Problem Set 6 — Due 21 September 2000

Problem 1.

Part A. Show that

\[ L_A = \{ \langle M, k \rangle : M \text{ is a TM which accepts at least one string of length } k \} \]

is not decidable.

Part B. Prove that

\[ L_B = \{ \langle M, k \rangle : M \text{ is a TM that loops on at least one string of length } k \} \]

is not decidable.

Part C. Let

\[ L_C = \{ \langle M, k \rangle : M \text{ is a TM which accepts some string of length } k, \]
\[ \quad \text{but } M \text{ loops on some (other) string of length } k \}. \]

Show that \( L_C \) is not r.e. (Assume the underlying alphabet has at least two characters.)

Part D. Show that \( L_C \) is not co-r.e., either.

Problem 2. Classify the following languages as decidable, r.e. (but not co-r.e.), co-r.e. (but not r.e.), or neither r.e. nor co-r.e.. Prove all your answers, giving decision procedures, acceptance procedures, or reductions.

A. \( L = \{ \langle M \rangle : M \text{ accepts some even-length string} \} \).

B. \( L = \{ \langle M \rangle : M \text{ accepts some palindrome} \} \).

C. \( L = \{ \langle M \rangle : M \text{ never prints a "0" (regardless of the input)} \} \).

D. \( L = \{ \langle \alpha \rangle : \alpha \text{ is shortest regular expression for } L(\alpha) \} \).