Local features and image matching

May 7th, 2015

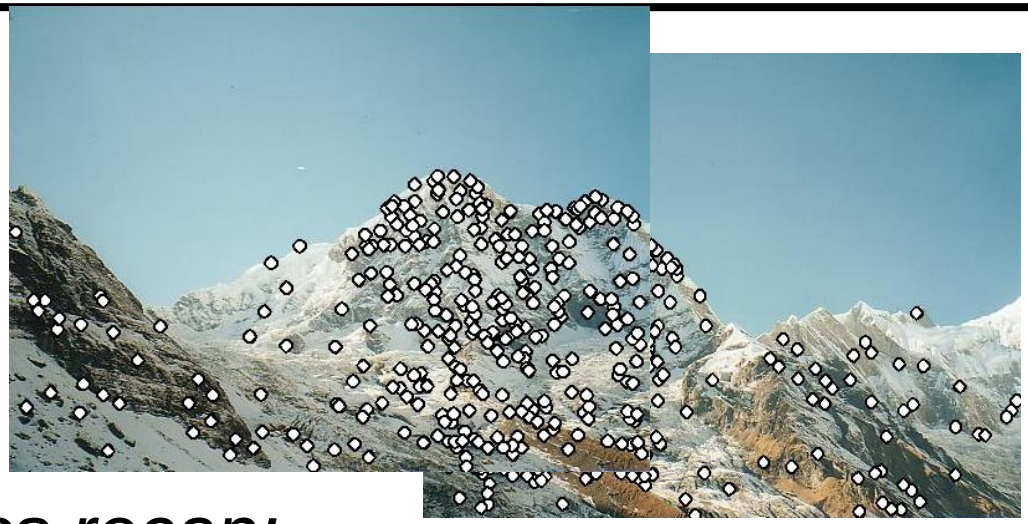
Yong Jae Lee

UC Davis

Last time

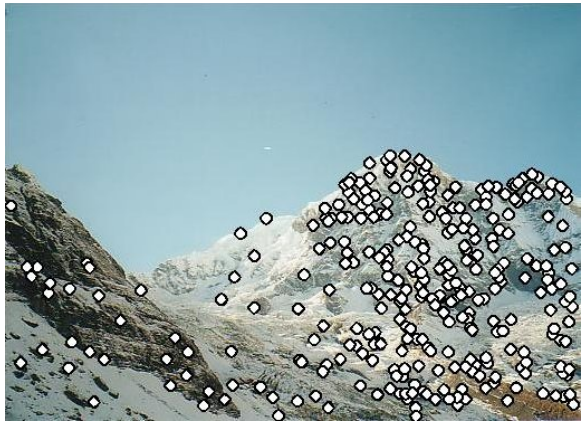
- RANSAC for robust fitting
  - Lines, translation
- Image mosaics
  - Fitting a 2D transformation
    - Homography

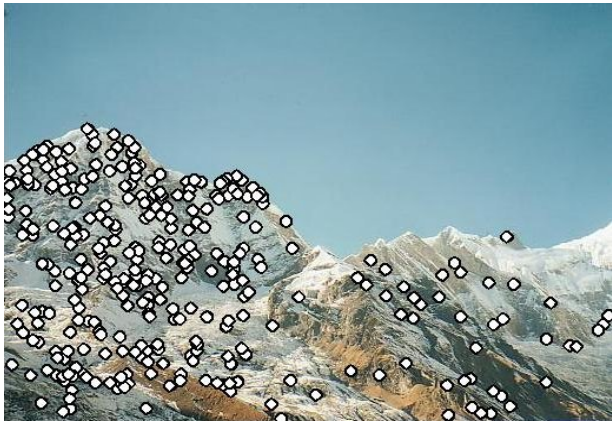
Today



Mosaics recap:

How to warp one image to the other, given H?





How to detect *which features* to match?

Motivation for feature-based alignment: Image mosaics

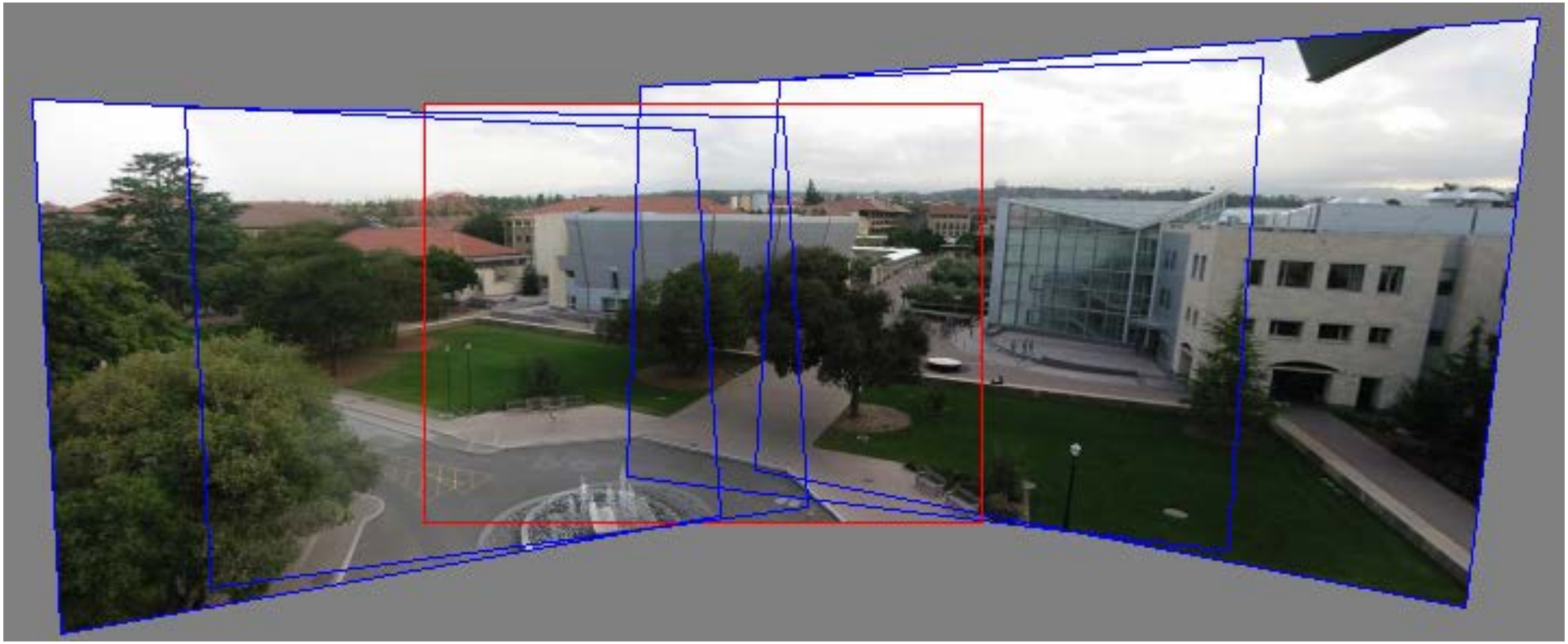


Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

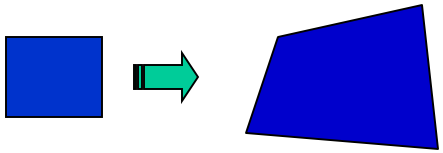
Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective transformations:

- Affine transformations, and
- Projective warps

Parallel lines do not necessarily remain parallel

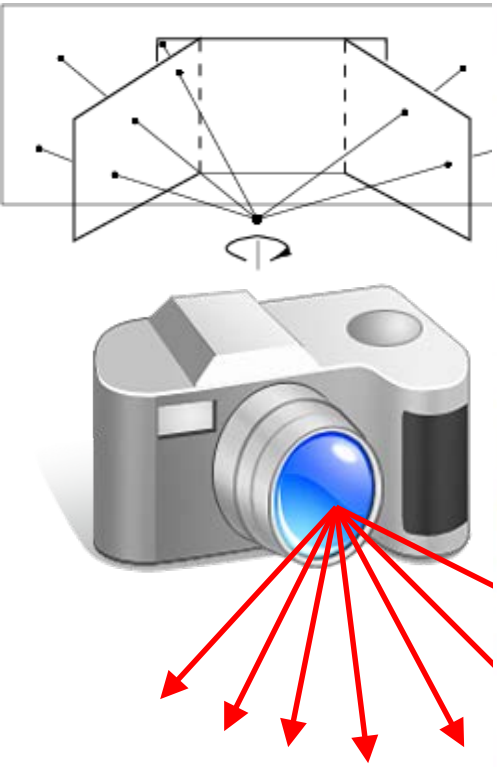


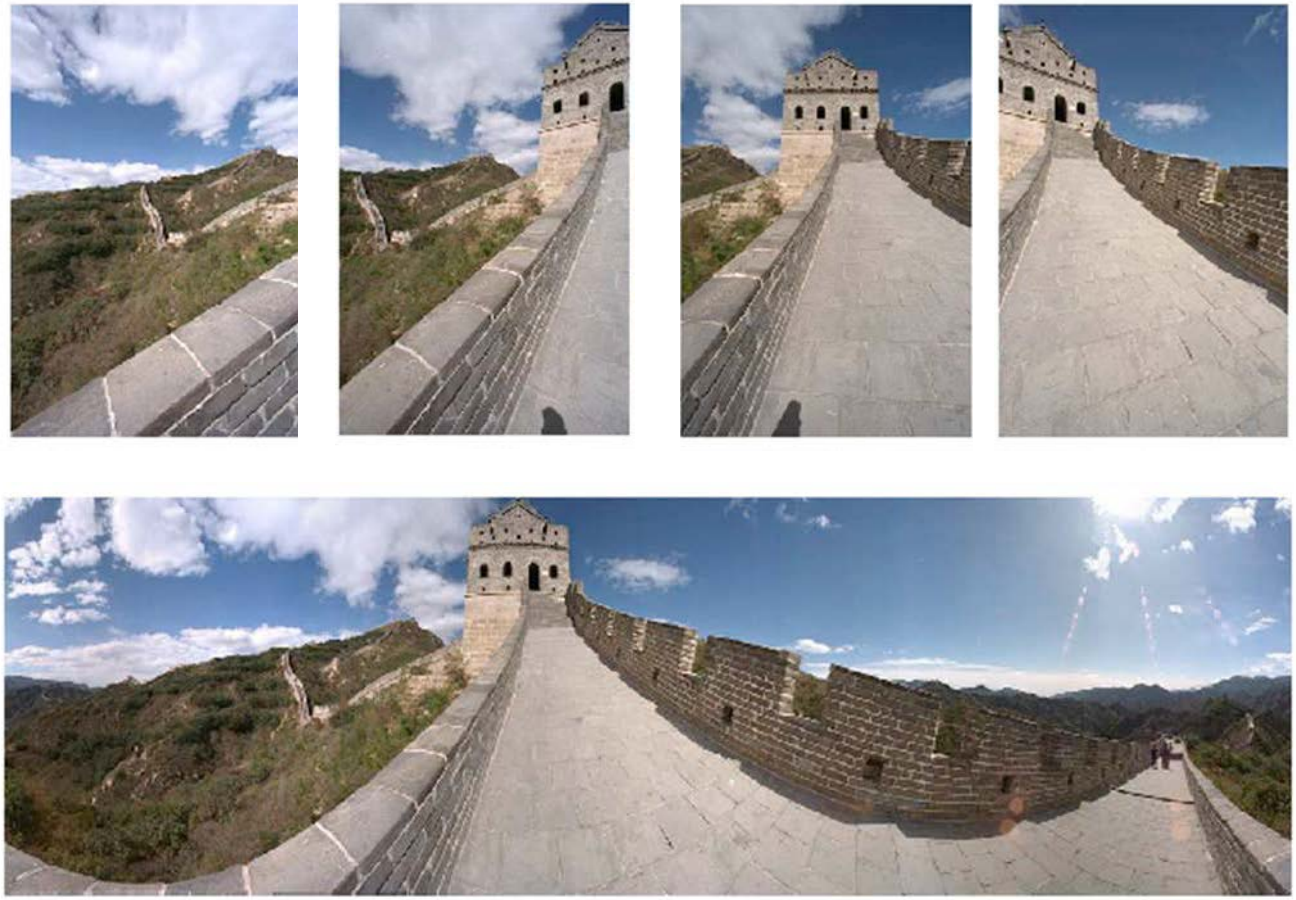
How to stitch together a panorama (a.k.a. mosaic)?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - (If there are more images, repeat)

Source: Steve Seitz

Mosaics





. . .

image from S. Seitz

Obtain a wider angle view by combining multiple images.

How to stitch together a panorama (a.k.a. mosaic)?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - (If there are more images, repeat)

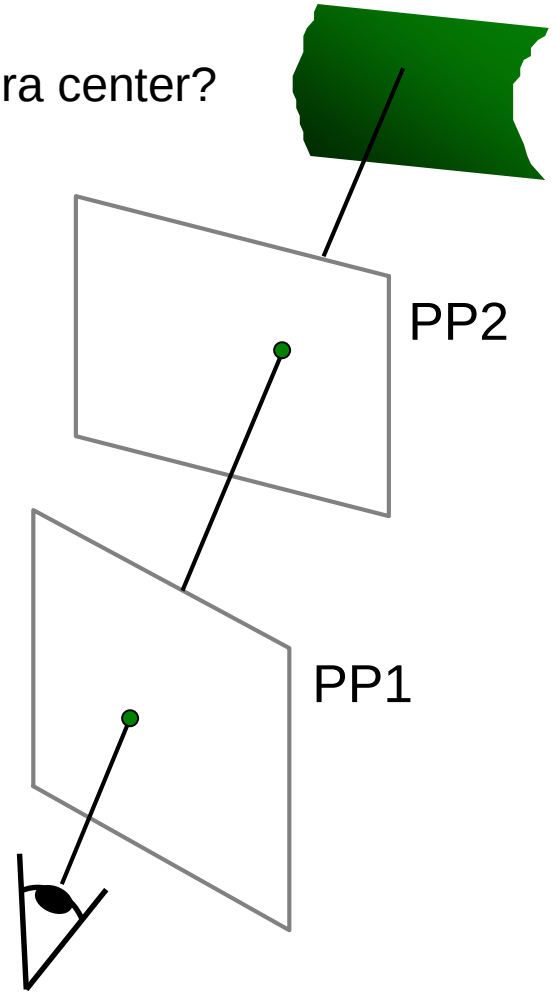
- …but **wait**, why should this work at all?
  - What about the 3D geometry of the scene?
  - Why aren't we using it?

Source: Steve Seitz

Image reprojection

Basic question

- How to relate two images from the same camera center?
  - how to map a pixel from PP1 to PP2



Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

Observation:
Rather than thinking of this as a 3D reprojection, think of it as a 2D **image warp** from one image to another.

Source: Alyosha Efros

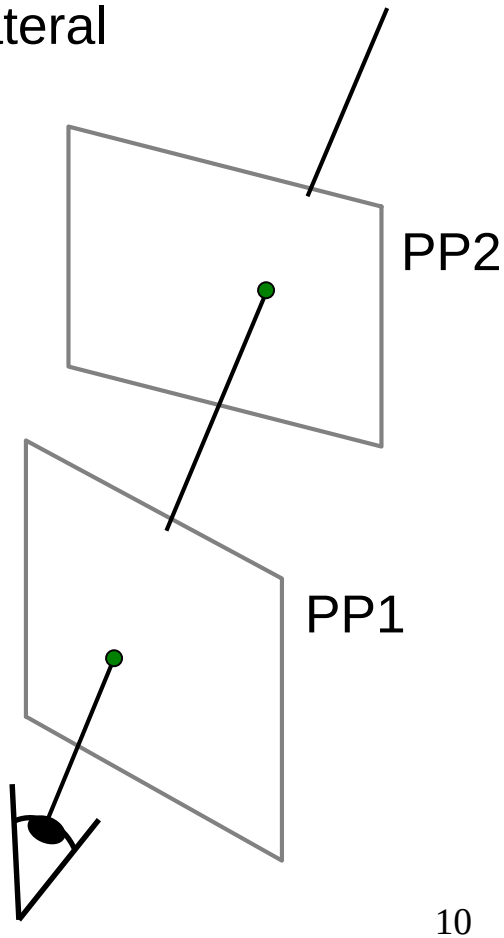
Image reprojection: Homography

A projective transform is a mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren't preserved
- but must preserve straight lines

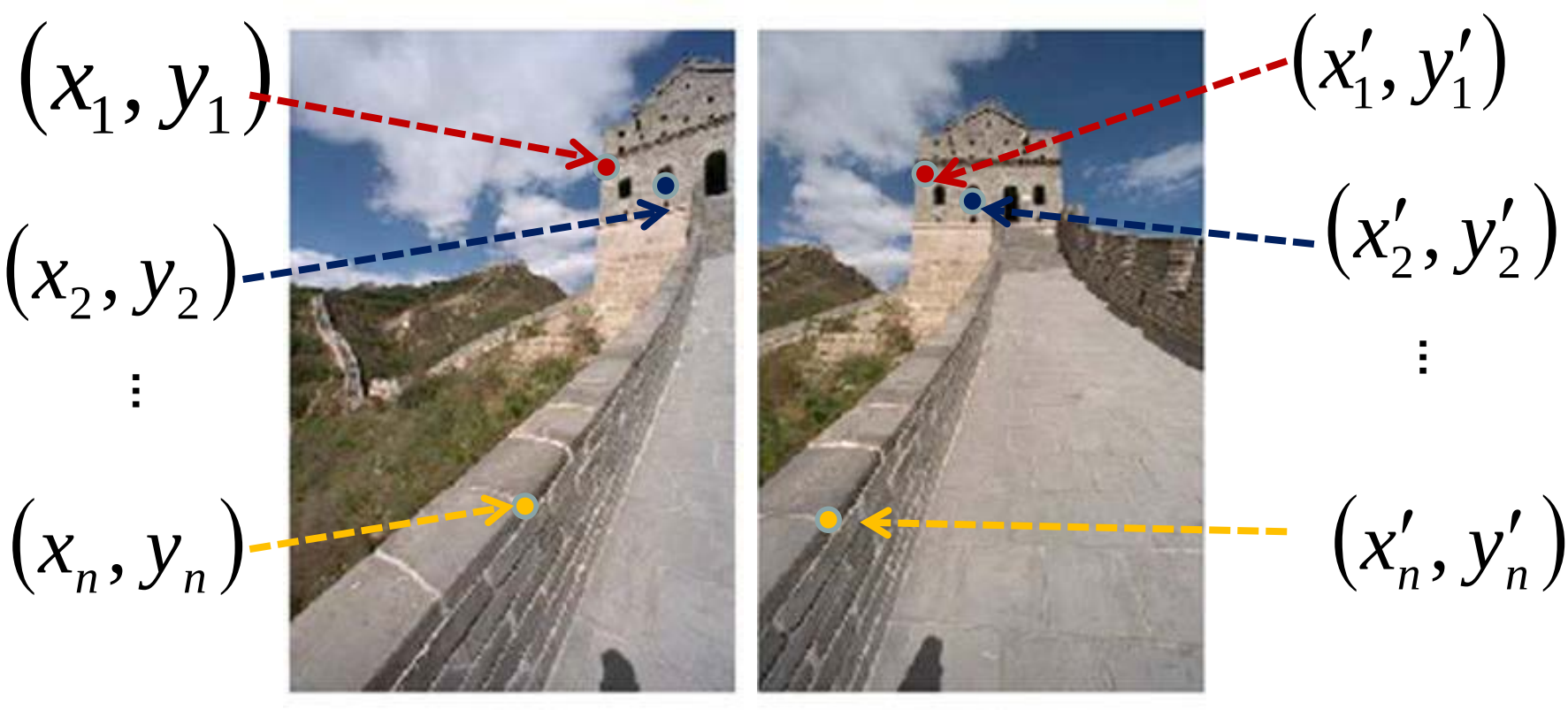
called **Homography**

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$



Source: Alyosha Efros

Homography



To **compute** the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of **H** are the unknowns…

Solving for homographies

p’ = Hp

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Defined up to a scale factor.

Constrain Frobenius norm of H to be 1.

Problem to be solved:

$$\min \|Ah - b\|^2$$

$$s.t. \quad \|h\|^2 = 1$$

where vector of unknowns h = $[h_{00}, h_{01}, h_{02}, h_{10}, h_{11}, h_{12}, h_{20}, h_{21}, h_{22}]^T$

Adapted from Devi Parikh

Solving for homographies

$$\begin{bmatrix} wx_i' \\ wy_i' \\ w \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$wx_i' = h_{00}x_i + h_{01}y_i + h_{02}$$
$$wy_i' = h_{10}x_i + h_{11}y_i + h_{12}$$
$$w = h_{20}x_i + h_{21}y_i + h_{22}$$

$$x_i' = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y_i' = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$\Updownarrow$$

$$x_i' = \frac{kh_{00}x_i + kh_{01}y_i + kh_{02}}{kh_{20}x_i + kh_{21}y_i + kh_{22}}$$

$$y_i' = \frac{kh_{10}x_i + kh_{11}y_i + kh_{12}}{kh_{20}x_i + kh_{21}y_i + kh_{22}}$$

There are 9 variables $h_{00},\ldots,h_{22}$.
Are there 9 degrees of freedom?

No. We can multiply all h_{ij} by nonzero scalar k without changing the equations:

Enforcing 8 DOF

One approach: set $h_{22} = 1$

$$x'_i = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + 1} \qquad y'_i = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + 1}$$

Second approach: impose unit vector constraint

$$x'_i = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}} \qquad y'_i = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

Subject to: $h_{00}^2 + h_{01}^2 + h_{02}^2 + h_{10}^2 + h_{11}^2 + h_{12}^2 + h_{20}^2 + h_{21}^2 + h_{22}^2 = 1$

Solving for homographies

$$\begin{bmatrix} wx_i' \\ wy_i' \\ w \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{aligned} wx_i' &= h_{00}x_i + h_{01}y_i + h_{02} \\ wy_i' &= h_{10}x_i + h_{11}y_i + h_{12} \\ w &= h_{20}x_i + h_{21}y_i + h_{22} \end{aligned}$$

$$\begin{aligned} x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\ y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12} \end{aligned}$$

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving for homographies

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

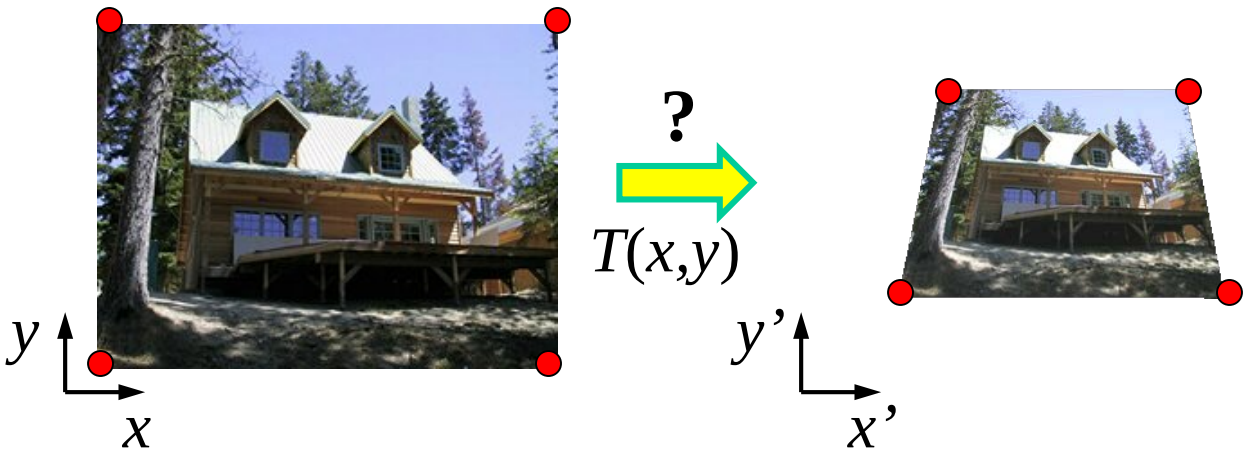
$$\underset{2n \times 9}{\mathbf{A}} \qquad \underset{9}{\mathbf{h}} \qquad \underset{2n}{\mathbf{0}}$$

Defines a least squares problem:

$$\text{minimize } \|\mathbf{A}\mathbf{h} - \mathbf{0}\|^2$$

- Since **h** is only defined up to scale, solve for unit vector $\hat{\mathbf{h}}$ $(i.e., \|h\|^2 = 1)$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Projective: # correspondences?

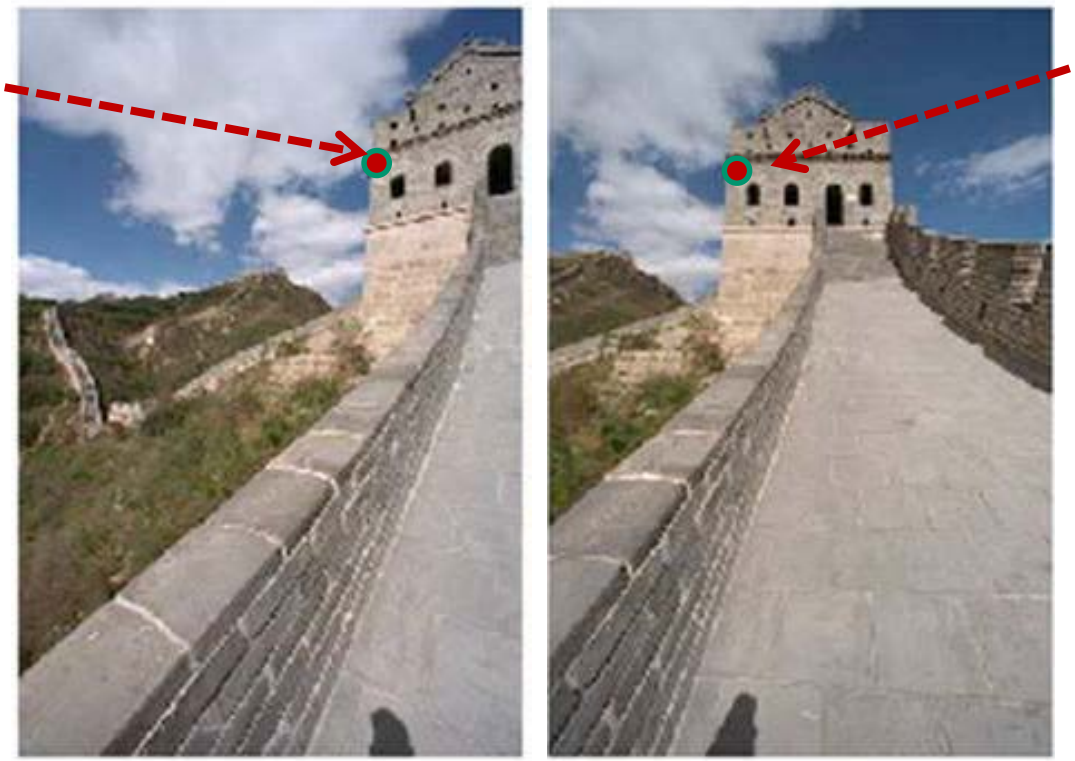


How many correspondences needed for projective?

Source: Alyosha Efros

Homography

(x, y)



$\left(wx'/w, \; wy'/w \right)$

$= (x', y')$

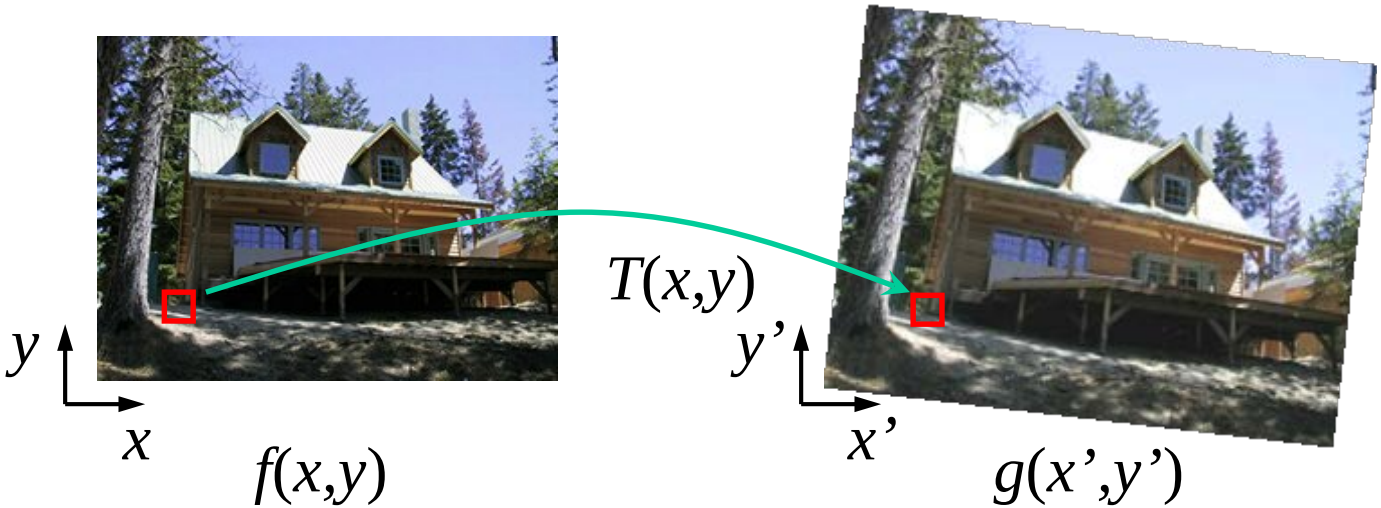
To **apply** a given homography **H**

- Compute **p'** = **Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$

Slide credit: Kristen Grauman

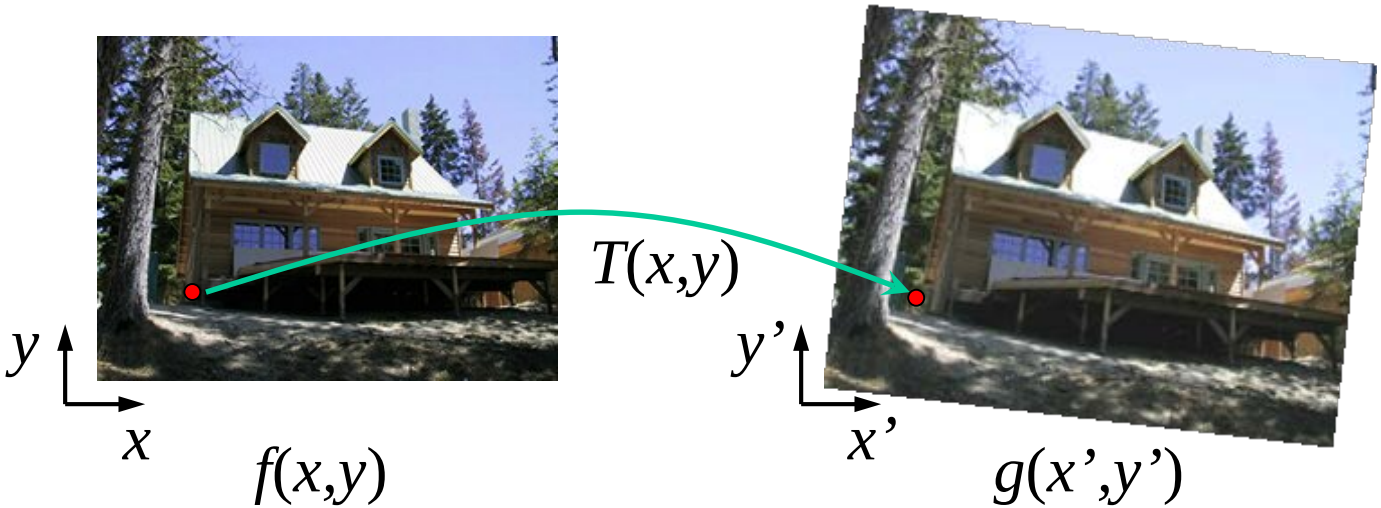
Image warping



Given a coordinate transform and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

Slide from Alyosha Efros, CMU

Forward warping



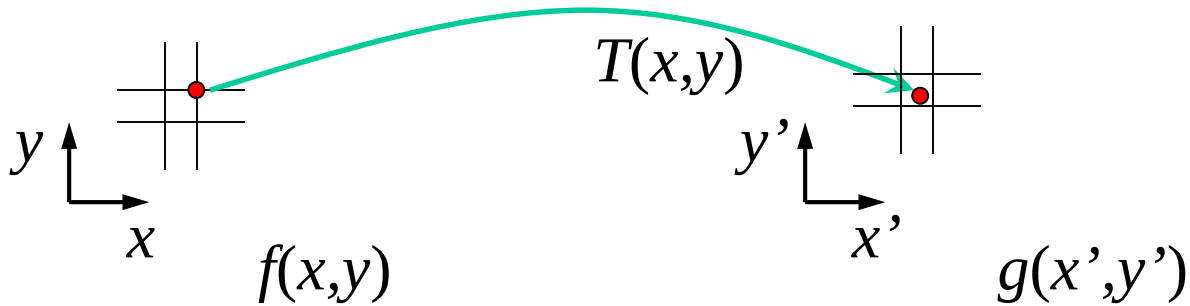
Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

Slide from Alyosha Efros, CMU

Forward warping



Send each pixel *f*(*x*,*y*) to its corresponding location

(*x'*,*y'*) = *T*(*x*,*y*) in the second image

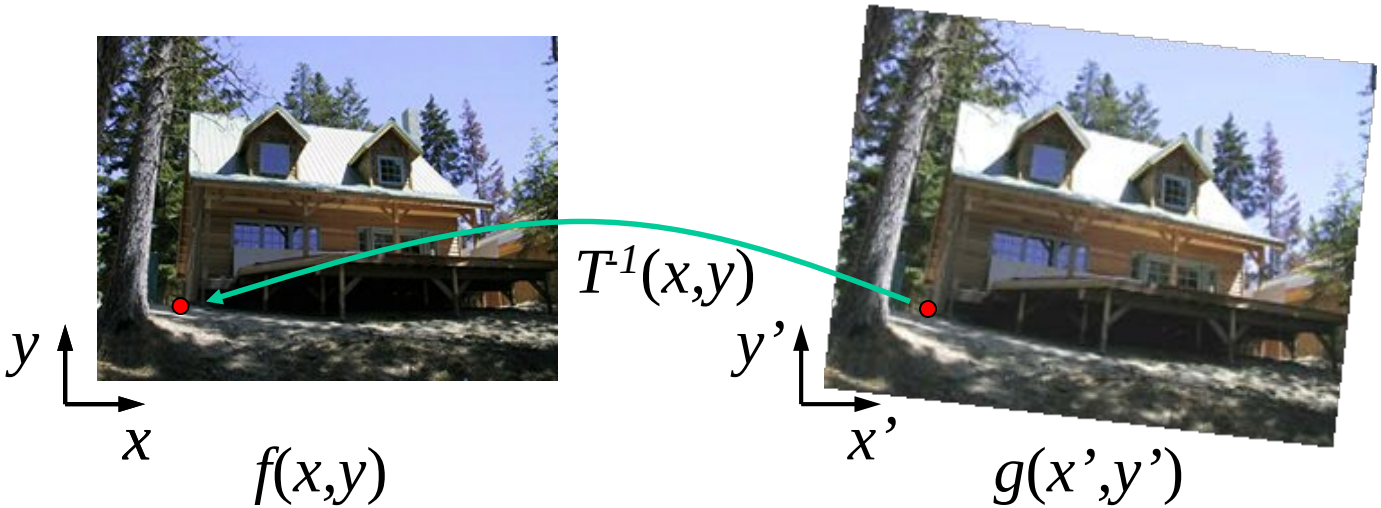
Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

– Known as "splatting"

Slide from Alyosha Efros, CMU

Inverse warping



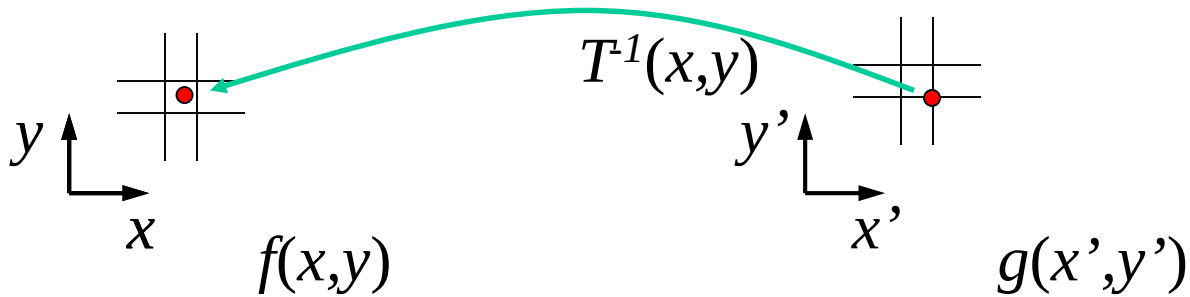
Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from “between” two pixels?

Slide from Alyosha Efros, CMU

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

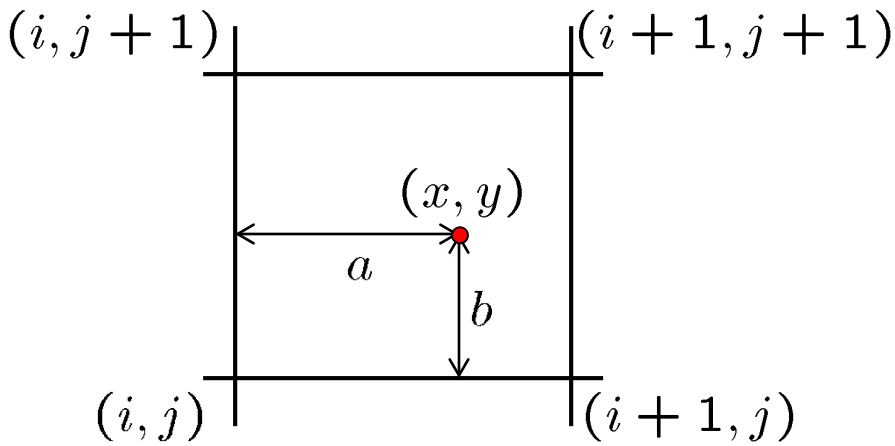
– nearest neighbor, bilinear…

`>> help interp2`

Slide from Alyosha Efros, CMU

Bilinear interpolation

Sampling at *f(x,y):*



$$
\begin{aligned}
f(x,y) = \quad & (1-a)(1-b) && f[i,j] \\
& +a(1-b) && f[i+1,j] \\
& +ab && f[i+1,j+1] \\
& +(1-a)b && f[i,j+1]
\end{aligned}
$$

Slide from Alyosha Efros, CMU

Image warping with homographies

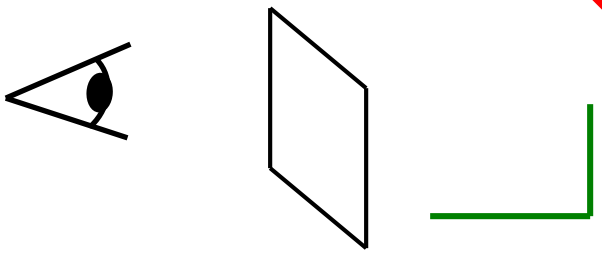
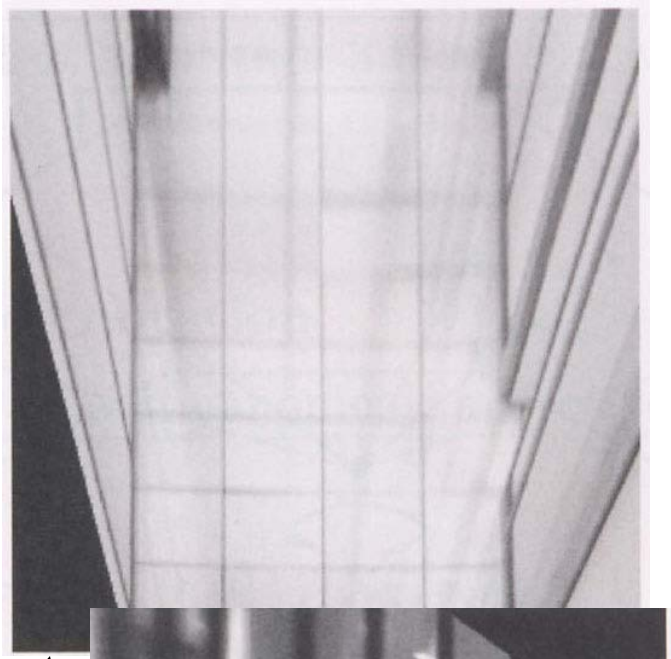
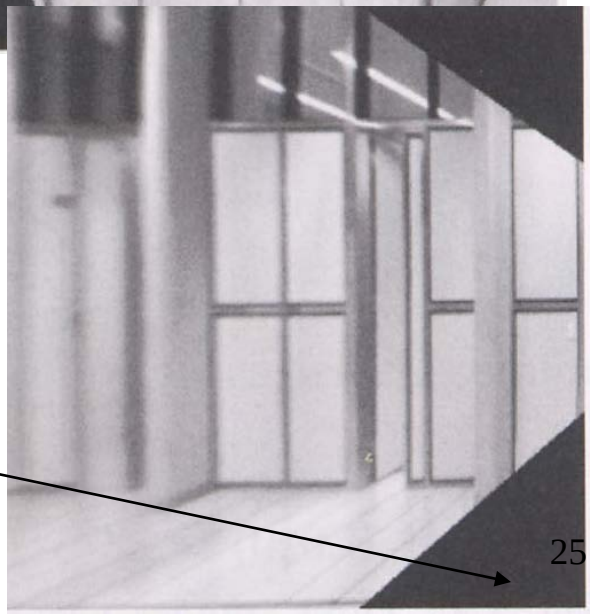


image plane in front

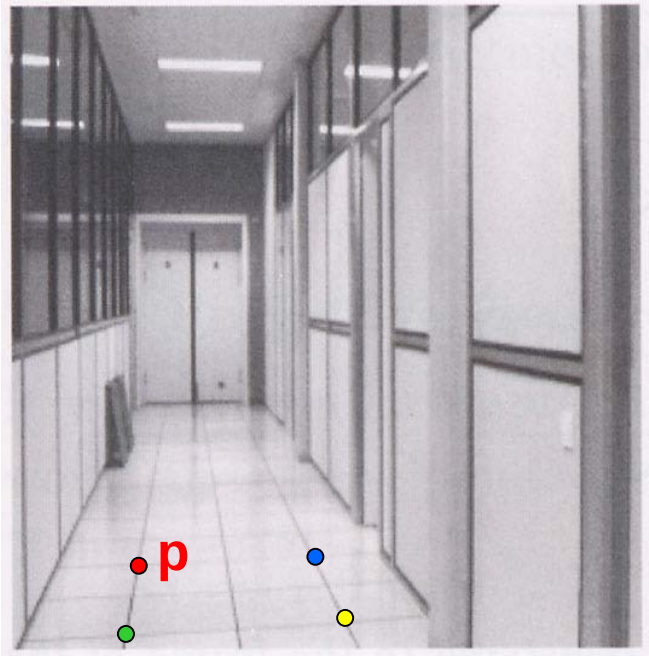


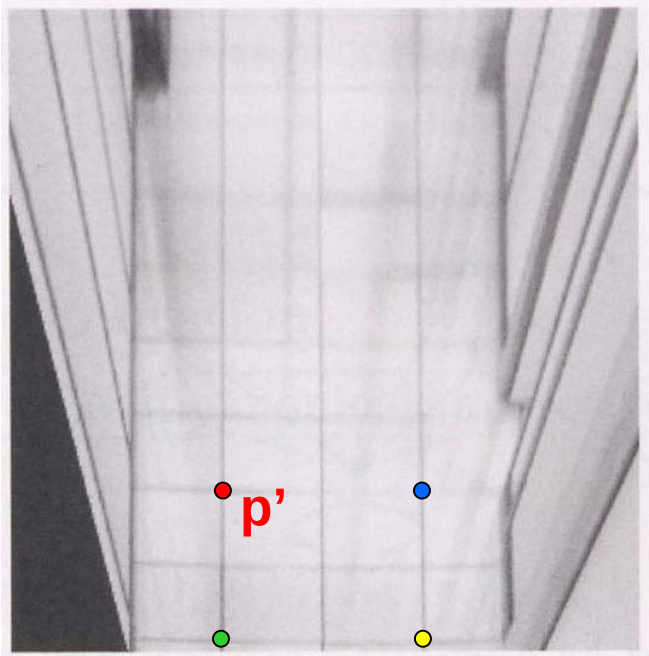


black area
where no pixel
maps to

Source: Steve Seitz

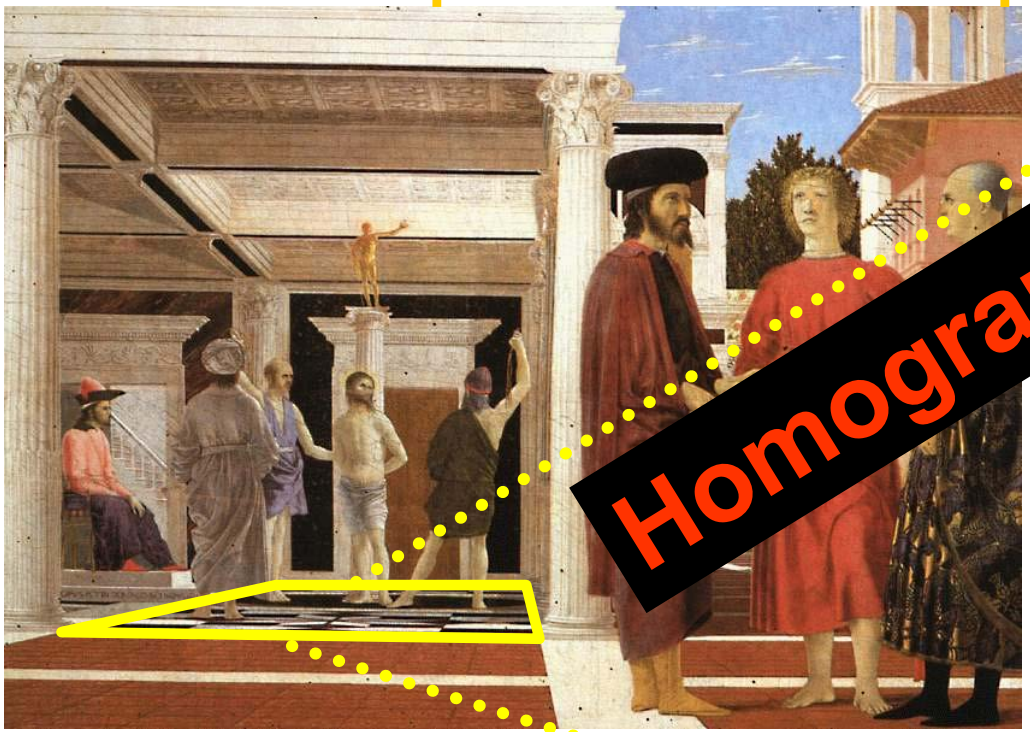
Image rectification





Analysing patterns and shapes

What is the shape of the b/w floor pattern?



Homography

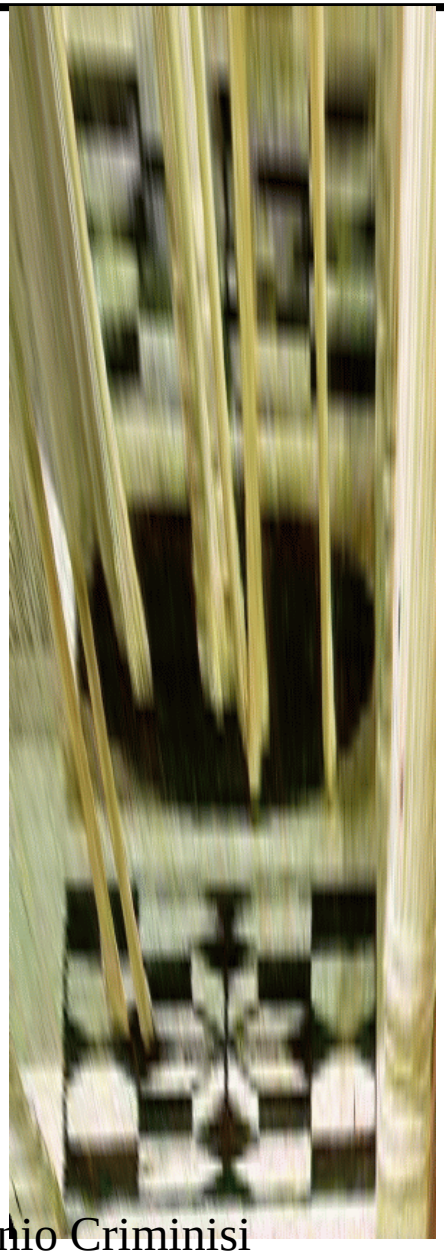
The floor (enlarged)

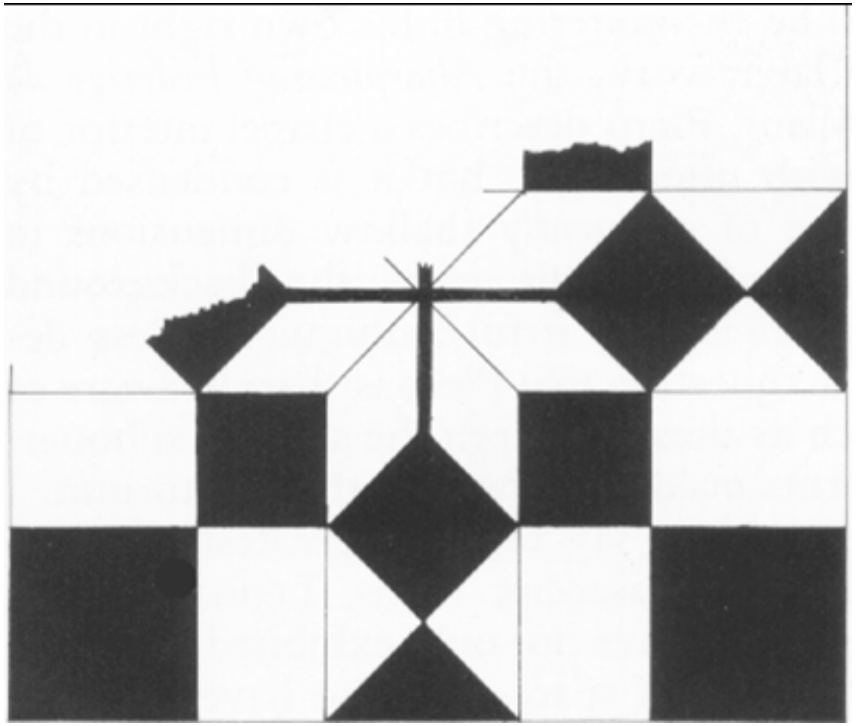
Automatically rectified floor

Slide from Antonio Criminisi

Analysing patterns and shapes

Automatic rectification

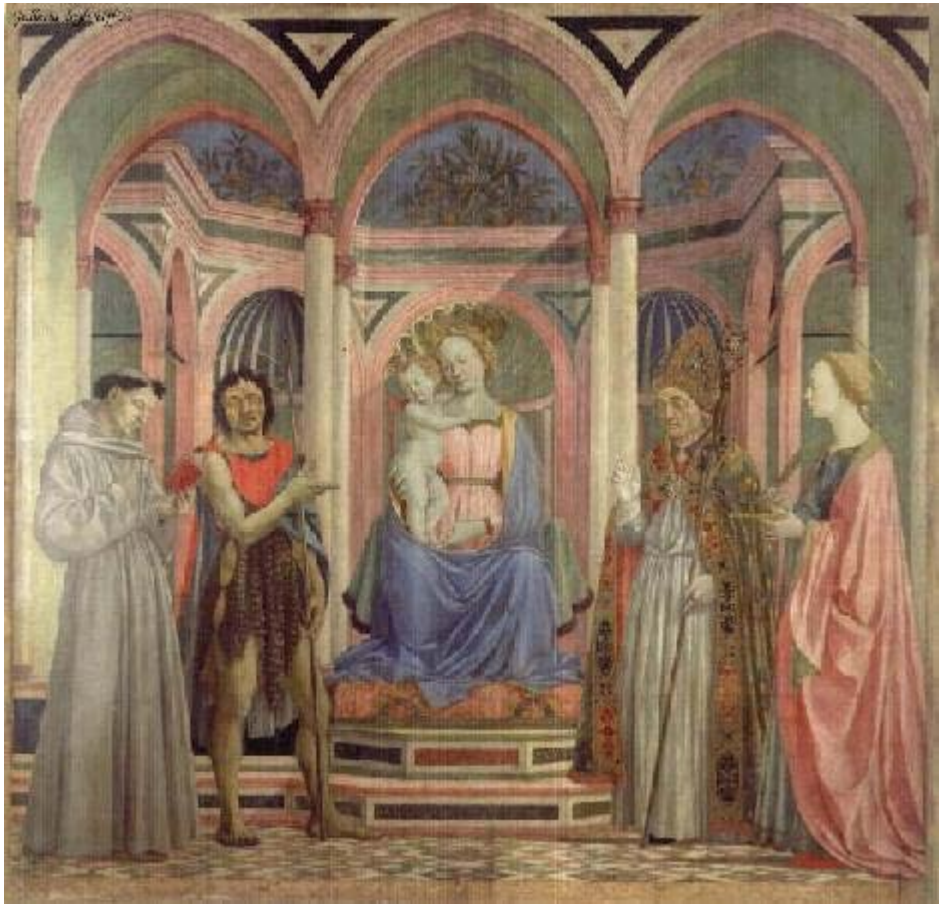




From Martin Kemp *The Science of Art*
(manual reconstruction)

Slide from Antonio Criminisi

Analysing patterns and shapes



St. Lucy Altarpiece, **D. Veneziano**

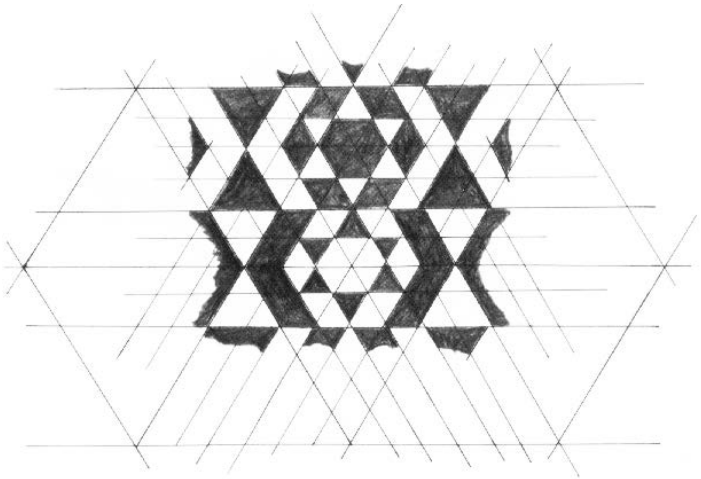
What is the (complicated) shape of the floor pattern?

Automatically rectified floor

Slide from Criminisi

Analysing patterns and shapes

Automatic rectification

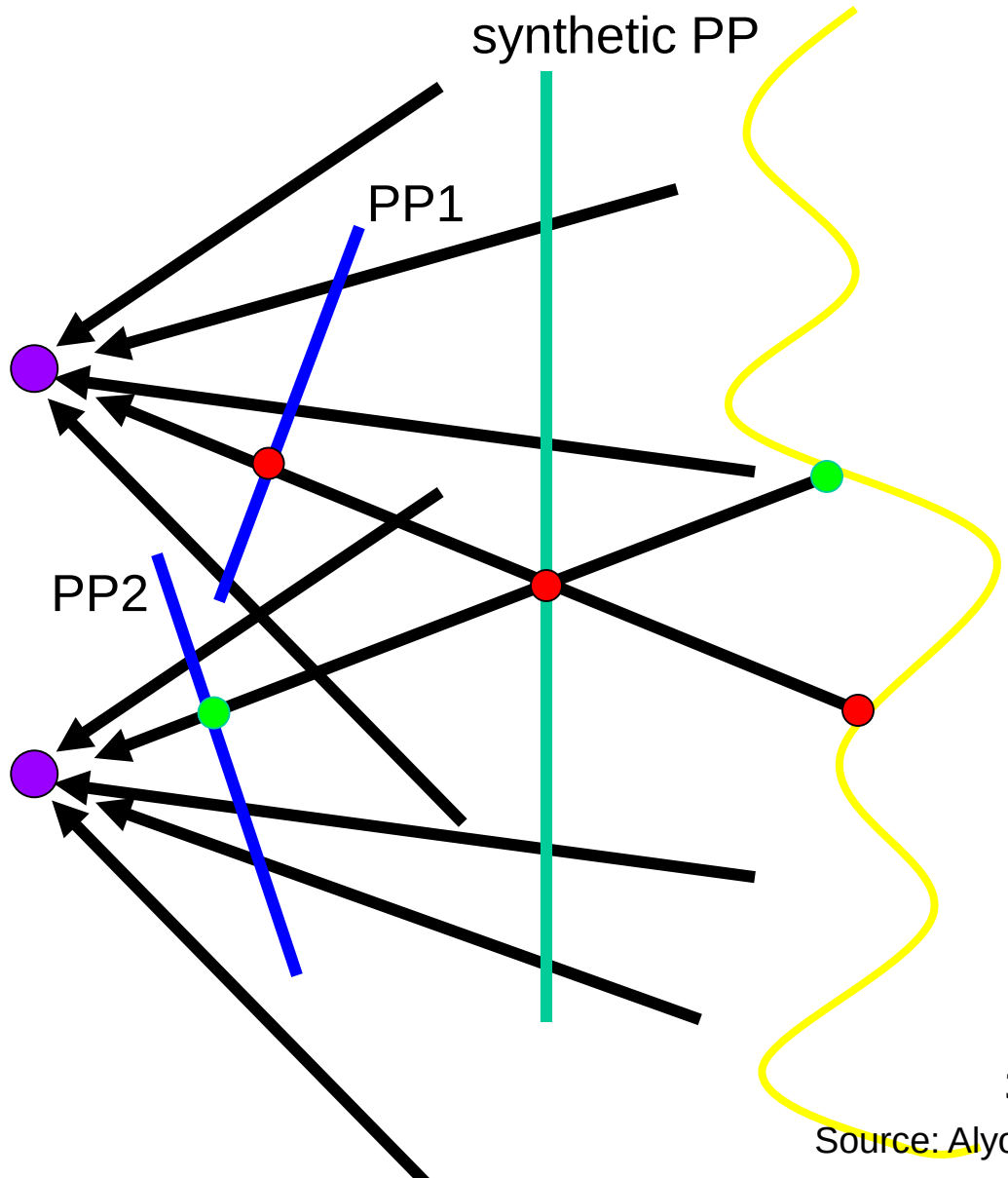


From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

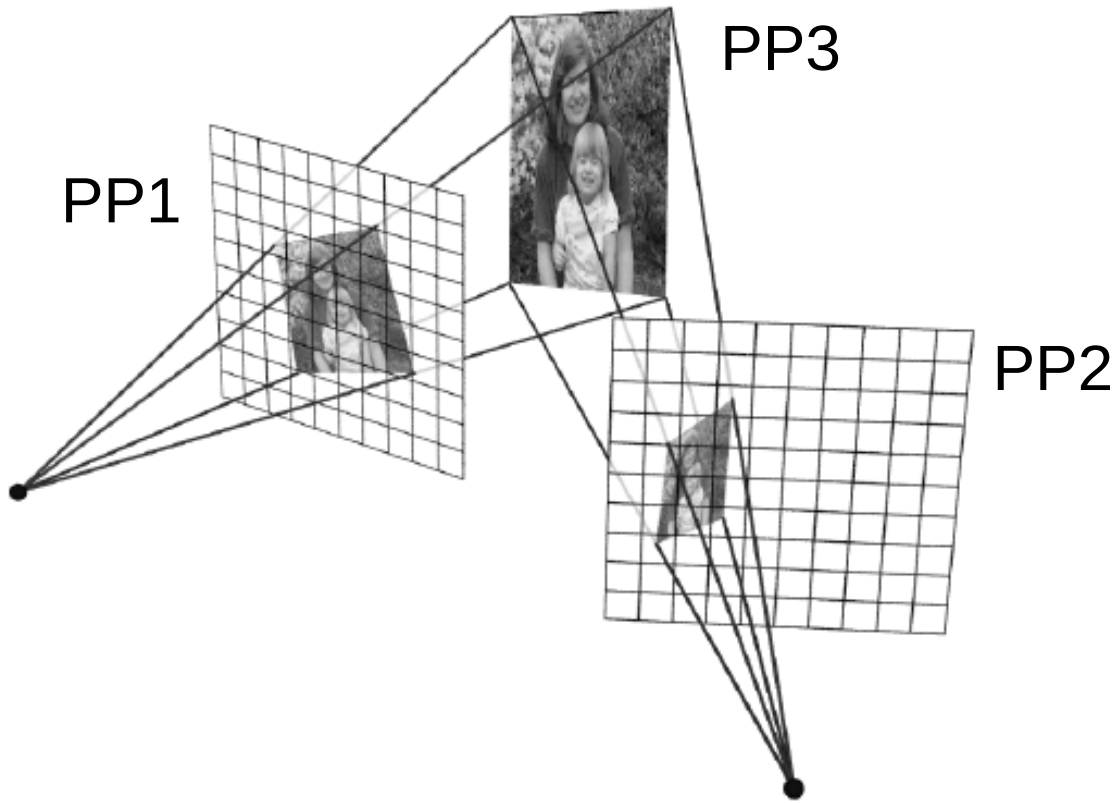
Changing camera center

Does it still work?



Source: Alyosha Efros

…Or: Planar scene (or far away)



PP3 is a projection plane of both centers of projection, so we are OK!

This is how big aerial photographs are made

Source: Alyosha Efros


Map
Satellite
Hybrid
200 ft
100 m
©2007 Google - Terms of Use

RANSAC for estimating homography

RANSAC loop:

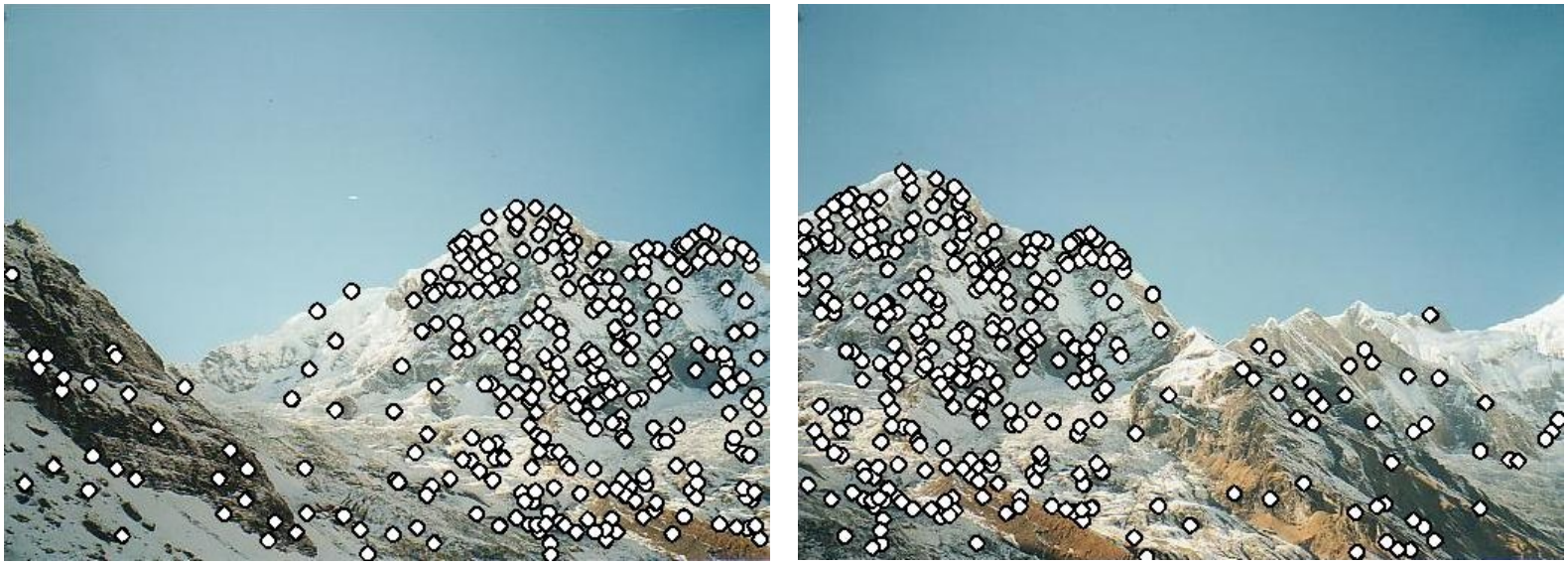
1. Select four feature pairs (at random)
2. Compute homography H (exact)
3. Compute *inliers* where $SSD(p_i', \boldsymbol{H}p_i) < \varepsilon$
4. Keep largest set of inliers
5. Re-compute least-squares H estimate on all of the inliers

Slide credit: Steve Seitz

Robust feature-based alignment

Source: L. Lazebnik

Robust feature-based alignment



- Extract features

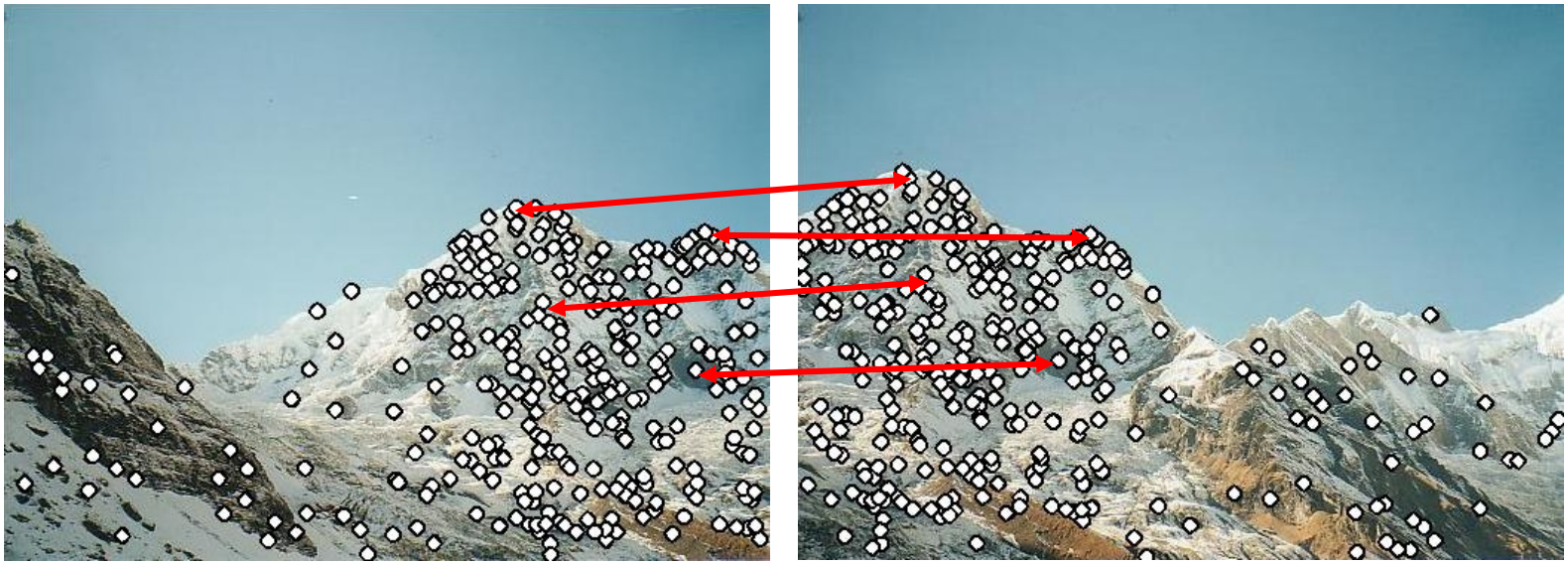
Source: L. Lazebnik

Robust feature-based alignment

- Extract features
- Compute *putative matches*

Source: L. Lazebnik

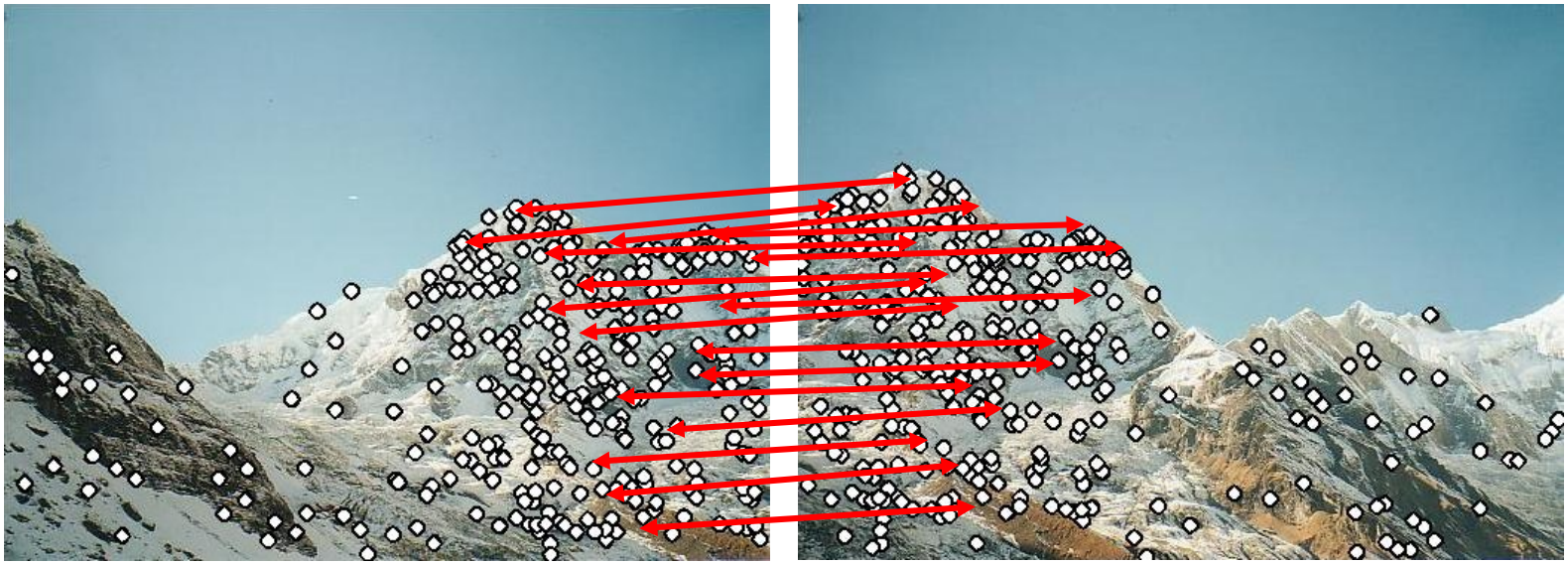
Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)

Source: L. Lazebnik

Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Source: L. Lazebnik

Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Source: L. Lazebnik

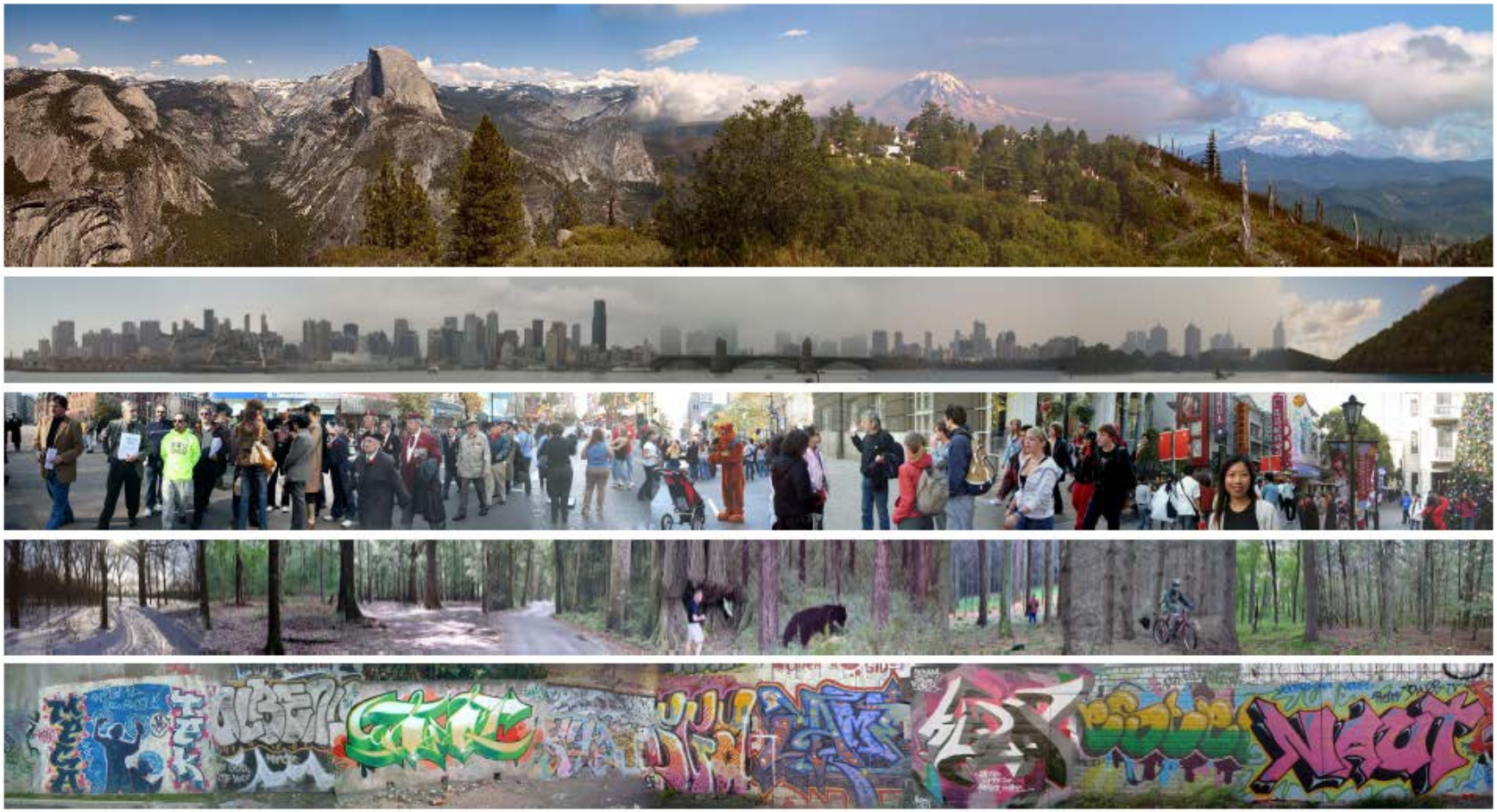
Summary: alignment & warping

- Write **2d transformations** as matrix-vector multiplication (including translation when we use homogeneous coordinates)
- **Fitting transformations**: solve for unknown parameters given corresponding points from two views (affine, projective (homography)).
- Perform **image warping** (forward, inverse)
- **Mosaics**: uses homography and image warping to merge views taken from same center of projection.

Boundary extension

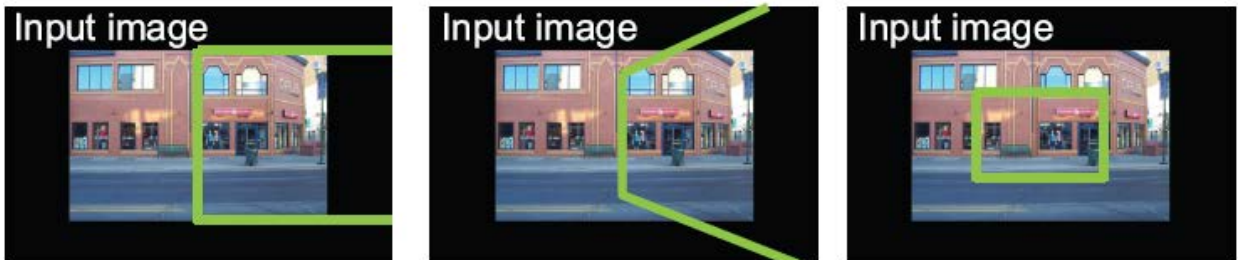
- Wide-Angle Memories of Close-Up Scenes, Helene Intraub and Michael Richardson, Journal of Experimental Psychology: Learning, Memory, and Cognition, 1989, Vol. 15, No. 2, 179-187

Creating and Exploring a Large Photorealistic Virtual Space



Josef Sivic, Biliana Kaneva, Antonio Torralba, Shai Avidan and William T. Freeman, Internet Vision Workshop, CVPR 2008.
http://www.youtube.com/watch?v=E0rboU10rPo

Creating and Exploring a Large Photorealistic Virtual Space

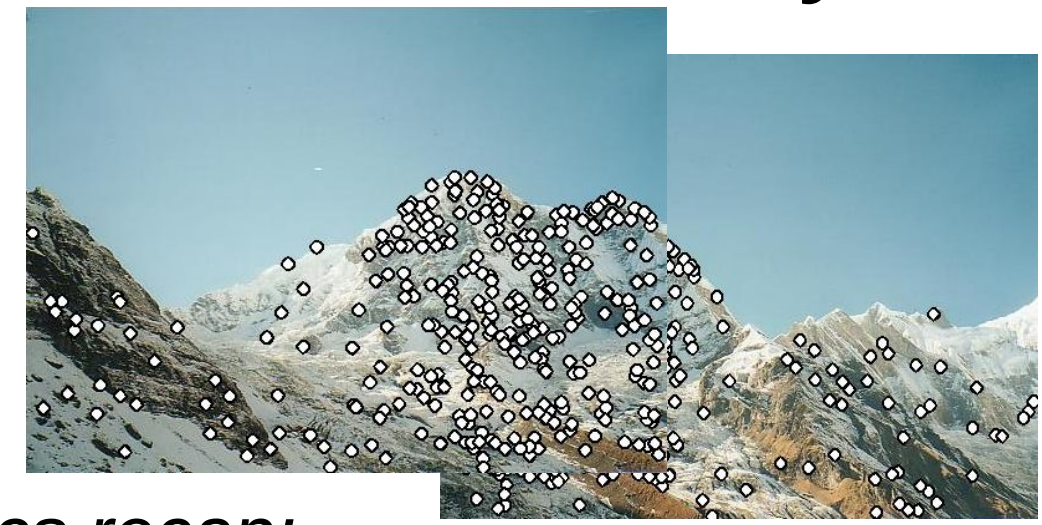


Current view, and desired view in green

Synthesized view from new camera

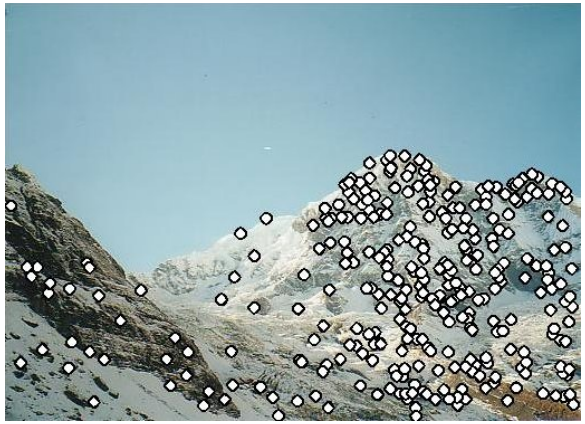
Induced camera motion

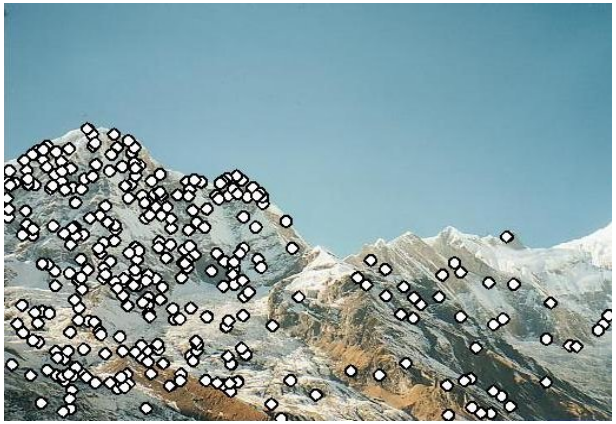
Today



Mosaics recap:

How to warp one image to the other, given H?





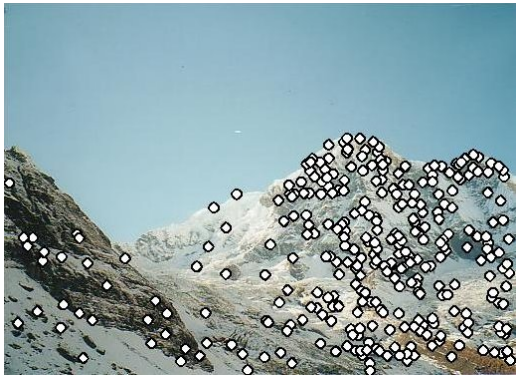
How to detect *which features* to match?

Detecting local invariant features

- Detection of interest points
  - Harris corner detection
  - (Scale invariant blob detection: LoG)
- (Next time: description of local patches)

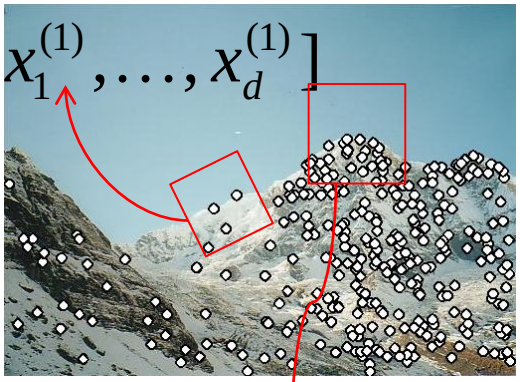
Local features: main components

1) Detection: Identify the interest points



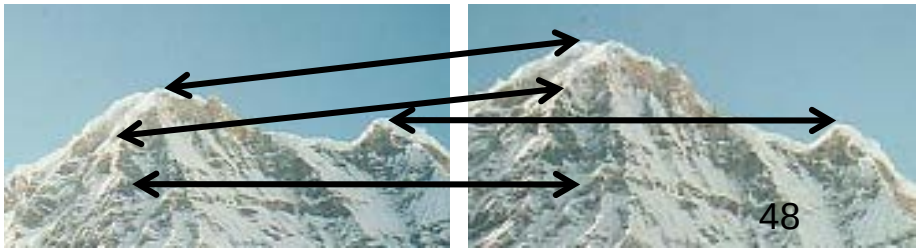
2) Description: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) Matching: Determine correspondence between descriptors in two views



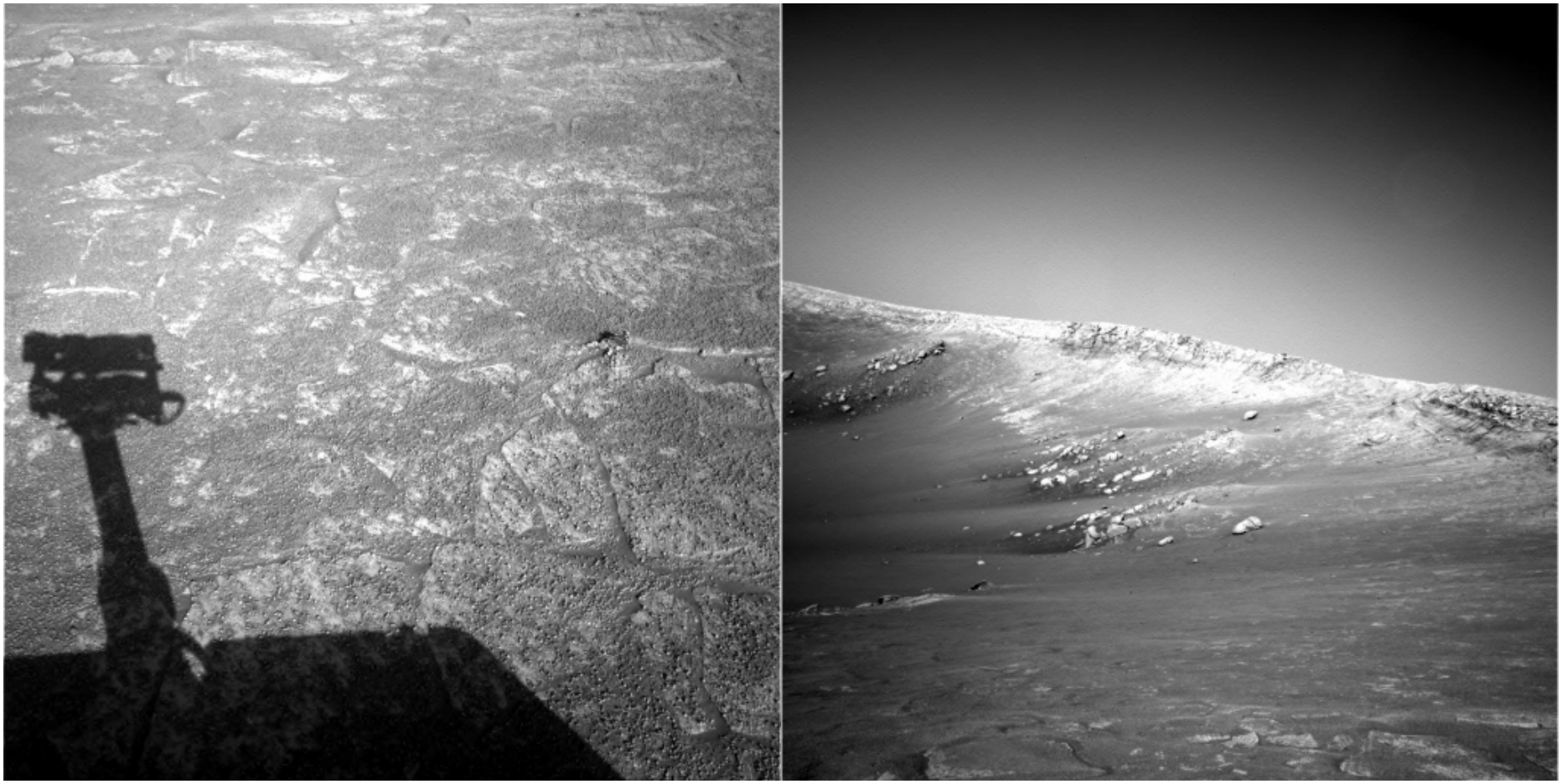
Local features: desired properties

- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature has a distinctive description
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Applications

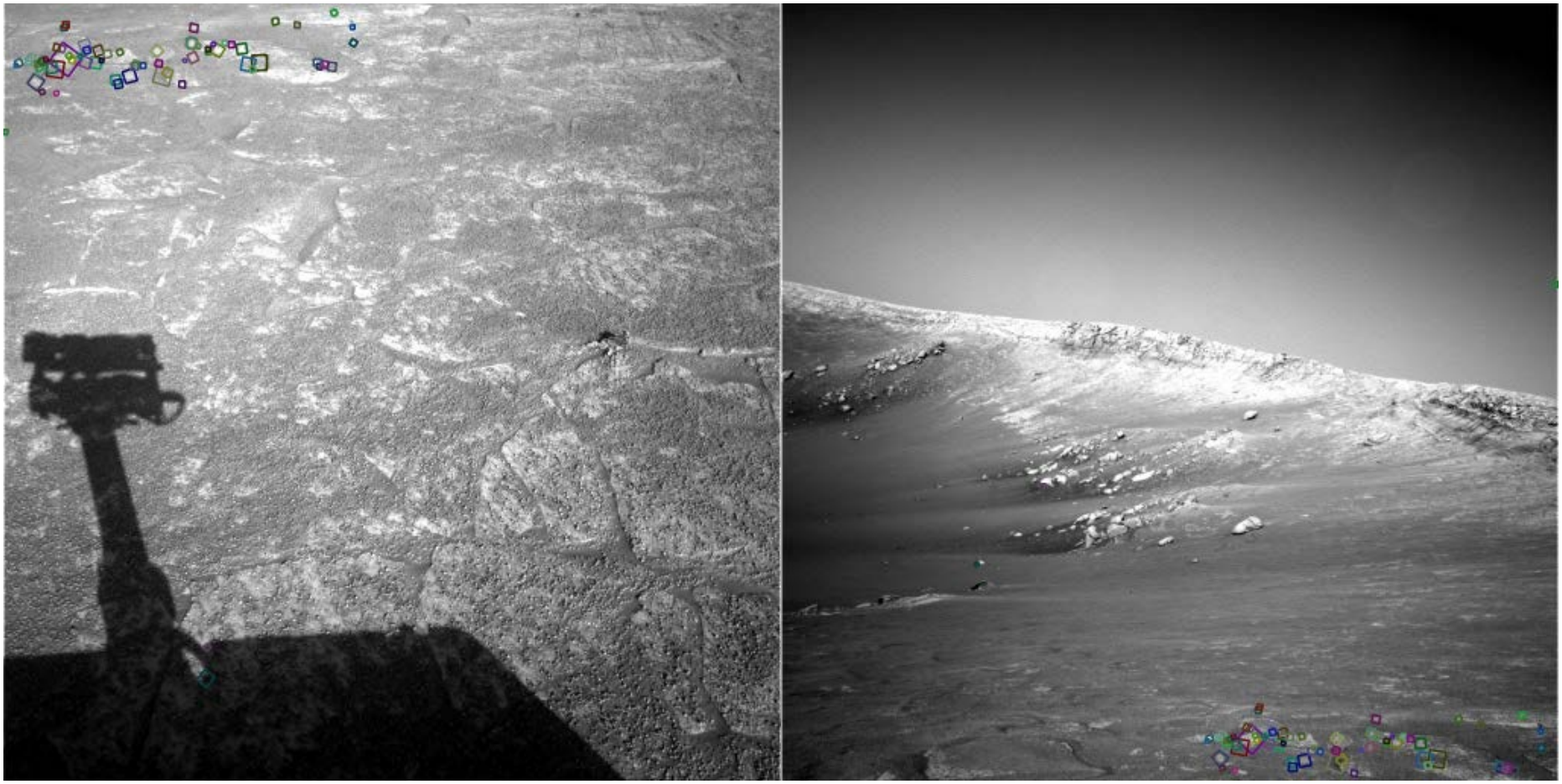
- Local features are used for:
  - Image alignment
  - 3D reconstruction
  - Motion tracking
  - Robot navigation
  - Indexing and database retrieval
  - Object recognition

A hard feature matching problem



NASA Mars Rover images

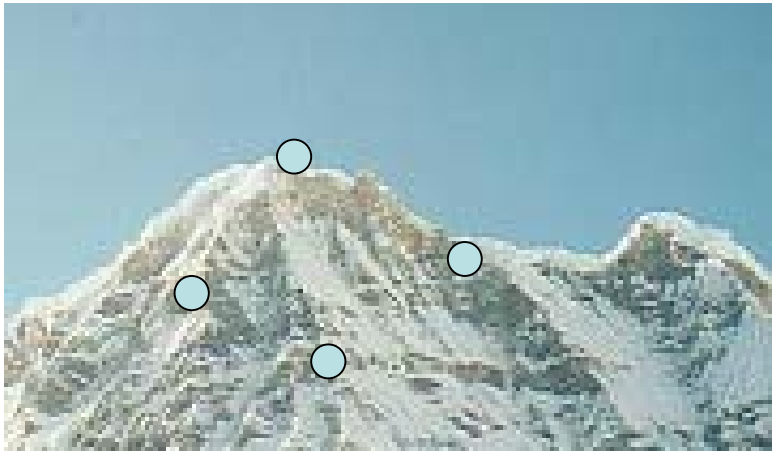
Answer below (look for tiny colored squares…)

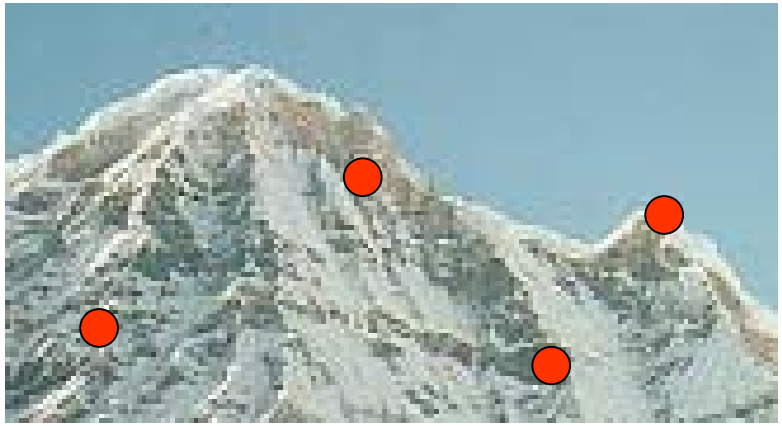


NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Goal: interest operator repeatability

- We want to detect (at least some of) the same points in both images.



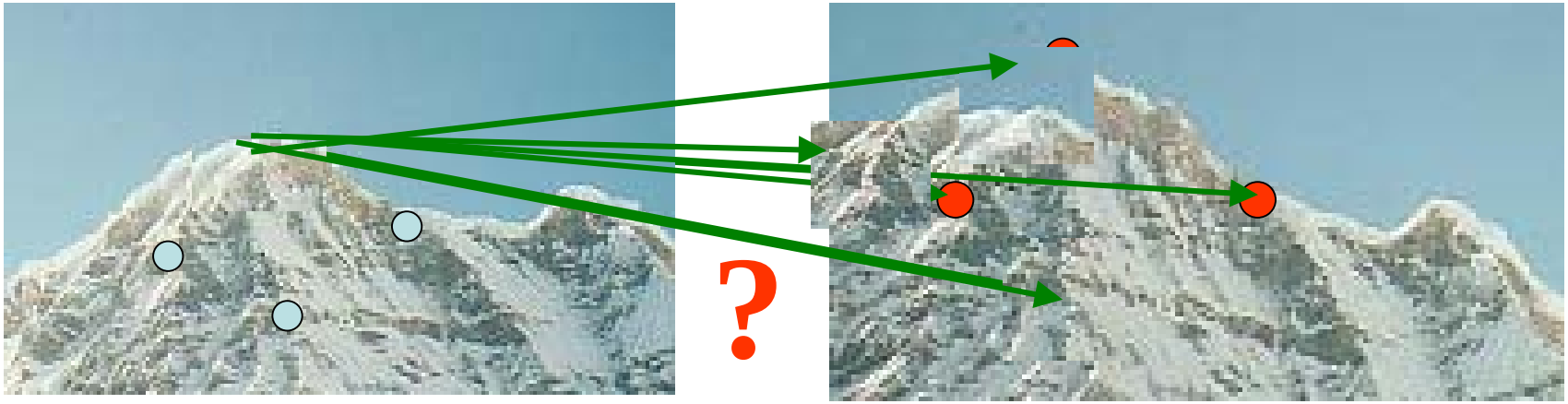


No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

Goal: descriptor distinctiveness

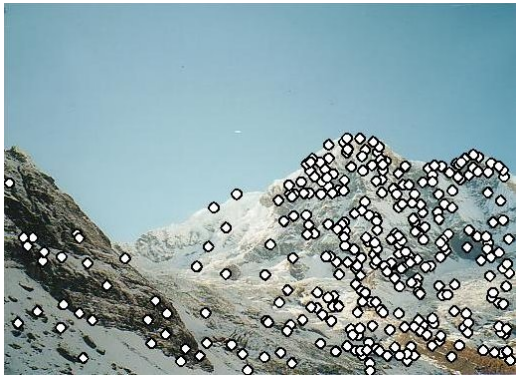
- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.

Local features: main components

1) Detection: Identify the interest points



2) Description:Extract vector feature descriptor surrounding each interest point.

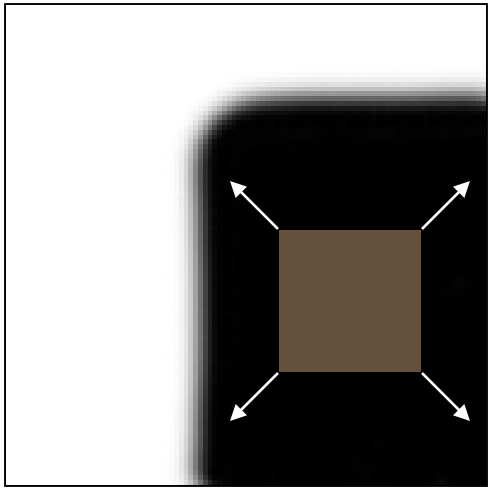
3) Matching: Determine correspondence between descriptors in two views

- What points would you choose?

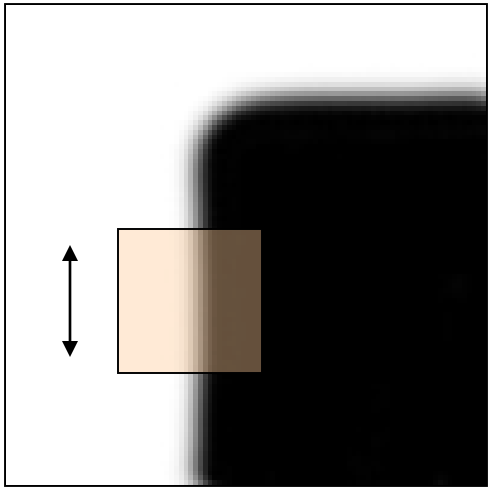
Corners as distinctive interest points

We should easily recognize the point by looking through a small window

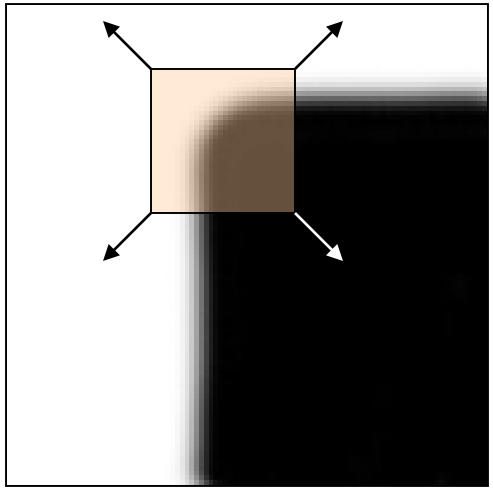
Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction



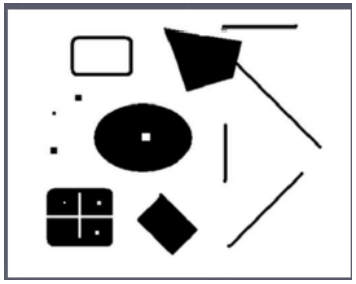
"corner":
significant change in all directions

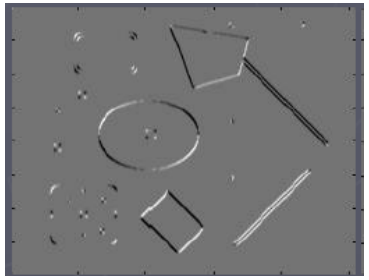
Slide credit: Alyosha Efros, Darya Frolova, Denis Simakov

Corners as distinctive interest points

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point).



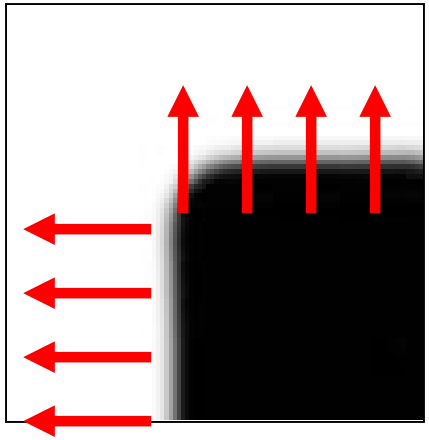


Notation:

$$I_x \Leftrightarrow \frac{\partial I}{\partial x} \qquad I_y \Leftrightarrow \frac{\partial I}{\partial y} \qquad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$$

What does this matrix reveal?

First, consider an axis-aligned corner:



What does this matrix reveal?

First, consider an axis-aligned corner:

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means dominant gradient directions align with x or y axis

Look for locations where **both** λ's are large.

If either λ is close to 0, then this is **not** corner-like.

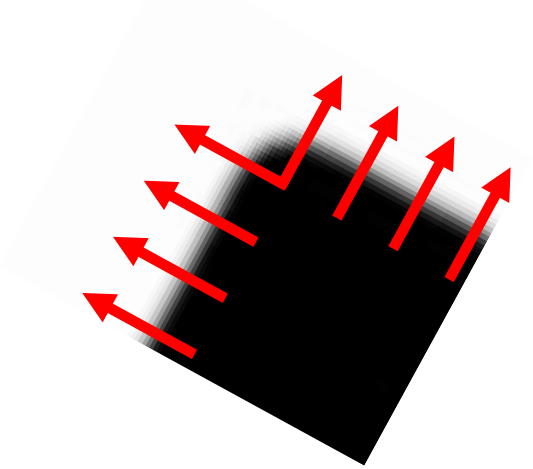
What if we have a corner that is not aligned with the image axes?

What does this matrix reveal?

Since *M* is symmetric, we have $M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$

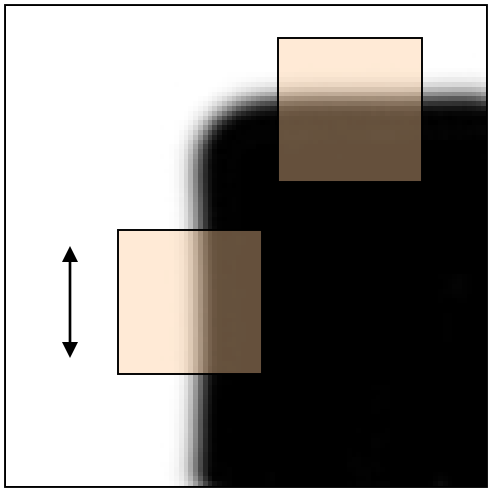
(Eigenvalue decomposition)

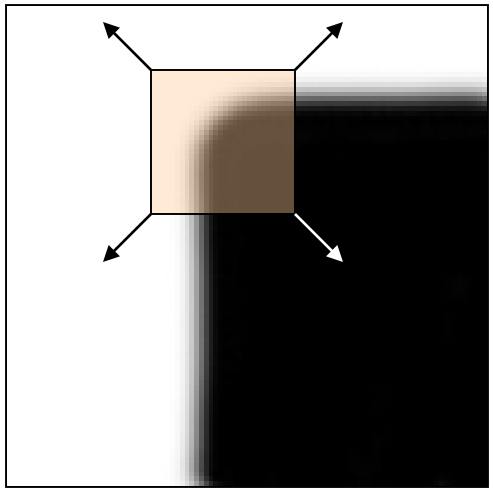
$$Mx_i = \lambda_i x_i$$

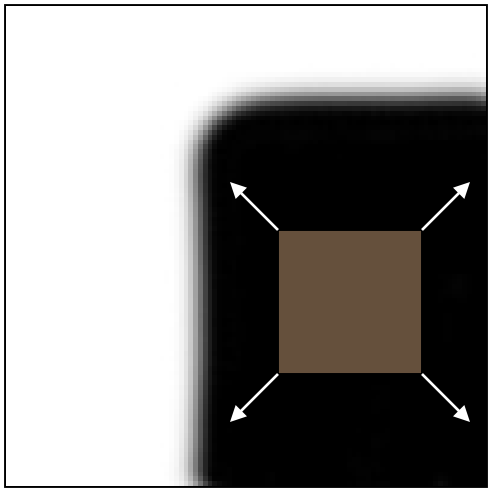


The *eigenvalues* of *M* reveal the amount of intensity change in the two principal orthogonal gradient directions in the window.

Corner response function







"edge":

$\lambda_1 >> \lambda_2$

$\lambda_2 >> \lambda_1$

"corner":

λ_1 and λ_2 are large, $\lambda_1 \sim \lambda_2$;

"flat" region

λ_1 and λ_2 are small;

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

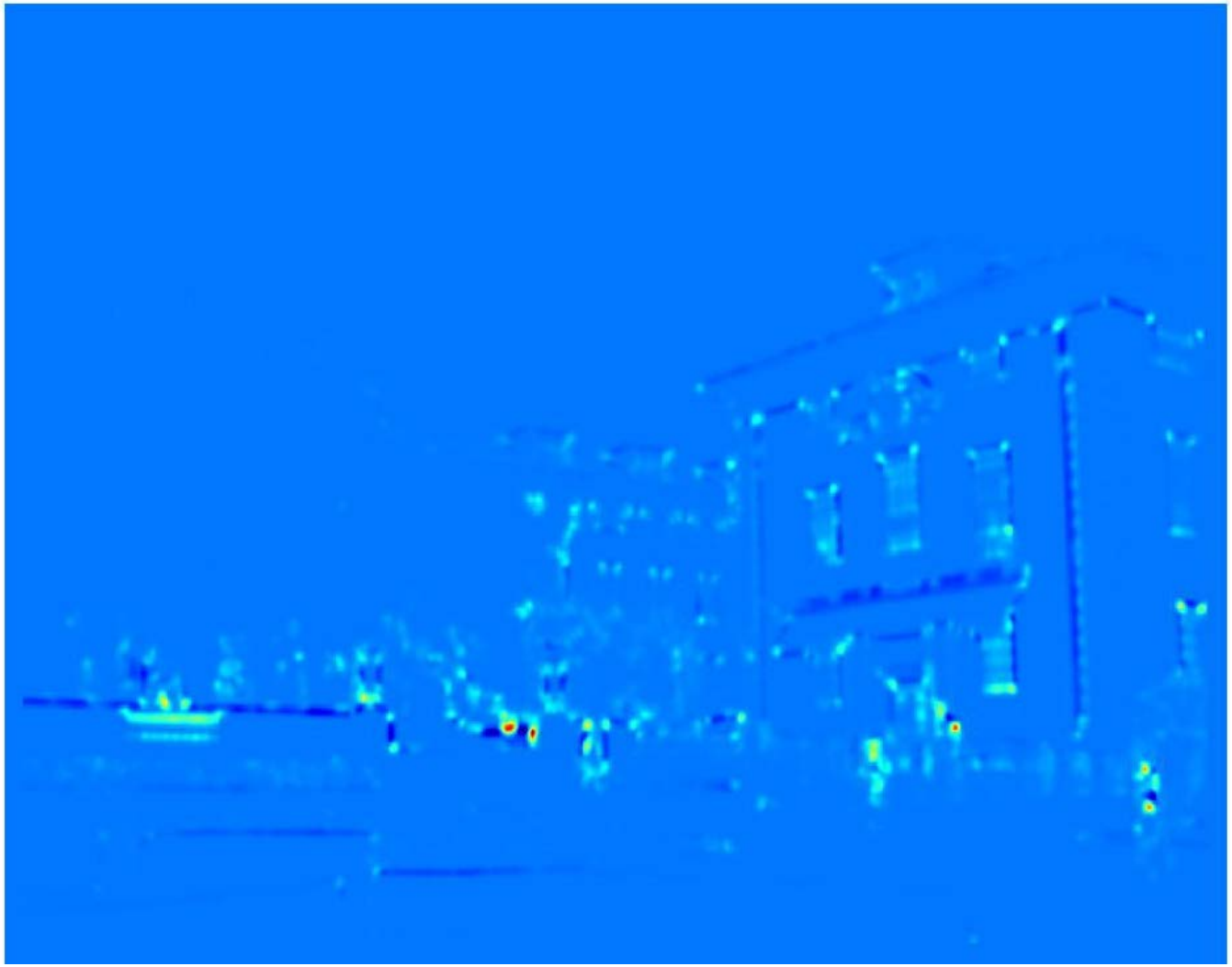
Harris corner detector

1) Compute *M* matrix for each image window to get their *cornerness* scores.
2) Find points whose surrounding window gave large corner response (f > threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression

Example of Harris application

Example of Harris application

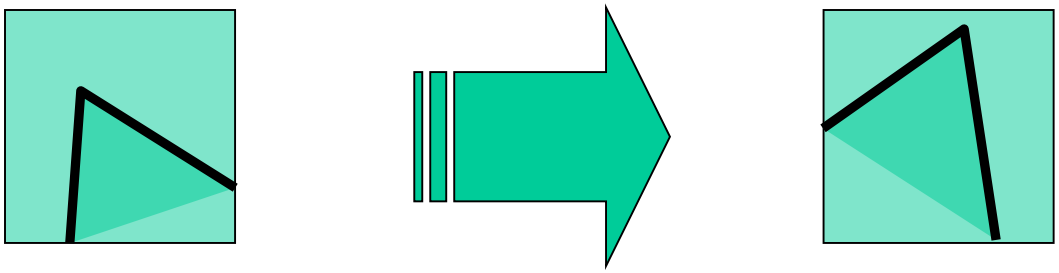
Compute corner response at every pixel.



Example of Harris application

Properties of the Harris corner detector

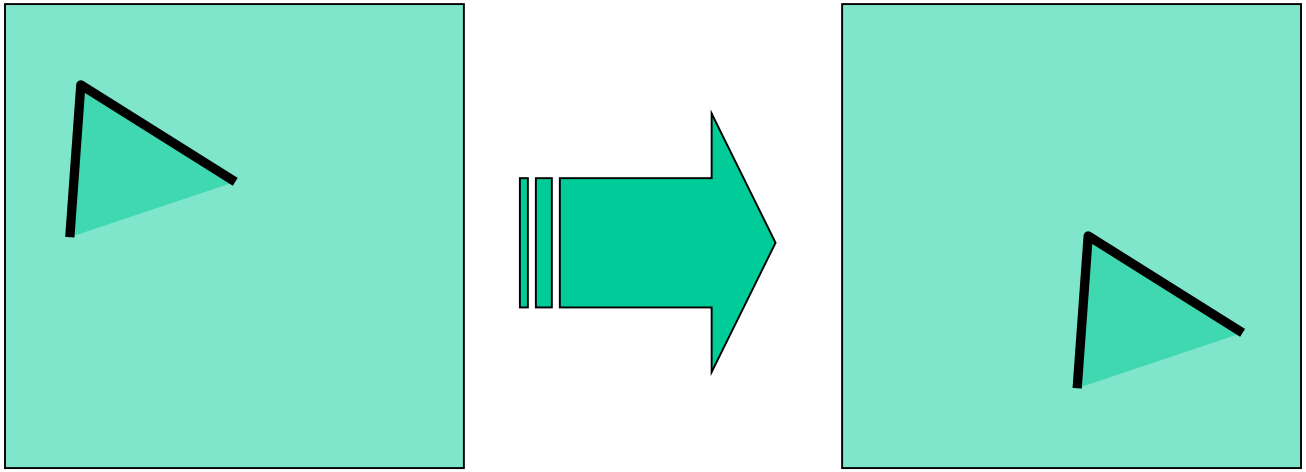
Rotation invariant? Yes



Properties of the Harris corner detector

Rotation invariant? Yes

Translation invariant? Yes

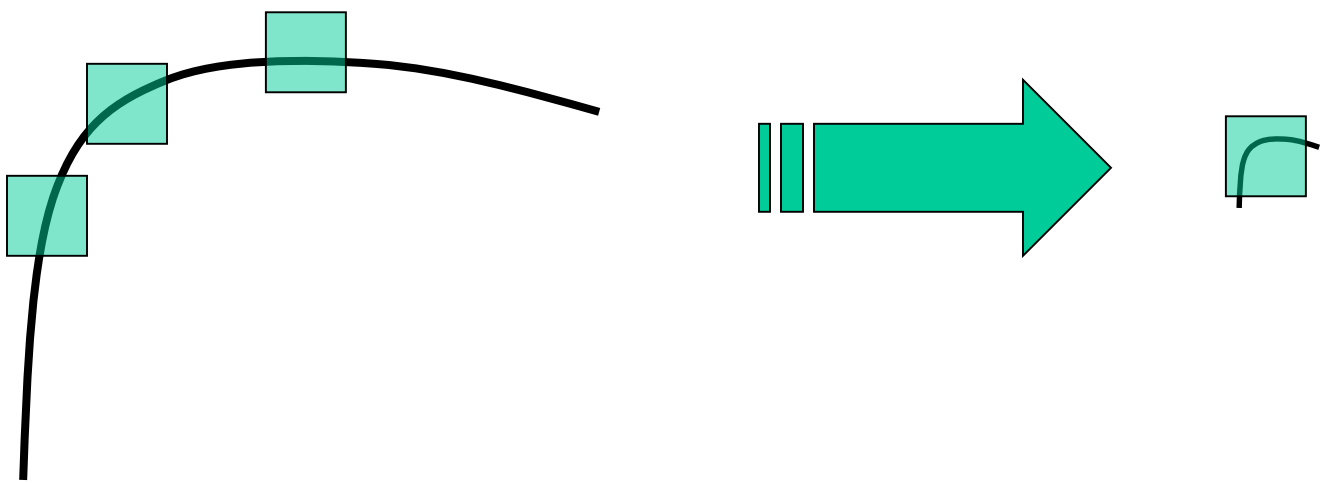


Properties of the Harris corner detector

Rotation invariant? Yes

Translation invariant? Yes

Scale invariant? No



All points will be classified as edges

Corner !

Summary

- Image warping to create mosaic, given homography

- Interest point detection
  - Harris corner detector
  - Next time:
    - Laplacian of Gaussian, automatic scale selection