Simplified Master Theorem

A recurrence relation of the following form:

$$
\begin{aligned}
T(n) &= c \quad n < c_1 \\
&= aT(n/b) + \Theta(n^i), \quad n \geq c_1
\end{aligned}
$$

Has as its solution:

1) If $a > b^i$ then $T(n) = \Theta(n^{\log_b a})$ (Work is increasing as we go down the tree, so this is the number of leaves in the recursion tree).

2) If $a = b^i$ then $T(n) = \Theta(n^i \log_b n)$ (Work is the same at each level of the tree, so the work is the height, $\log_b n$, times work/level).

3) If $a < b^i$ then $T(n) = \Theta(n^i)$ (Work is going down as we go down the tree, so dominated by the initial work at the root).