = \{x_1, x_2, x_3, \ldots\}$ and $x_1 < x_2 < x_3 < \cdots$ under the lexicographic ordering of strings.)

Problem 8.2. (Counts as two problems)

A. Show that

$$L_A = \{\langle M, k \rangle : M \text{ is a TM which accepts at least one string of length } k\}$$

is not decidable.

B. Prove that

$$L_B = \{\langle M, k \rangle : M \text{ is a TM that loops on at least one string of length } k\}$$

is not decidable.

C. Let

$$L_C = \{\langle M, k \rangle : M \text{ is a TM which accepts some string of length } k, \text{ but } M \text{ loops on some (other) string of length } k\}.$$ 

Show that $L_C$ is not r.e.. (Assume the underlying alphabet has at least two characters.)

D. Show that $L_C$ is not co-r.e., either.

Problem 8.3. (Counts as two problems) Classify the following languages as decidable, r.e. (but not decidable), co-r.e. (but not decidable), or neither r.e. nor co-r.e.. Prove all your answers, giving decision procedures or reductions.

A. $L = \{\langle M \rangle : M \text{ accepts some even-length string}\}$.

B. $L = \{\langle M, w \rangle : M \text{ is a TM which uses at most 17 tape squares when run on } w\}$

C. $L = \{\langle M \rangle : M \text{ accepts some palindrome}\}$.

D. $L = \{\langle M \rangle : M \text{ never prints a “0” (regardless of the input)}\}$.

E. $L = \{\langle \alpha \rangle : \alpha \text{ is shortest regular expression for } L(\alpha)\}$.