Problem Set 3 – Due Tuesday, January 31, 2012

Problem 1. Consider trying to show that the NFA-acceptable languages are closed under \( * \) (Kleene closure) by way of the following construction: add \( \varepsilon \)-arrows from every final state to the start state; then finalize the start state, too. Show, by finding a small counterexample, that the proposed construction does not work.

Problem 2. Let \( M = (Q, \Sigma, \delta, q_0, F) \) be an NFA. We say that \( M \) accepts a string \( x \) in the all-paths sense if every computation of \( M \) on \( x \) ends in a state in \( F \). Let \( L'(M) \) denote the set of all \( x \in \Sigma^* \) such that \( M \) accepts \( x \) in the all-paths sense. Show that \( L \) is regular iff \( L = L'(M) \) for some NFA \( M \).

Problem 3. Prove that the following languages are not regular.
Part A. \( L = \{www : w \in \{a,b\}^*\} \).
Part B. \( L = \{a^{2^n} : n \geq 0\} \).

Problem 4. Decide if the following languages are regular or not, proving your answers either way.
Part A. \( L = \{w \in \{0,1\}^* : w \text{ is not a palindrome}\}. \)
Part B. \( L = \{w \in \{0,1\}^* : w \text{ has an equal number of 01's and 10's}\}. \)
Part C. \( L = \{w \in \{0,1,2\}^* : w \text{ has an equal number of 01's and 10's}\}. \)

Problem 5. Describe a decision procedure to solve the following problem: given a regular expression \( \alpha \), is there a shorter regular expression for the same language?

Problem X. The following question is for, at most, the top 2–3 students in the class; other students should spend their time elsewhere. If you solve it, please email a solution directly to Prof. Rogaway. Show that if \( L \subseteq 1^* \), then \( L^* \) is regular.