Problem Set 7 — Due Tuesday, February 21, 2006

Problem 1. Show that the collection of decidable languages is closed under the operations of

- Union.
- Concatenation.
- Star.
- Complementation.
- Intersection.

Problem 2. Show that the collection of Turing-recognizable languages is closed under the operations of

- Union.
- Concatenation.
- Star.
- Intersection.

Problem 3. Show that the problem of testing whether a CFG generates some string in $1^*$ is decidable. In other words, show that $\{\langle G \rangle \mid G$ is a CFG over $\{0, 1\}^*$ and $1^* \cap L(G) \neq \emptyset\}$ is decidable.

Problem 4. Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

a. Can a TM ever write the blank symbol on its tape?

b. Can the tape alphabet $\Gamma$ be the same as the input alphabet $\Sigma$?

c. Can a TMs head ever be in the same location in two successive steps?

d. Can a TM contain just a single state?

Problem 5. Give an implementation-level description of a Turing machine that decides $L = \{w \mid w$ contains an equal number of 0s and 1s$\}$ over the alphabet $\{0, 1\}$.

Problem 6. Say that a write-once Turing machine is a single-tape Turing machine that can alter each tape square at most once (including the input portion of the tape). Altering a tape square means overwriting the symbol it currently contains with some other symbol; merely moving over a tape square, reading and rewriting the symbol it contains, as when $\delta(q_i, 0) = (q_j, 0, R)$, is allowed.

Show that this variant Turing machine model is equivalent to the ordinary Turing machine model. (Hint: As a first step consider the case whereby the Turing machine may alter each tape square at most twice. Use lots of tape.)