Problem 1. Let $A$ and $B$ be two disjoint languages. The recursive language $A$ *recursively separates* $A$ and $B$ if $A \subseteq A$ and $B \cap A = \emptyset$.

A. Show that any two disjoint co-r.e. languages are separable by some decidable language.
B. Show that $A = \{\langle M \rangle : M(\langle M \rangle) = \text{Yes}\}$ and $B = \{\langle M \rangle : M(\langle M \rangle) = \text{No}\}$ are not recursively separable.

Problem 2.
A. Show that $L_A = \{\langle M, k \rangle : M$ is a TM which accepts at least one string of length $k\}$ is not decidable.
B. Prove that $L_B = \{\langle M, k \rangle : M$ is a TM that loops on at least one string of length $k\}$ is not decidable.
C. Let $L_C = \{\langle M, k \rangle : M$ is a TM which accepts some string of length $k$, but $M$ loops on some (other) string of length $k\}$.
   Show that $L_C$ is not acceptable. (Assume the underlying alphabet has at least two characters.)
D. Show that $L_C$ is not co-acceptable, either.

Problem 3. Counts as two problems. Classify the following languages as decidable, acceptable (but not decidable), co-acceptable (but not decidable), or neither acceptable nor co-acceptable. Prove all your answers, giving decision procedures or reductions.

A. $L = \{\langle M \rangle : M$ accepts some even-length string\}.
B. $L = \{\langle M, w \rangle : M$ is a TM which uses at most 17 tape squares when run on $w\}$
C. $L = \{\langle M \rangle : M$ accepts some palindrome\}.
D. $L = \{\langle M \rangle : M$ never prints a “0” (regardless of the input)\}.
E. $L = \{\langle \phi \rangle : \phi$ is a Boolean formula which has no satisfying assignment\}.
F. $L = \{\langle G_1, G_2 \rangle : G_1$ and $G_2$ are CFGs which generate the same CFL.\} *You may use the fact that $\{\langle G \rangle : G$ is a CFG and $L(G) = \Sigma^*\}$ is undecidable.*
G. $L = \{\langle \alpha \rangle : \alpha$ is shortest regular expression for $L(\alpha)\}.$