Midterm Exam

Instructions: Please ask me if you do not understand something. I want you to understand.

You may not use the help of any notes or books or friends. Please!

Good luck, my students.
— Phil Rogaway

Your Name in English:

Your Name in Thai:

Your Nickname in Thai:

Your Student Number:

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1 Short Answer

1. Draw a 3-state DFA for the language of all binary strings which encode numbers divisible by 3; that is, \( L = 0^*\{\varepsilon, 11, 110, 1001, \cdots\} \).

2. Draw a 4-state DFA for the language:
\[ L = \{x \in \{a, b\}^*: \text{the number of times } ab \text{ appears in } x \text{ is even}\} \].
3. Write a regular expression for the language:

\[ L = \{ x \in \{a, b\}^* : |x| \text{ is even and its first character is } a \}. \]

4. Write the following language as simply as you can:

\[(a \cup abb^*a \cup bba \cup bb^*)^* = \]

5. List the first five strings, in lexicographic order, in the complement of \((a \cup ab)^*\).

(For taking the complement, the alphabet is \(\Sigma = \{a, b\}\). Remember that the lexicographic order of \(\{a, b\}^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, \ldots \}\)).
6. Using the construction seen in class, convert the following NFA into a DFA for the same language:

7. Using the construction seen in class, convert the following NFA into a regular expression for the same language:
8. You are given DFAs \( M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1) \) and \( M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2) \). Suppose you want to construct a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) for the language \( L(M_1) - L(M_2) \).

(By \( L(M_1) - L(M_2) \) I mean set difference — the set of all strings in \( L(M_1) \) which are not in \( L(M_2) \).)

Suppose you use the product construction, so \( Q = Q_1 \times Q_2 \). Then the final state set, \( F \), for the machine \( M \) will be:

\[
F = \{(p, q) \in Q_1 \times Q_2 : \text{[condition]}\}.
\]

9. Given a DFA \( M = (Q, \Sigma, \delta, q_0, F) \), describe a DFA or NFA, \( M' = (Q', \Sigma, \delta', q'_0, F') \), such that \( L(M') = L(M) \cup \{\epsilon\} \). (Please say if you are trying to give a DFA or an NFA.)
2 Justified True or False [20 points]

Put an X through the correct box. When it says “Explain” provide a brief (but convincing) justification. Where appropriate, make this justification a counterexample.

1. For every $n$, the language $L_n = \{0^i1^i : i \leq n\}$ is regular. 
   \[ \text{True} \] \[ \text{False} \]

   Explain:

2. If $L^*$ is regular, then $L$ is regular, too. 
   \[ \text{True} \] \[ \text{False} \]

   Explain:

3. $L = \{a^ib^{i+j}c^j : i, j \geq 0\}$ is regular. 
   \[ \text{True} \] \[ \text{False} \]

   Explain:

4. Every regular language can be accepted by some NFA which has exactly 15 final states. 
   \[ \text{True} \] \[ \text{False} \]

   Explain:
3 A Lower Bound on DFA Size [15 points]

Let \( L = \{a, aa, aaa\} \). Prove that \( L \) can not be recognized by a 4-state DFA. The alphabet is \( \Sigma = \{a\} \).
4 A Closure Property [15 points]

Let \( L \) be a language over \( \Sigma \). Define

\[
\text{Enlarge}(L) = \{ x \in \Sigma^* : \text{for some } y \in \Sigma^*, \ xy \in L \}
\]

Prove that if \( L \) is regular, then so is \( \text{Enlarge}(L) \).

To do this, let \( M = (Q, \Sigma, \delta, q_0, F) \) be a DFA for \( M \). Construct a DFA \( M' = (Q', \Sigma, \delta', q'_0, F') \) for \( \text{Enlarge}(L) \). Show the machine \( M' \) that you get if \( M \) is the following machine: