PRECIMONIOUS: Tuning Assistant for Floating-Point Precision

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Oracle

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Motivation

- Floating-point arithmetic used in wide variety of domains
- Reasoning about these programs is difficult
  - Large variety of numerical problems
  - Most programmers not experts in FP
- Common practice: use highest available precision
  - Disadvantages: more expensive!
- Automated technique for tuning precision
  - Search over variable types → type configurations
  - Program produces “accurate enough” answer and runs faster
Consider the problem of finding the arc length of the function

\[ g(x) = x + \sum_{0 \leq k \leq 5} 2^{-k} \sin(2^k x) \]

Summing for \( x_k \in (0, \pi) \) into n subintervals

\[
\sum_{k=0}^{n-1} \sqrt{h^2 + (g(x_{k+1}) - g(x_k))^2} \quad \text{with} \quad h = \pi/n \quad \text{and} \quad x_k = kh
\]

<table>
<thead>
<tr>
<th>Precision</th>
<th>Slowdown</th>
<th>Result</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>double-double</td>
<td>20X</td>
<td>5.795776322412856</td>
<td>✔</td>
</tr>
<tr>
<td>double</td>
<td>1X</td>
<td>5.795776322413031</td>
<td>✖</td>
</tr>
<tr>
<td>Summation variable is double-double</td>
<td>&lt; 2X</td>
<td>5.795776322412856</td>
<td>✔</td>
</tr>
</tbody>
</table>
long double fun(long double x) {
    int k, n = 5;
    long double t1 = x;
    long double d1 = 1.0L;
    
    for(k = 1; k <= n; k++) {
        ...
    }
    return t1;
}

int main() {
    int i, n = 1000000;
    long double h, t1, t2, dppi;
    long double s1;
    ...
    for(i = 1; i <= n; i++) {
        t2 = fun(i * h);
        s1 = s1 + sqrt(h*h + (t2 - t1)*(t2 - t1));
        t1 = t2;
    }
    // final answer stored in variable s1
    return 0;
}
Example (D.H. Bailey)

```c
long double fun(long double x) {
    int k, n = 5;
    long double t1 = x;
    long double d1 = 1.0L;
    for(k = 1; k <= n; k++) {
        ...
    }
    return t1;
}

int main() {
    int i, n = 1000000;
    long double h, t1, t2, dpPi;
    long double s1;
    ...
    for(i = 1; i <= n; i++) {
        t2 = fun(i * h);
        s1 = s1 + sqrtf(h*h + (t2 - t1)*(t2 - t1));
        t1 = t2;
    }
    // final answer stored in variable s1
    return 0;
}
```

Original Program

```c
double fun(double x) {
    int k, n = 5;
    double t1 = x;
    float d1 = 1.0f;
    for(k = 1; k <= n; k++) {
        ...
    }
    return t1;
}

int main() {
    int i, n = 1000000;
    double h, t1, t2, dpPi;
    long double s1;
    ...
    for(i = 1; i <= n; i++) {
        t2 = fun(i * h);
        s1 = s1 + sqrtf(h*h + (t2 - t1)*(t2 - t1));
        t1 = t2;
    }
    // final answer stored in variable s1
    return 0;
}
```

Tuned Program
Searching efficiently over variable types and function implementations
- Naïve approach → exponential time
  - 19,683 configurations for arc length program \(3^9\)
  - 11 hours 5 minutes
- Global minimum vs. a local minimum

Evaluating type configurations
- Less precision → not necessarily faster
- Based on run time, energy consumption, etc.

Determining accuracy constraints
- How accurate must the final result be?
- What error threshold to use?
Precimonious
“Parsimonious or frugal with precision”

Dynamic Program Analysis for Floating-Point Precision Tuning

Annotated Program

Test Inputs

Annotated with Error Threshold

Less Precision

Speedup

Type Configuration

Modified Program
Analysis Components

Original Program

Create Search Type Configuration

Search Configuration

Precimonious

LCCSEARCH Algorithm

A Type Configuration

Program Transformation

Tuned Program

Original Program

Evaluate Type Configuration

Proposed type Configuration
Searching: Delta Debugging

- Delta Debugging Search Algorithm [Zeller et. al]
  - An approach to debugging
  - Isolates failures systematically
    - Failing test ➔ Isolate the change(s) that introduced failure

- Main idea:
  - We can do better than making a change at the time
  - Start by dividing the change set in two equally sized subsets
  - Narrow the search to the subset that still causes the failure
  - Otherwise, increase the number of subsets

- Efficient search algorithm
  - Average time complexity: $O(n \log n)$
  - Worst case: $O(n^2)$
LCCSEARCH Algorithm

- Based on the Delta-Debugging Search Algorithm [Zeller et. Al]
- Our definition of a change
  - Lowering the precision of a floating-point variable in the program
    - Example: double x → float x
- Our success criteria
  - Resulting program produces an “accurate enough” answer
  - Resulting program is faster than the original program
- Main idea:
  - Start by associating each variable with set of types
    - Example: x → {long double, double, float}
  - Refine set until it contains only one type
- Find a local minimum
  - Lowering the precision of one more variable violates success criteria
Searching for Type Configuration

double precision

single precision
Searching for Type Configuration

- **Double Precision**
  - [ ] ✔
  - [ ] ✗
  - [ ] ✗

- **Single Precision**
  - [ ] ✗
  - [ ] ✗
  - [ ] ✗
Searching for Type Configuration

double precision

single precision

✔
✘
✔
✘
✔
Searching for Type Configuration

double precision

single precision
Searching for Type Configuration

double precision

single precision
Searching for Type Configuration

double precision

single precision
Searching for Type Configuration

double precision

Proposed configuration

Failed configurations

single precision
Program Transformation

- Automatically generate program variants
  - Reflect type configurations produced by LCCSEARCH algorithm

- Intermediate representation
  - LLVM IR

- Define transformation rules for each LLVM instruction
  - alloca, load, store, fpext, fptrunc, fadd, fsub, etc.
  - Changes equivalent to modifying the program at the source level

- Able to run resulting modified program
### Source Code

```c
1 long double fun(long double x) {
2   int k, n = 5;
3   long double t1;
4   long double d1 = 1.0L;
5   t1 = x;
6   for(k = 1; k <= n; k++) {
7     d1 = 2.0 * d1;
8     t1 = t1 + sin(d1 * x) / d1;
9   }
10  return t1;
11}
```

### LLVM IR

```llvm
define x86_fp80 @fun(x86_fp80) {
  ;
  3:
  long
double
t1;
  4:
  long
double
d1 = 1.0L;
  5:
  long
double
t1 = x;
  6:
  int
k, n = 5;
  7:
  double
d1 = 2.0 * d1;
  8:
  long
double
t1 = t1 + sin(d1 * x) / d1;
  9:
  double
return t1;
}
```
**Experimental Setup**

- **Benchmarks**
  - 8 GSL programs
  - 2 NAS Parallel Benchmarks: *ep* and *cg*
  - 2 other numerical programs

- **Test inputs**
  - Inputs Class A for *ep* and *cg* programs
  - 1000 random floating-point inputs for the rest

- **Error thresholds**
  - Multiple error thresholds: $10^{-10}$, $10^{-8}$, $10^{-6}$, and $10^{-4}$
  - User can evaluate trade-off between accuracy and speedup
## Experimental Results

### Original Type Configuration

<table>
<thead>
<tr>
<th>Program</th>
<th>L</th>
<th>D</th>
<th>F</th>
<th>C</th>
</tr>
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<tbody>
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<td>0</td>
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<td>0</td>
<td>4</td>
</tr>
<tr>
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<td>0</td>
<td>3</td>
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<tr>
<td>simpsons</td>
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<td>0</td>
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</tr>
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</table>

### Proposed Type Configuration

<table>
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<tr>
<th></th>
<th>L</th>
<th>D</th>
<th>F</th>
<th>S</th>
<th># Config</th>
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<tr>
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<td>3</td>
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<tr>
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<td>336</td>
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<tr>
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<td>23:53</td>
</tr>
<tr>
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<td>3</td>
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<tr>
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<td>0</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>0:07</td>
</tr>
</tbody>
</table>
Maximum speedup observed across all error thresholds: 41.7%
Calculating infinite sum \[
\sum_{k>0} \left( \frac{3465}{(k + 1/2)^2 - 1/16} + \frac{3465}{k^2 - 1/16} \right) = 9240
\]

```c
int main() {
    double sum, oldsum;
    long int k;
    sum = 0.0;
    oldsum = -1.0;
    k = 0;
    while(sum > oldsum) {
        oldsum = sum;
        k = k + 1;
        sum = sum + term(k);
    }
    // infinite sum
    sum = sum + tail(k);
    return 0;
}
```

- If single precision is sufficient, number of terms reduces from 87,290,410 to only 3,768
- Unlike other programs, the arithmetic’s precision determines number of iterations
- Speedup as high as 5000x
Limitations and Future Work

- Type configurations rely on program inputs tested
  - No guarantees if worse conditioned input
  - Additional experiments to assess inputs used in evaluation

- Getting trapped in local minimum
  - Improve search by exploiting relationships among variables

- Providing support for other data types such as double-double as implemented in the QD library
  - Emulating higher precision in software is significantly more expensive
Devised a dynamic analysis for tuning the precision of floating-point programs

Implemented in an efficient and publicly available tool named PRECIMONIOUS

https://github.com/corvette-berkeley/precimionious

Initial evaluation on 12 programs shows encouraging speedups of up to 41%

PRECIMONIOUS is under active development
  ▪ Feature suggestions and programs to analyze are welcome!