Announcements

• PS1 due tomorrow, 11:59 pm
  – Write your name on the answer sheet
  – Must write and implement own solutions (list names of anyone you discussed with)

Recap so far:
Grouping and Fitting

Goal: move from array of pixel values (or filter outputs) to a collection of regions, objects, and shapes.
Grouping: Pixels vs. regions

By grouping pixels based on Gestalt-inspired attributes, we can map the pixels into a set of regions.

Each region is consistent according to the features and similarity metric we used to do the clustering.

Fitting: Edges vs. boundaries

Edges useful signal to indicate occluding boundaries, shape.

Here the raw edge output is not so bad…

…but quite often boundaries of interest are fragmented, and we have extra “clutter” edge points.

Given a model of interest, we can overcome some of the missing and noisy edges using fitting techniques.

With voting methods like the Hough transform, detected points vote on possible model parameters.
Voting with Hough transform

- Hough transform for fitting lines, circles, arbitrary shapes

\[ y = mx + b \]

\[ (x_0, y_0) \]

In all cases, we knew the explicit model to fit.

Today

- Fitting an *arbitrary* shape with “active” deformable contours

Deformable contours

a.k.a. active contours, snakes

*Given:* initial contour (model) near desired object

Slide credit: Kristen Grauman

Figure credit: Yuri Boykov
Deformable contours
a.k.a. active contours, snakes

**Given:** initial contour (model) near desired object

**Goal:** evolve the contour to fit exact object boundary

**Main idea:** elastic band is iteratively adjusted so as to
- be near image positions with high gradients, and
- satisfy shape "preferences" or contour priors

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Deformable contours: intuition

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Deformable contours vs. Hough

Like generalized Hough transform, useful for shape fitting; but

<table>
<thead>
<tr>
<th>Hough</th>
<th>Deformable contours</th>
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</thead>
<tbody>
<tr>
<td>Rigid model shape</td>
<td>Prior on shape types, but shape iteratively adjusted (deforms)</td>
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<tr>
<td>Single voting pass can detect multiple instances</td>
<td>Requires initialization nearby</td>
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<tr>
<td></td>
<td>One optimization “pass” to fit a single contour</td>
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</table>
Why do we want to fit deformable shapes?

- Some objects have similar basic form but some variety in the contour shape.

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- Non-rigid, deformable objects can change their shape over time, e.g. lips, hands...

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Why do we want to fit deformable shapes?

- Non-rigid, deformable objects can change their shape over time.

Aspects we need to consider

- Representation of the contours
- Defining the energy functions
  - External
  - Internal
- Minimizing the energy function
- Extensions:
  - Tracking
  - Interactive segmentation

Representation

- We’ll consider a discrete representation of the contour, consisting of a list of 2d point positions ("vertices").

\[ v_i = (x_i, y_i), \]

for \( i = 0, 1, \ldots, n-1 \)

- At each iteration, we’ll have the option to move each vertex to another nearby location ("state").
Fitting deformable contours

How should we adjust the current contour to form the new contour at each iteration?

- Define a cost function ("energy" function) that says how good a candidate configuration is.
- Seek next configuration that minimizes that cost function.

Energy function

The total energy (cost) of the current snake is defined as:

\[ E_{\text{total}} = E_{\text{internal}} + E_{\text{external}} \]

**Internal** energy: encourage *prior* shape preferences: e.g., smoothness, elasticity, particular known shape.

**External** energy ("image" energy): encourage contour to fit on places where image structures exist, e.g., edges.

A good fit between the current deformable contour and the target shape in the image will yield a **low** value for this cost function.

External energy: intuition

- Measure how well the curve matches the image data
- "Attract" the curve toward different image features – Edges, lines, texture gradient, etc.
External image energy

- How do edges affect “snap” of rubber band?
- Think of external energy from image as gravitational pull towards areas of high contrast

\[
G(I)^2 + G_x(I)^2 - (\nabla G)^2
\]

External image energy

- Gradient images \( G_x(x, y) \) and \( G_y(x, y) \)

- External energy at a point on the curve is:
\[
E_{\text{external}}(\nu) = -\left( |G_x(\nu)|^2 + |G_y(\nu)|^2 \right)
\]

- External energy for the whole curve:
\[
E_{\text{external}} = -\sum_{i,j \in \Omega} |G_x(x_i, y_i)|^2 + |G_y(x_j, y_j)|^2
\]

Internal energy: intuition

- What are the underlying boundaries in this fragmented edge image?
- And in this one?
A priori, we want to favor smooth shapes, contours with low curvature, contours similar to a known shape, etc. to balance what is actually observed (i.e., in the gradient image).

Internal energy: intuition

For a continuous curve, a common internal energy term is the "bending energy".

At some point $v(s)$ on the curve, this is:

$$E_{\text{internal}}(v(s)) = \alpha \frac{d^2 v}{ds^2} + \beta \left| \frac{d^2 v}{ds^2} \right|^2$$

• For our discrete representation, $v_i = (x_i, y_i)$, $i = 0, \ldots, n-1$

$$\frac{dv}{ds} = v_{i+1} - v_i$$

$$\frac{d^2 v}{ds^2} = (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - v_i - 2v_i + v_{i-1}$$

Main energy for our discrete curve is also position—not spatial image gradients.

$$E_{\text{internal}} = \sum_{i=0}^{n-1} \alpha \left[ v_{i+1} - v_i \right]^2 + \beta \left[ v_{i+1} - 2v_i + v_{i-1} \right]^2$$

Why do these reflect tension and curvature?
Example: compare curvature

\[ E_{\text{curvature}}(v_i) = \|v_{i+1} - 2v_i + v_{i-1}\|^2 \]

\[ = (x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2 \]

(2,5)

(2,2)

(1,1) (3,1) (1,1) (3,1)

\[ (3 - 2(2) + 1)^2 + (1 - 2(5) + 1)^2 \]

\[ = (-6)^2 = 36 \]

\[ (3 - 2(2) + 1)^2 + (1 - 2(2) + 1)^2 \]

\[ = (-2)^2 = 4 \]

Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

\[ E_{\text{elastic}} = \sum_{i=0}^{n-1} \alpha \|v_{i+1} - v_i\|^2 \]

\[ = \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 \]

What is the possible problem with this definition?

Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

\[ E_{\text{elastic}} = \sum_{i=0}^{n-1} \alpha \|v_{i+1} - v_i\|^2 \]

Instead:

\[ = \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - d^2 \]

where \( d \) is the average distance between pairs of points – updated at each iteration.
Dealing with missing data

• The preferences for low-curvature, smoothness help deal with missing data:

![Illusory contours found!](Figure from Kass et al. 1987)

Extending the internal energy: capture shape prior

• If object is some smooth variation on a known shape, we can use a term that will penalize deviation from that shape:

\[ E_{\text{internal}} = \alpha \sum_{i=0}^{n-1} (v_i - \hat{v}_i)^2 \]

where \( \{ \hat{v}_i \} \) are the points of the known shape.

Total energy: function of the weights

\[ E_{\text{total}} = E_{\text{internal}} + \gamma E_{\text{external}} \]

\[ E_{\text{external}} = -\sum_{i=0}^{n-1} |G_i(x_i, y_i)|^2 + |G_i(x_i, y_i)|^2 \]

\[ E_{\text{internal}} = \sum_{i=0}^{n-1} \alpha (d - \|v_i - v_i\|)^2 + \beta \|v_i - 2v_i + v_i\|^2 \]
Recap: deformable contour

- A simple elastic snake is defined by:
  - A set of $n$ points,
  - An internal energy term (tension, bending, plus optional shape prior)
  - An external energy term (gradient-based)

- To use to segment an object:
  - Initialize in the vicinity of the object
  - Modify the points to minimize the total energy

Energy minimization

- Several algorithms have been proposed to fit deformable contours.
- We’ll look at two:
  - Greedy search
  - Dynamic programming (for 2d snakes)
Energy minimization: greedy

- For each point, search window around it and move to where energy function is minimal
  - Typical window size, e.g., 3 x 3 pixels
- Stop when predefined number of points have not changed in last iteration, or after max number of iterations
- Note:
  - Convergence not guaranteed
  - Need decent initialization

Energy minimization

- Several algorithms have been proposed to fit deformable contours.
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Energy minimization: dynamic programming

With this form of the energy function, we can minimize using dynamic programming, with the Viterbi algorithm.

Iterate until optimal position for each point is the center of the box, i.e., the snake is optimal in the local search space constrained by boxes.
Energy minimization: dynamic programming

- Possible because snake energy can be rewritten as a sum of pair-wise interaction potentials:
  \[ E_{\text{total}}(v_1, \ldots, v_n) = \sum_{i=1}^{n-1} E_i(v_i, v_{i+1}) \]
  Or sum of triple-interaction potentials.
  \[ E_{\text{total}}(v_1, \ldots, v_n) = \sum_{i=1}^{n-1} E_i(v_{i-1}, v_i, v_{i+1}) \]

Snake energy: pair-wise interactions

\[ E_{\text{total}}(x_1, \ldots, x_n, y_1, \ldots, y_n) = -\sum_{i=1}^{n} \| G_i(x_i, y_i) \|^2 + \alpha \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 \]

Re-writing the above with \( v_i = (x_i, y_i) \):

\[ E_{\text{total}}(v_1, \ldots, v_n) = -\sum_{i=1}^{n} \| G_i(v_i) \|^2 + \alpha \sum_{i=1}^{n-1} \| v_{i+1} - v_i \|^2 \]

\[ E_{\text{total}}(v_1, \ldots, v_n) = E_1(v_1, v_2) + E_2(v_2, v_3) + \ldots + E_{n-1}(v_{n-1}, v_n) \]

where \( E_i(v_i, v_{i+1}) = -\| G_i(v_i) \|^2 + \alpha \| v_{i+1} - v_i \|^2 \)

Viterbi algorithm

Main idea: determine optimal position (state) of predecessor, for each possible position of self. Then backtrack from best state for last vertex.

\[ E_{\text{total}} = E_1(v_1, v_2) + E_2(v_2, v_3) + \ldots + E_{n-1}(v_{n-1}, v_n) \]

Complexity: \( O(nm^2) \) vs. brute force search?
With this form of the energy function, we can minimize using dynamic programming, with the Viterbi algorithm.

Iterate until optimal position for each point is the center of the box, i.e., the snake is optimal in the local search space constrained by boxes.

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DP can be applied to optimize an open ended snake

\[ E_1(v_1, v_2) + E_2(v_2, v_3) + \ldots + E_{n-1}(v_{n-1}, v_n) \]

For a closed snake, a "loop" is introduced into the total energy.

\[ E_1(v_1, v_2) + E_2(v_2, v_3) + \ldots + E_{n-1}(v_{n-1}, v_n) + E_n(v_n, v_1) \]

Work around:
1) Fix \( v_1 \) and solve for rest.
2) Fix an intermediate node at its position found in (1), solve for rest.

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- Extensions:
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  - Interactive segmentation
Tracking via deformable contours

1. Use final contour/model extracted at frame $t$ as an initial solution for frame $t+1$
2. Evolve initial contour to fit exact object boundary at frame $t+1$
3. Repeat, initializing with most recent frame.

Tracking Heart Ventricles (multiple frames)

Applications:

- Traffic monitoring
- Human-computer interaction
- Animation
- Surveillance
- Computer assisted diagnosis in medical imaging

Limitations

- May over-smooth the boundary
- Cannot follow topological changes of objects
Limitations

- External energy: snake does not really "see" object boundaries in the image unless it gets very close to it.

\[ \nabla \text{image gradients} \]

are large only directly on the boundary

Distance transform

- External image can instead be taken from the distance transform of the edge image.

Value at \((x, y)\) tells how far that position is from the nearest edge point (or other binary image structure).

Deformable contours: pros and cons

Pros:
- Useful to track and fit non-rigid shapes
- Contour remains connected
- Possible to fill in "subjective" contours
- Flexibility in how energy function is defined, weighted.

Cons:
- Must have decent initialization near true boundary, may get stuck in local minimum
- Parameters of energy function must be set well based on prior information
Summary

• Deformable shapes and active contours are useful for
  – Segmentation: fit or "snap" to boundary in image
  – Tracking: previous frame’s estimate serves to initialize the next
• Fitting active contours:
  – Define terms to encourage certain shapes, smoothness, low curvature, push/pulls, …
  – Use weights to control relative influence of each component cost
  – Can optimize 2d snakes with Viterbi algorithm.

Questions?

See you Thursday!