Simplification of Closed Triangulated Surfaces

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1 Introduction

Recently, data visualization has become increasingly important in several fields. Every year, the quality of imaging and computational simulation technology improves substantially. This results in an enormous increase in the amount of data produced. However, it has become apparent that, for many applications large parts of data sets are often not necessary for generating a "good picture". The goal was and still is to reduce data sets in such a way that the pictures generated from a reduced data set are highly similar to those produced from the original one.

We are concerned with polygonal surfaces and their compression. Furthermore, we focus on triangulated two-dimensional (2D) manifolds with no boundaries. For an extensive overview of the field of polygonal surface simplification, we refer to Heckbert and Garland [2] and Rossignac [4]. We present a randomized algorithm that preserves a specified error bound.

2 Related Work

When approximating a polygonal surface using the min-ϵ approach, one has to determine, for a given number n, an approximation that consists of n vertices and minimizes the approximation error. Many of the common algorithms use min-ϵ optimization, and several references are given in [2],[4]. Of special interest is Kreylos and Hamann [3], since they use a method closely related to the one presented here.

Using a min-# approximation approach, one tries to find an approximation with the minimal number of vertices that satisfies a tolerance condition [1]. Our algorithm ensures that no point of the approximating surface deviates more than ϵ from the original surface. This requires us to consider an "offset" around the original surface, and the approximation surface must stay inside this offset. Such an approach was first proposed by Cohen et al. [1] and was called a *simplifiction envelope*. A simplification envelope is a linearized and, in some respects, simplified version of the exact offset.

The simplification envelope of a triangulated surface is constructed in the following way: For each vertex, one computes its normal n as a combination of the normals of the surrounding triangles, normalized to length ϵ; one defines two offset vertices, the $(+\epsilon)$-offset and the $(-\epsilon)$-offset vertices, by adding/subtracting n to/from the original vertex. This defines a so-called *fundamental prism*.

Another problem is caused by self-intersections: Cohen et al. [1] require a simplification envelope that does not self-intersect. They use the global ϵ-value whenever possible and decrease it in areas of possible self-intersections. Our approach is not impacted by self-intersections and can handle every ϵ-value at any given vertex.

3 Atomic Envelopes

To satisfy an a-priori error bound, we define *atomic envelopes*. For each triangle, we construct an atomic envelope such that the simplification envelope equals the union of atomic envelopes. Our implementation uses fundamental prisms as atomic envelopes but different constructions are possible when higher accuracy is desired.

During simplification, we have to decide whether a triangle lies inside the simplification envelope. To answer this query we decide whether there exists a set of atomic envelopes whose union contains the triangle. First we find all atomic envelopes that might intersect the triangle. Then we search for points where the triangle might leave the envelope. Finally, these points are separated into critical and non-critical exit points. If a triangle has no critical exit points it lies inside the simplification envelope.

The same basic algorithm can also be used with more complicated atomic envelopes. For example, a construction that does not only use the vertex normal but also the normal of the triangle to create the atomic envelope. This new atomic envelope approximates the exact non-linear offset much better, especially in regions of high curvature.

4 Simplification

We simplify the given surface using a *simulated annealing* algorithm, also called *Metropolis algorithm*.

Simulated annealing models the state transition from fluid to crystalline state of metals. From the algorithmic point of view, this process is an optimization process of extremely high dimensionality. For our application, we interpret the configuration of a polygonal surface as the configuration of metal molecules. Our internal energy is represented by a target function, and the random heat movement of molecules is represented by random changes in the configuration.

The target function describes the quality of an approximation. Furthermore, the target function should not only prefer configurations that consist of few vertices but also configurations that lead to vertex removals. We use the sum of the square roots of the angles between triangle normals as the target function. This function is highly related to the number of vertices. It also prefers planar surfaces, since a large number of small angles has a higher target function value than a smaller number of large angles. This leads to near-planar platelets of triangles, where we can delete vertices. This target function is also easy to compute and can be recomputed locally after local changes.

To change a configuration, we use the method of Kreylos and Hamann [3], adapted to our problem. We use three different operations: edge rotation, vertex removal, and vertex movement. The edge rotation only changes the triangulation of two neighboring triangles. To move a vertex we randomly choose a new position inside a small sphere around the original one. To check the validity of the resulting triangulation we project all involved triangles onto the plane defined by the vertex normal of the changing vertex. This can lead to degenerate triangles or triangles with wrong orientations. We resolve these conflicts by swapping the appropriate edges. To remove a vertex we collapse the shortest edge emanating from it.

5 Future Research

The main drawback of our algorithm is its lack of computational efficiency. The simulated annealing is especially expensive. However, it provides a means to use any point inside the envelope as a possible vertex position. Future work will be done to replace the simulated annealing approach, while keeping this advantage.

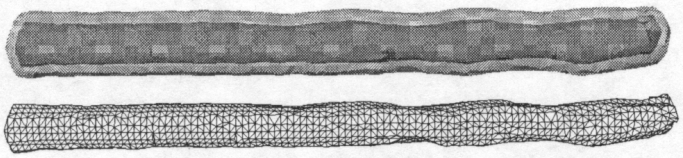

Figure 1: Drill bit data set original (1964 vertices)

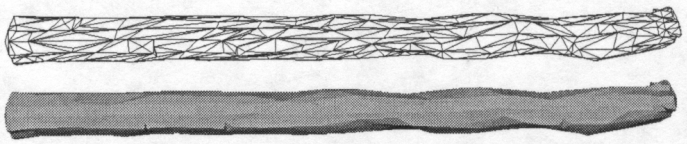

Figure 2: Drill bit data set simplified using 1/2% error bound (430 vertices)

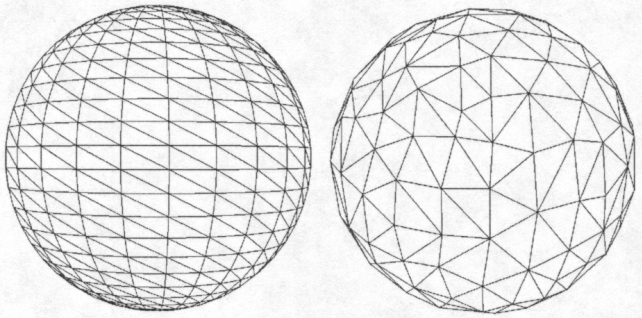

Figure 3: Sphere data set original (602 vertices) and simplified using 1/2% error bound (200 vertices)

References

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[4] J. Rossignac. Interactive Exploration of Distributed 3D Databases over the Internet. *CGI Proceedings* pp. 324-335, 1998.