Problem Set 10 — Due Tuesday, March 14, 2006

Problem 1. The following theorem was presented in class: A language $L$ is decidable iff there exists an enumerator $E$ that lists it in lexicographic order. Prove it.

Problem 2. Finish the proof of Rice’s theorem in your handout by arguing the case when the emptyset does have property $P$.

Problem 3. Suppose you are given a polynomial time algorithm $D$ that, on input of a Boolean formula $\phi$, decides if $\phi$ is satisfiable. Describe an efficient procedure $S$ that finds a satisfying assignment for $\phi$. How many calls to $D$ do you make?

Problem 4. Let $\text{MULT-SAT} = \{\langle \phi \rangle \mid \phi \text{ has at least ten satisfying assignments}\}$. Show that $\text{MULT-SAT}$ is NP-complete.

Problem 5. A graph $G = (V, E)$ is said to be $k$-colorable if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When $k$ is a number, by $k\text{COLOR}$ we denote the language of (encodings of) $k$-colorable graphs. The language $3\text{COLOR}$ is NP-Complete. (You can assume this.) Use this to prove that the language $4\text{COLOR}$ is NP-Complete, too.

Problem 6. Let

$$D = \{\langle p \rangle : p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root}\}$$

Prove that $3\text{SAT} \leq_p D$. 