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Interactive Construction of B–Spline Approximations of Surfaces and a Tesselation Algorithm for the Representation of Trimmed Surfaces for Use in Numerical Grid Generation

Grid generation is concerned with discretizing surfaces and surrounding volumes in three–dimensional space. Often, the originally given surfaces contain errors, such as gaps between surfaces, intersecting surfaces, or overlapping surfaces. A method is presented for the correction of such errors. The method replaces a faulty, discontinuous geometry by a new set of continuous surface patches.

The second part of this paper deals with the representation of trimmed surfaces, i.e., surfaces with holes. Many applications, including surface rendering and grid generation, require an approximation of the complement of the part of a trimmed surface that is "cut out" by the holes. An approach for decomposing the valid part of a trimmed surface into a set of four–sided surfaces (without holes) is discussed.

1. Constructing B–spline approximations for geometries with discontinuities

In most computer–aided design (CAD) systems three–dimensional geometries are represented by parametric surfaces, *e.g.*, Bézier, B–spline, or non–uniform rational B–spline (NURBS) surfaces. Often, the geometries created by a design engineer are incomplete or are faulty, *i.e.*, they contain discontinuities. Examples thereof are surface patches that do not connect properly or patches that intersect each other. Such data are unacceptable for further processing, in particular grid generation (see [10]), flow simulation, and manufacturing.

The technique used for removing "gaps"/"holes," "overlaps," and transverse surface intersections is based on the idea of projecting local surface approximations onto the given, faulty geometry. The overall approximation process of an entire geometry relies on various geometric modeling and approximation techniques (see [1] and [8]). The technique is based on this sequence of steps:

- Definition of patch *boundary curves* (for generation of Coons patches)
- Construction of bilinear *Coons patches* (local surface approximations)
- Perpendicular *projection* of points on Coons patches onto original surface patches
- Approximation of *"artificial projections"* if projections cannot be found for certain points on Coons patches
- Computation of *B–spline surfaces* by interpolating projections and "artificial projections"
- *Error computation* based on distance between original surface patches and approximating B–spline surfaces
- Generation of *overall geometry approximation* by uniting all local B–spline approximations

"Artificial projections," *i.e.*, points that correspond to points on Coons patches for which no projection on the original patches can be found, are computed using a bivariate scattered data interpolation scheme applied to the projections that can be found (*Hardy's multiquadric method*, see [2]). The error estimate considers shortest distances between points on the approximating B–spline surfaces and the original surface patches. Due to the fact that all local surface approximations (the B–spline surfaces) are constructed independently, the control information of B–spline surfaces sharing boundary curves must be adjusted in order to ensure continuity of the overall geometry approximation. The method is described in detail in [3]. Fig. 1 (left) shows an example for the approximation of an entire car body (top: given geometry with gaps; bottom: continuous approximation).

2. A tesselation algorithm for the representation of trimmed surfaces

The tesselation (tiling) algorithm discussed is concerned with the representation of trimmed parametric surfaces by sets of approximating surfaces without trimming curves. Trimmed surfaces are the result of a surface–surface intersection algorithm applied to multiple, intersecting surfaces. Complex real–world geometries typically contain thousands of trimmed surfaces. The algorithm presented assumes that all intersection curves are given in parameter space (see [1]). Trimmed surfaces cause problems when exchanging data among different CAD systems. The method represents the valid part of the parameter space (*i.e.*, the part that remains when disregarding all holes) by a set of *basic parametric surfaces* (*i.e.*, surfaces without trimming curves, see [11]). The representation by *basic surfaces* simplifies the grid generation process, which is extremely important for many applications.

The approach utilizes a generalized Voronoi diagram for the tesselation of the valid part of a trimmed surface in parameter space (see [5], [6], [7], and [9]). This paper presents a method for constructing the boundary (bisecting) curves of the tiles around each trimming curve, defining a general Voronoi diagram. The tile boundary curves are obtained from the perpendicular bisectors of all possible point pairs lying on different trimming curves. Once the tile boundary curves around each trimming curve are generated, they are used to subdivide the tiles into four-sided surfaces (in parameter space) whose union represents the valid part of the trimmed surface. These four-sided surfaces are constructed by splitting the tile boundary curves into several segments and connecting the end points of these segments with *significant points* on the trimming curves. *Significant points* on a trimming curve (in parameter space) are end points of horizontal line segments, local extrema in the second parameter direction, and points where slope discontinuities occur. Segments of the *medial axis* of the tiles can be used as boundary curves for the four-sided surfaces as well (see [4] for a more detailed description of the overall algorithm). Fig. 1 (right) shows an example of the decomposition of a trimmed surface with multiple trimming curves.

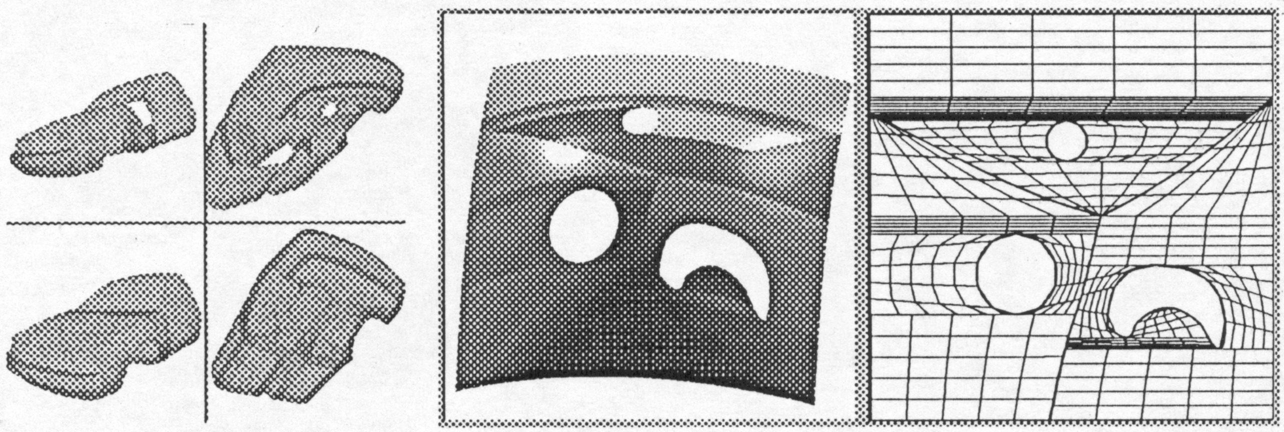

Fig. 1. B-spline approximation of car body (left) and decomposed trimmed surface (right).

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3. References

1 FARIN, G.: Curves and Surfaces for Computer Aided Geometric Design, Third Edition, Academic Press, San Diego 1993.
2 FRANKE, R.: Scattered data interpolation: Tests of some methods; Math. Comp. **38** (1982), 181–200.
3 HAMANN, B., JEAN, B. A.: Interactive surface correction based on a local approximation scheme; Computer Aided Geometric Design **12** (1995), to appear.
4 HAMANN, B., TSAI, P.-Y.: A tesselation algorithm for the representation of trimmed surfaces with arbitrary trimming curves; Computer-Aided Design **27** (1995), to appear.
5 KLEIN, R., MEHLHORN, K., MEISER, S.: Randomized incremental construction of abstract Voronoi diagrams, Computational Geometry: Theory and Applications **3** (1993), 157–184.
6 LEE, D. T., DRYSDALE, R. L.: Generalization of Voronoi diagrams in the plane, SIAM Journal on Computing **10** (1981), 73–87.
7 LEVEN, D., SHARIR, M.: Intersection and proximity problems and Voronoi diagrams, in Advances in Robotics, Vol. I, Schwartz, J., Yap, C. K., eds., Lawrence Erlbaum, London 1986, 187–228.
8 PIEGL, L. A.: Rational B-spline curves and surfaces for CAD and graphics; in State of the Art in Computer Graphics, Rogers, D. F., Earnshaw, R. A., eds., Springer-Verlag, New York 1991, 225–269.
9 SRINIVASAN, V., NACKMAN, L. R., TANG, J. M., MESHKAT, S. N.: Automatic mesh generation using the symmetric axis transformation of polygonal domains, Proceedings of the IEEE **80** (1992), 1485–1501.
10 THOMPSON, J. F., WARSI, Z. U. A., MASTIN, C. W.: Numerical Grid Generation, North-Holland, New York 1985.
11 VRIES-BAAYENS, A. E., SEEBREGTS, C. H.: Exact conversion of a trimmed nonrational Bézier surface into composite or basic nonrational Bézier surfaces, in Topics in Surface Modeling, Hagen, H., ed., SIAM, Philadelphia 1992, 115–143.

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