CS 222 HW 7 Due Thursday March 31.

The definition of $W*$ is in the lecture and videos (both the old video on sports elimination and the new video). In the old video W^* is discussed at the end of the lecture and in the new video, it is discussed at the start of the video.

Problems:

1. In the case of baseball, prove that W^* is the minimum number of games that any team must win in order not to be eliminated. In more detail, $W^*(i)$ is the minimum number of games that team i must win in order not to be eliminated. W^* is the minimum $W^*(i)$ over all teams.

2. Show how to compute W^* using network flow in the case of baseball. Is the computation done in polynomial time?

3. In the lecture on the problem of determining whether a team can be the undisputed winner under some scenario in baseball, we proved a structural result that shows how *all* such teams can be identified together. I claimed, in both the new video and in the lecture, that *all* such teams can be identified together with a *single* network, once W^* is known. Show how to do this. That is, show what the single flow problem is that identifies *all* such teams - give the details of the network and the capacities etc. and justify your answer.

4. In one of the lectures I showed that in a directed graph where each edge is given a weight and there are two designated nodes s and t, the problem of finding the set of edges with minimum total weight whose removal disrupts all paths from s to t can be solved by finding a minimum s, t cut. The proof of this had to go in two directions, and the conceptually hard part was that in our definition of an s, t cut, a cut is a partition of the *nodes*, while in this issue we are concerned with a set of edges.

Write up clearly and completely the proof, paying particular attention to the issue that the definition of an s, t cut is as a partition of nodes, while now we are concerned with a selection of a set of edges.