Homework 1 – ECS 289, Winter 2016

Say a tile assembly system $\mathcal{T} = (T, \sigma, \tau)$ is *singly-seeded* if $|\text{dom } \sigma| = 1$.

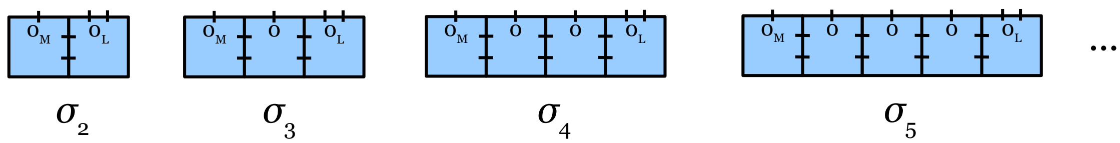

Figure 1: Double-notches represent strength-2 glues and single-notches represent strength-1 glues.

1. Design a tile set to "spell Γ." More precisely, for each $n \geq 2$, let σ_n be the assembly shown in Figure 1. Design a tile set T such that for all $n \geq 2$, the tile assembly system $\mathcal{T} = (T, \sigma_n, 2)$ produces the shape Γ_n shown in Figure 2.

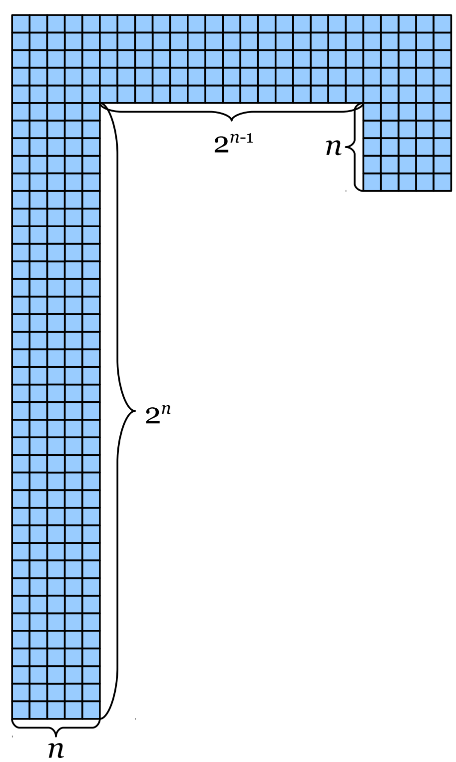

Figure 2: Γ_n shape.

2. Let $\mathcal{D} = \{D_0, D_1, D_2, D_3, \ldots\}$ be the infinite set of shapes shown in Figure 3, where for each $n \in \mathbb{N}$, $D_n = \left\{ (x,y) \in \mathbb{Z}^2 \;\middle|\; |x| + |y| \leq n \right\}$ is the set of points with Manhattan distance (a.k.a., L_1 distance) at most n from the origin (i.e., the "circle" of radius n in the L_1 metric, which looks like a diamond).

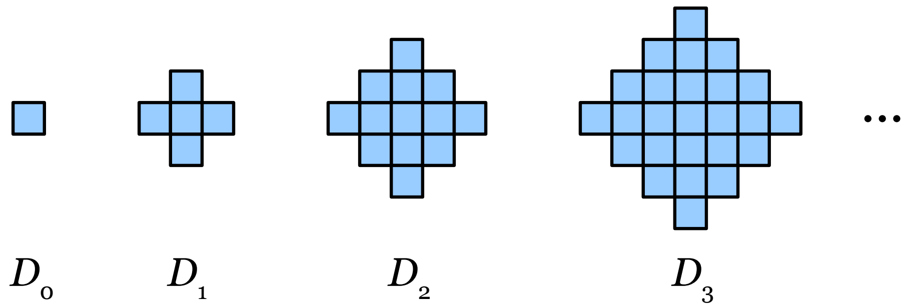

Figure 3: Infinite family of diamond shapes.

Design a singly-seeded tile assembly system $\mathcal{T} = (T, \sigma, \tau)$ such that, defining

$$A = \{ \; S_\alpha \;\mid \alpha \in \mathcal{A}_\Box[\mathcal{T}] \text{ and } |S_\alpha| < \infty \; \},$$

we have $|A| = \infty$ and $A \subseteq \mathcal{D}$. In other words, the set of shapes of finite terminal assemblies of $\mathcal{T}$ is an infinite subset of $\mathcal{D}$.

Make A the largest subset of $\mathcal{D}$ that you can. Explain the difficulty with trying to make it larger.

3. **(extra credit)** Show that for any tile assembly system $\mathcal{T}$ satisfying problem 2, there is an infinite terminal producible assembly $\alpha \in \mathcal{A}_\Box[\mathcal{T}]$.

4. Design a tile assembly system to simulate cellular automaton Rule 110:

   `https://en.wikipedia.org/wiki/Rule_110#Definition`

   Defining what exactly it means to "simulate" a cellular automaton is part of the problem. Explain the sense in which your tile system "simulates" Rule 110, and in particular, if there are any ways it could be improved that you couldn't figure out how to do. (Think of this part as practice for the art of stating open questions at the end of a research paper.)

5. Design a tile set so that, at temperature 2, if initialized with a horizontal, 1D seed assembly of length $\approx \log n$ representing a positive integer n in binary, computes successive applications of the *Collatz function* $f : \mathbb{N} \to \mathbb{N}$ defined for all $n \in \mathbb{N}$ by:

$$f(n) \begin{cases} 3n + 1, & \text{if } n \text{ is odd;} \\ n/2, & \text{if } n \text{ is even.} \end{cases}$$

   What this means is: if the seed row represents n, then after some amount of growth, eventually a row appears representing $f(n)$, then after some more growth, a row appears representing $f(f(n))$, and so on.

   The tile system should *stop* growing if ever it reaches the value 1.

6. Show that there is a finite shape $S \subset \mathbb{Z}^2$ so that every singly-seeded tile assembly system that strictly self-assembles S has more than one terminal assembly sequence. In other words, there will always be nondeterministic order of binding. Note that this is *different* from having more than one terminal assembly: a TAS can have multiple assembly sequences that all result in the same terminal assembly.

7. **(extra credit)** Characterize exactly the set of finite shapes that can be assembled by a singly-seeded tile assembly system with exactly one terminal assembly sequence.

8. Show that every finite shape can be strictly self-assembled by a singly-seeded tile assembly system $\mathcal{T} = (T, \sigma, \tau)$ such that every tile binds with strength exactly τ (in other words, there are never any binding events with *excessive* binding strength). Note that this refers only to initial binding; the tile may *later* have tiles attach adjacent to it so that its total bond strength with the rest of the assemble exceeds τ.

9. **(extra credit)** Show that there is a finite shape $S \subset \mathbb{Z}^2$ such that the *smallest* singly-seeded tile assembly system $\mathcal{T} = (T, \sigma, \tau)$ strictly self-assembling S permits tiles to bind with strength strictly larger than τ. (In other words, although it is possible to assemble S while guaranteeing no excessive strength binding, one can use fewer tile types if excessive strength binding is allowed.)

10. Show that the following definitions are all equivalent. A tile assembly system $\mathcal{T} = (T, \sigma, \tau)$ is *directed* if and only if

    (a) $|\mathcal{A}_\Box[\mathcal{T}]| = 1$.

    (b) For all $\alpha, \beta \in \mathcal{A}[\mathcal{T}]$, there exists $\gamma \in \mathcal{A}[\mathcal{T}]$ such that $\alpha \to^{\mathcal{T}} \gamma$ and $\beta \to^{\mathcal{T}} \gamma$.

    (c) For all $\alpha, \beta \in \mathcal{A}[\mathcal{T}]$ and $p \in S_\alpha \cap S_\beta$, $\alpha(p) = \beta(p)$.

11. Show that every set weakly self-assembled by a tile assembly system is computably enumerable.

12. **(extra credit)** Give an example of a computably enumerable set that is not weakly self-assembled by any tile assembly system.

13. Show that for any tile assembly system $\mathcal{T}$, the relation $\to^{\mathcal{T}}$ is transitive. In other words, for assemblies α, β, and γ, if $\alpha \to^{\mathcal{T}} \beta$ and $\beta \to^{\mathcal{T}} \gamma$, then $\alpha \to^{\mathcal{T}} \gamma$. (This is straightforward if $S_\beta \setminus S_\alpha$ is finite; the trick is to show it holds even if β is infinitely larger than α.)

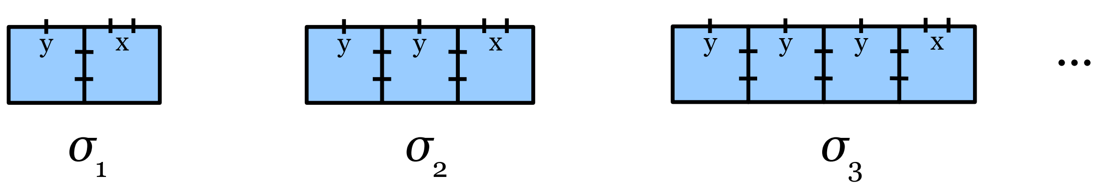

Figure 4: Infinite family of "floor" seeds. Double-notches represent strength-2 glues and single-notches represent strength-1 glues.

14. **(extra credit)** Let $\sigma_1, \sigma_2, \sigma_3, \ldots$ be a infinite sequence of assemblies of the form shown in Figure 4, where in each case, assume the leftmost tile is at the origin.

    Call the three tile types used t_1, t_2, t_3. For any tile set T and any $n \in \mathbb{Z}^+$, define the tile assembly system $\mathcal{T}_{T,n} = (T \cup \{t_1, t_2, t_3\}, \sigma_n, 2)$.

    Imagine the following goal: to design a single tile set T such that, for all $n \in \mathbb{Z}^+$, $\mathcal{T}_{T,n}$ is guaranteed to place a tile at position $(-1, 0)$ (i.e., just to the left of the leftmost seed tile); T is trying to figure out where the floor ends and place a tile there.

    Show that for any such tile set T, there exists $n \in \mathbb{Z}^+$ such that $\mathcal{A}_{\Box}[\mathcal{T}]$ has an infinite assembly. (In other words, there is no way to "search" for where the floor ends, "sense" it, and stop growing; the only way to guarantee a tile is placed at $(-1, 0)$ is to place infinitely many tiles.)