Topographic Distance Functions for Interpolation of Meteorological Data

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Abstract: In agriculture and water resource management an important meteorological quantity is the reference evapotranspiration ET_0. It is usually estimated from other meteorological quantities measured at weather stations. To estimate ET_0 at an arbitrary geographical position these quantities have to be interpolated. The Center for Spatial Technologies And Remote Sensing (CSTARS) at UC Davis uses the DayMet approach for this task.

We discuss some inconsistencies within the DayMet approach and suggest improvements. One of the problems of DayMet is the lack of consideration of terrain topography. We define new distance functions that are elevation-dependent and show preliminary results of the comparison of the classic and the improved DayMet approach.

1 Introduction

To set up near-optimal irrigation schedules the amount of water that has vanished into the atmosphere during the day has to be known. This is important to farmers that have to replace this amount of water and to administrators of water management systems so they can provide adequate water supplies for agricultural and urban needs. Also for setting up a land use plan it can be a base for decisions.

One important quantity is the evapotranspiration that is the combination of evaporation, i.e., the loss of water from the surface of the plants, and the soil and the transpiration, i.e., the loss of water from inside the plants to the atmosphere. This quantity depends on several factors, such as weather variables, soil conditions, and type of vegetation.

For determining the evapotranspiration ET for a certain region, there is a reference evapotranspiration ET_0 defined. This is defined to be the evapotranspiration above a defined reference vegetation (uniform closely-cropped grass), and therefore only depends on the weather conditions. The evapotranspiration ET_c for a specific vegetation or surface type is

$$ET_c = K_c \cdot ET_0, \tag{1}$$

where K_c is the crop coefficient and can be determined from a table.

To estimate ET_0 for every place in California the California Department of Water Resource and the University of California, Davis developed the CIMIS project (California Irrigation Management Information System). They established about 120 automated weather stations all over California, each of which measures several climate values (such as solar radiation, relative humidity, wind speed, temperature) under defined reference conditions (2m above a dense grass surface). This data is collected and stored in a database. From these measured values ET_0 can be estimated.

This approach leads to estimated values for ET_0 only for the locations of the CIMIS weather stations. For every other place, the weather values have to be estimated by combining the measured values of nearby weather stations (interpolation), and then ET_0 can be estimated from these.

In the CIMIS project, a map is created that contains the estimated value for every point on a dense grid with grid distance of 2km. To find estimates for each grid point, two different methods are used: Some of the weather values are interpolated using regularized splines with tension ([MM93], [MH93], [HPMM02]), for others the DayMet interpolation method ([TRW97]) is used.

We focus on the DayMet interpolation approach. In Section 2, we give a formal definition of this approach and show some of its deficits. In Section 3, we suggest some improvements and present some first results in Section 4. Section 5 contains a list of further research that can be done in this field.

2 The DayMet interpolation method

We provide an overview of the DayMet approach in Section 2.1, give a short overview of its implementation within the CIMIS project in Section 2.2, and point out some weaknesses of the approach and the implementation in Section 2.3.

2.1 Definition

As input to the DayMet interpolation we have n weather stations W_i $(i = 1, \ldots, n)$ corresponding to two-dimensional observation points $p_i \in \mathbb{R}^2$ on a planar map, elevation $z_i \in \mathbb{R}$ and the associated weather data $f_i \in \mathbb{R}$. Examples for possible weather data are temperature, solar radiation, precipitation, humidity, or wind speed, as measured at W_i.

To interpolate the value at an arbitrary query point Q with two-dimensional coordinates $q \in \mathbb{R}^2$, we define a weight function as a truncated Gaussian filter,

$$w(q,r) = \begin{cases} 0; & r > R(q) \\ \exp\left(-\left(\frac{r}{R(q)}\right)^2 \alpha\right) - e^{-\alpha}; & r \leq R(q) \end{cases}, \tag{2}$$

where r is the radial distance arround q, $R(q)$ is the truncation distance of q, and α is a unitless shape parameter.

We define the weights of the weather station W_i at a query point Q as

$$w_{q,i} = w(q, \|q - p_i\|_2). \tag{3}$$

If the truncation distance were constant, there would be a large number of observation points with non-zero weight at dense regions, whereas in regions with a sparse number of observation points all weights could be zero. Therefore $R(q)$ depends on the local density of weather stations arround q, and an iterative approach is used to find a value for $R(q)$:

1. Start with $R(q) = R$ with R a user-specified value.

2. Use $R(q)$ to calculate the weights $w_{q,i}$ of all W_i ($i = 1, \ldots, n$) using Equation (2), and calculate the local station density $D(q)$ (number of stations / area) as

$$D(q) = \frac{\sum_{i=1}^{n} \frac{w_{q,i}}{\overline{w}}}{\pi R(q)^2}, \tag{4}$$

   where $\overline{w}$ is the average weight over the untruncated region of the kernel, defined as

$$\overline{w} = \frac{\int_0^{R(q)} w(q,r)\mathrm{d}r}{\pi R(q)^2} = \left(\frac{1 - e^{-\alpha}}{\alpha}\right) - e^{-\alpha}. \tag{5}$$

3. With a user-specified desired average number of observations N and the calculated value of $D(q)$, we can calculate a new value for $R(q)$ as

$$R(q) = \sqrt{\frac{\hat{N}}{D(q)\pi}}, \tag{6}$$

   where $\hat{N} = 2N$ is chosen for every iteration except the last one, for which $\hat{N} = N$.

4. Perform I (I being user-specified) iterations of step 2. and 3. to get the final value of $R(q)$.

The value $f(q)$ at the arbitrary query point Q at two-dimensional coordinates $q \in \mathbb{R}^2$ is now estimated as

$$f(q) = \frac{\sum_{i=1}^{n} w_{q,i} f_i}{\sum_{i=1}^{n} w_{q,i}}. \tag{7}$$

For temperature data there exists a relationship between elevation and temperature. In [TRW97] the use of a correction term to take elevation into account is suggested. First, one estimates regression coefficients β_0 and β_1 that describe the correlation between elevation z and temperature t in absence of any other meteorological effect

$$t = \beta_0 + \beta_1 z. \tag{8}$$

To calculate the values for β_0 and β_1, a weighted least squares regression is used on every pair of observation points (W_i, W_j), weighted by the product of the interpolation weights $w(p_i, \|p_i - p_j\|_2)w(p_j, \|p_i - p_j\|_2)$ of one to the other. But instead of calculating the regression directly as in Equation (8), it was suggested to do this regression for the differences of temperature $(t_i - t_j)$ and elevation $(z_i - z_j)$

$$(t_i - t_j) = \beta_0 + \beta_1(z_i - z_j). \tag{9}$$

With temperatures $t_i = f_i$ $(i = 1, \ldots, n)$ and the estimated values of β_0 and β_1, the temperature $t(q) = f(q)$ for a query point Q at two-dimenisional coordinates $q \in \mathbb{R}^2$ with elevation $z \in \mathbb{R}$ is now calculated as

$$t(q) = \frac{\sum\limits_{i=1}^{n} w_{q,i}\left[t_i + \beta_0 + \beta_1(z - z_i)\right]}{\sum\limits_{i=1}^{n} w_{q,i}}. \tag{10}$$

2.2 Implementation of DayMet within the CIMIS project

Within the CIMIS project, the DayMet interpolation method is implemented as a GRASS module. For temperature interpolation, an elevation map of California is used. The module reads a sites file with the values of the weather stations, including exact positions, and interpolates the value for every point on a regular grid of 500×550 points. The grid distance is 2km. The resulting interpolated values are written to the GRASS database as a raster file.

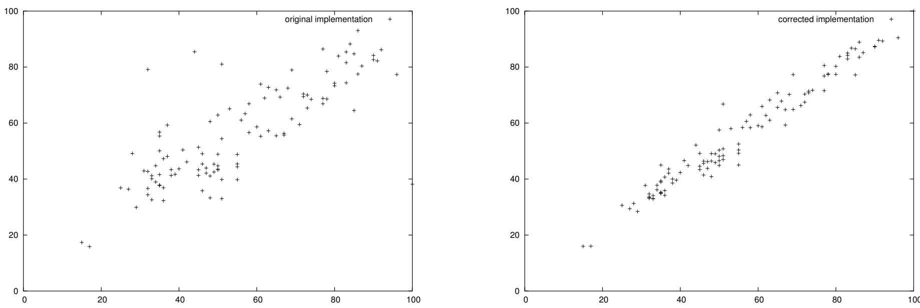
To find suitable values for the free parameters I (number of iterations for calculating $R(q)$), N (desired average number of observation points) and α (shape parameter for weight $w(q, r)$), a range for each of the three parameters is specified and every combination of values is checked via cross validation: For every observation point p_i the interpolation $f(p_i)$ is calculated, using only the $(n - 1)$ other observation points $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$. The cross validation root-mean-square error (RMSE) is

$$E_{\text{RMSE}} = \sqrt{\sum_{i=1}^{n}(f(p_i) - f_i)^2}. \tag{11}$$

The combination of values for I, N and α producing the least error E_{RMSE} is used for the interpolation procedure.

2.3 Deficits of the CIMIS DayMet implementation

In the CIMIS implementation the valid ranges for I and N were interchanged: The ranges were set to $I \in \{3, \ldots, 5\}$ and $N \in \{30, \ldots, 50\}$. With these ranges the maximum number of iterations I to calculate $R(q)$ was five, too few iterations to make the calculation converge. On the other hand, the minimum number of average observations N that are taken into account was 30 and therefore too high. Figure 1(a) shows the correspondence between the measured and the interpolated values at the positions of the observation points. They are nearly unrelated. After interchanging the ranges to $I \in \{30, \ldots, 50\}$ and $N \in \{3, \ldots, 5\}$ the results were better, see Figure 1(b).



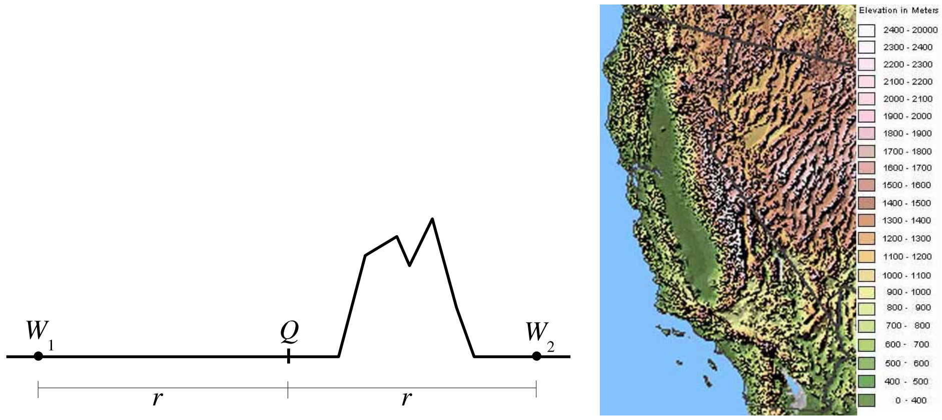
(a) Original CIMIS implementation. (b) Corrected ranges of I and N.

Figure 1: Measured value (horizontal axis) against interpolated value (vertical axis) at observation points.

Although being called an "interpolation method" in [TRW97] the DayMet approach is not an interpolation, but only an approximation of scattered data. Calculating the value $f(p_i)$ at an observation point p_i results in a value close to f_i, but in general does not reproduce f_i exactly. This can be seen in Equation (7), since there are in general several non-zero weights w_j, so that $f(p_i)$ does not only depend on f_i but also on other observation values. If it were an interpolation method, the points of Figure 1(b) would all lie on the line $y = x$.

There is another shortcoming of the method in [TRW97] related to using the regression Equation (9) for calculating the values of β_0 and β_1. When substracting two instances of the original Equation (8) $t_i = \beta_0 + \beta_1 z_i$ and $t_j = \beta_0 + \beta_1 z_j$, the absolute term β_0 vanishes, resulting in $t_i - t_j = \beta_1(z_i - z_j)$. From this equation only β_1 can be estimated by a least squares regression. With the argument of symmetry one can also conclude that $\beta_0 = 0$, because the indices of the weather stations are artificial. Having two weather stations, either of them can be (z_i, t_i) or (z_j, t_j). Only $\beta_0 = 0$ can then fulfill Equation (9).

Another drawback of the DayMet approach is the way the weights in Equation (2) are calculated: The distance r only takes the x- and y-coordinate of the query position q and the weather station position p_i into account. For temperature interpolation, also the elevation of z and z_i influences the result, see Equation (10). But the topographic structure of the terrain between q and p_i does not play any role. (Think of a terrain with a cross section as in Figure 2(a), build of a plane adjacent to a mountain. To interpolate the value at the query point Q with two-dimensional coordinates q the weights for the weather stations W_1 and W_2 have to be calculated. Since their radial distances $r = \|q - p_1\|_2 = \|q - p_2\|_2$ from Q are the same, they have the same weight $w_1 = w(q, r) = w_2$ from Equation (2). Obviously the influence of W_2 is lower than the influence of W_1 since the mountain divides the terrain into two different regions and inhibits air exchange across it. Therefore, the interpolation weight w_1 should be bigger than w_2. Since California has a diverse topographic structure containing high mountains and large flat valleys, see Figure 2(b), these constellations are common.)



(a) A plane adjacent to a mountain. (b) Map of California.

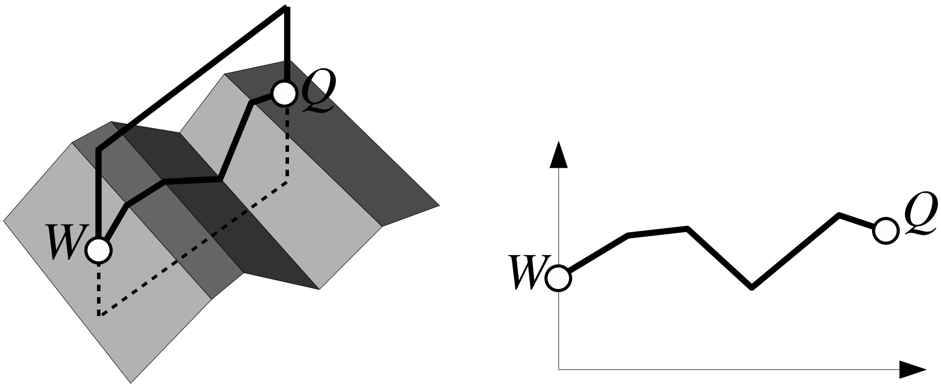
Figure 2: DayMet neglects the topographic structure of the terrain.

In the following, we introduce a way to take the topographic structure of the terrain into account to improve interpolation quality.

3 Improvement of DayMet

To take the topographic structure of the terrain into account we keep the general concept of DayMet interpolation, but change the way the distance r in Equation (2) is calculated, so that the terrain elevation influences the weights. If on the path from the weather station W to the query point Q a mountain has to be crossed, the distance should be larger, resulting in a smaller weight and therefore in a smaller influence on the overall result.

We only take the direct path from W to Q into account, i.e., we calculate the intersection of the terrain surface with a plane that contains W and Q and contains the ray from W to the center of the Earth as illustrated in Figure 3(a). This intersection is a planar curve representing the profile of the direct path from W to Q as illustrated in Figure 3(b). This profile is a function $P : [0, S] \to \mathbb{R}$, returning for every (horizontal) position s on the path from W to Q the elevation at that point. The distance r from W to Q is calculated by



(a) Intersection of terrain and plane. (b) Terrain profile between W and Q.

Figure 3: Direct path from W to Q.

using a function $d : ([0, S] \to \mathbb{R}) \to \mathbb{R}^+$ that uses the profile as input and returns r. We call such a function a *distance function*.

One can think of the original DayMet definition as the distance function d_{xy} with the property that for an arbitrary profile $P : [0, S] \to \mathbb{R}$ we have $d_{xy}(P) = S$. Thus, d_{xy} does not take the profile into account, but just returns the horizontal distance S between W and Q. We now define two different distance functions that do take the profile into account.

A mountain ridge with a height of 1000m has a larger impact on the influence of a weather station than a horizontal distance of 1000m has. Thus, the distance in vertical direction must be amplified to make it comparable to horizontal distances. For this reason, we introduce an exaggeration factor z_{exag}. We define the exaggerated profile $\hat{P}$ as

$$\hat{P}(s) = z_{\text{exag}} P(s). \tag{12}$$

3.1 Arc length of convex hull: d_{ch}

The first distance function returns as the distance of a profile the length of the shortest path through the air from the start to the end point. More formally speaking, this shortest path is the upper convex hull of the profile and the distance is its arc length.

Let $\hat{P}(s) : [0, S] \to \mathbb{R}$ be an exeggerated profile as defined above. Its *upper convex hull* is the function $\hat{P}_{ch}(s) : [0, S] \to \mathbb{R}$ that fulfills the following three conditions:

1. $\forall s \in [0, S] : \hat{P}_{ch}(s) \geq \hat{P}(s)$.
2. $\forall s_1, s_2 \in [0, S], s_1 < s_2 : \hat{P}'_{ch}(s_1) \geq \hat{P}'_{ch}(s_2)$.
3. $\forall \tilde{P} : [0, S] \to \mathbb{R}$ fulfilling condition 1 and 2, $s \in [0, S] : \hat{P}_{ch}(s) \leq \tilde{P}(s)$.

While the first condition ensures that our path is always above ground, the second (monotonic decreasing derivative) ensures that the path does not have unnecessary waves, and the third ensures that the path is as low above ground as possible. The three conditions together ensure that $\hat{P}_{ch}$ is the shortest path through the air from one end to the other.

The distance $d_{ch}(P)$ is now the arc length of that shortest path,

$$d_{ch}(P) = \int_0^T \sqrt{1 + (\hat{P}'_{ch})^2}.$$

Figure 4(a) demonstrates how the distance d_{ch} for a profile is calculated.

While crossing a mountain increases the distance reported by d_{ch}, crossing a canyon has no effect.

3.2 Radial distance plus highest peak: d_p

The second distance function d_p we define determines the highest peak that has to be crossed and adds this height to the radial distance between start and end point.

We define two slightly different functions d_{p1} and d_{p2}, differing in the base to which the height of the peak is measured. Let $P(s) : [0, S] \to \mathbb{R}$ be a profile, then

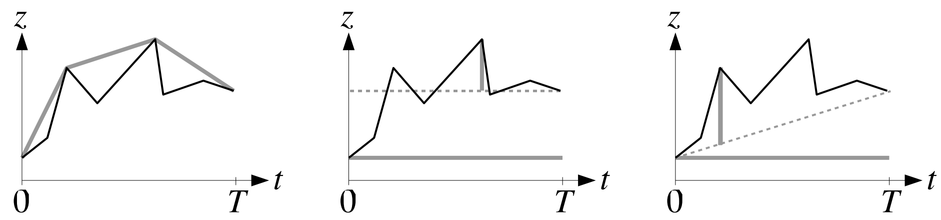
$$\begin{aligned} d_{p1}(P) =& S + \max_{s\in[0,S]} (\hat{P}(s) - \max\{\hat{P}(0), \hat{P}(S)\}) \\ =& S + z_{\text{exag}} \max_{s\in[0,S]} (P(s) - \max\{P(0), P(S)\}), \end{aligned} \tag{13}$$

and

$$\begin{aligned} d_{p2}(p) =& S + \max_{s\in[0,S]} \left(\hat{P}(s) - \left(\frac{s}{S}\hat{P}(0) + \frac{S-s}{S}\hat{P}(S) \right) \right) \\ =& S + z_{\text{exag}} \max_{s\in[0,S]} \left(P(s) - \left(\frac{s}{S}P(0) + \frac{S-s}{S}P(S) \right) \right). \end{aligned} \tag{14}$$

While d_{p1} takes the height of the highest peak relative to the higher of the start and the end point, d_{p2} takes it relative to the linear interpolation between the start and the end point. Figures 4(b) and 4(c) show examples of how the distance with these two distance functions is calculated.

It can be seen that $\forall P : [0, S] \to \mathbb{R} : d_{xy}(P) \leq d_{p1}(P) \leq d_{p2}(P)$ and $d_{xy}(P) \leq d_{ch}(P)$. It depends on the actual profile how $d_{ch}(P)$ compares to $d_{p1}(P)$ and $d_{p2}(P)$.



(a) d_{ch} (b) d_{p1} (c) d_{p2}

Figure 4: Three new distance functions. The black line is the profile, the (sum of the) length of thick gray line(s) is its distance. The dashed line is the base for measuring the height of the peaks for d_{p1} and d_{p2}.

4 Preliminary results

To compare the results of our distance functions with that used in the original DayMet implementation of CIMIS, we used a data set of relative humidity (values in the range from zero to 100) of the 108 weather stations that were measuring data that day. We used crossvalidation as described in Section 2.2.

Table 1 shows the overall results for E_{RMSE} as defined in Equation (11) for the different distance functions and different values of z_{exag}. These results do not show an advantage of the new distance functions when compared to the original implementation d_{xy}. Only d_{p2} with $z_{\text{exag}} = 50$ shows a slightly improved result, but this might also be random.

z_{exag}	d_{xy}	d_{ch}	d_{p1}	d_{p2}
5		12.98	12.95	12.99
50	12.96	13.09	13.32	12.64
500		13.03	13.21	12.88

Table 1: E_{RMSE} values for the different distance functions.

To understand in more detail the errors for specific structures, we searched for each distance function at what weather station it had the maximal improvement over the original implementation and at what it had the most degradation. These experiments were done with $z_{\text{exag}} = 50$. Figure 5(a) provides an overview of the positions of the three weather stations W_{88}, W_{113}, and W_{35} we studied in detail.

The first weather station we considered was W_{88}. Figure 5(b) shows its neighbourhood, Table 2 lists the relative interpolation weights. While d_{xy} and d_{p1} produced poor results, d_{ch} and d_{p2} have reasonable values. This is the situation we wanted to improve: a mountain ridge divides the terrain into two parts, the dry northern part (W_{54}, W_{146}, W_5, W_{138}, and W_{125}) and the humid southern part (W_{64}, W_{94}, and W_{107}). W_{88} belongs to the northern part, and therefore the southern weather stations should not have any influence on it. Note that d_{p1} produced the same result as d_{xy} since W_{88} is near a mountain top, so any profile ending in W_{88} has W_{88} as highest peak. Therefore, d_{xy} and d_{p1} produce the same distances.

Weather station	W_5	W_{64}	W_{94}	W_{107}	W_{125}	W_{138}	W_{146}	W_{88}
measured rel. hum.	55.0	69.0	86.0	87.0	35.0	45.0	46.0	32.0
d_{xy}	.05	.24	.27	.28			.16	74.1
d_{ch}	.28				.22	.15	.35	45.9
d_{p1}	.05	.25	.27	.27			.16	74.1
d_{p2}	.29	.13			.17	.13	.28	49.5

Table 2: Weights used for interpolation at weather station W_{88} (d_{ch} and d_{p2} having their best results there). The last column contains the measured value and the predicted values using the different distance functions at position of W_{88}.

d_{p1} has its best result for W_{113}, see Figure 5(c) for the neighbourhood and Table 3 for the interpolation weights. At first sight it seems we achived the opposite of what we wanted to do: d_{p1} used W_{124} and W_{190} that are hidden behind a mountain. But it also used W_{163} with a higher weight than the other functions, which is the station at the other end of the long, small valley, resulting in better results.

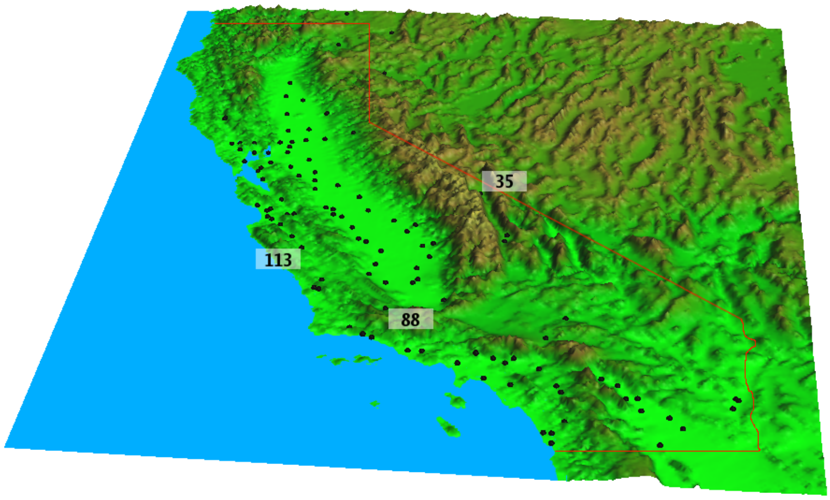
Weather station	W_{89}	W_{105}	W_{114}	W_{116}	W_{126}	W_{143}	W_{163}	W_{190}	others	W_{113}
measured rel. hum.	83.0	32.0	80.0	91.0	61.0	72.0	51.0	32.0		65.0
d_{xy}	.18		.77				.02	.03		78.7
d_{ch}	.25		.64	.07			.04			80.3
d_{p1}	.16	.05	.45	.05	.05	.05	.06	.07	.06	70.0
d_{p2}	.26		.66	.06			.02			80.8

Table 3: Weights used for interpolation at weather station W_{113} (d_{p1} having its best result there). The last column contains the measured value and the predicted values using the different distance functions at the position of W_{113}.

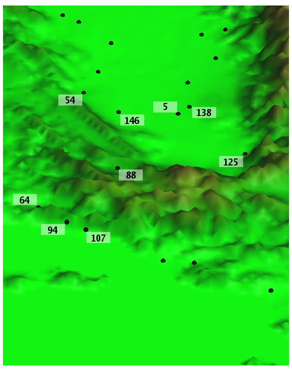
W_{35} is now the station, where d_{ch}, d_{p1}, and d_{p2} produced worse results than the original d_{xy}. Table 4 reveals that our new distance functions have a heigher weight on W_{183} and W_{189}, following exactly what we wanted: The stations to the west (W_{80}, W_{39}, W_{142}, W_{33}, and W_{86}) are completely out of sight, since they are behind a high mountain. The results are so bad as a consequence of the fact that the important W_{183} and W_{189}, which share the same valley with W_{35}, show very dry weather (only 15 and 17, respectively), while the weather station W_{35} reports rain with a relative humidity of 100. Possibly, W_{35} suffered from a very local weather phenomenon (like a local thunderstorm), or the reported value was wrong; d_{xy} is the winner by mere chance.

Weather stations	W_{33}	W_{39}	W_{80}	W_{86}	W_{142}	W_{183}	W_{189}	others	W_{35}
measured rel. hum.	47.0	48.0	34.0	37.0	31.0	15.0	17.0		100.0
d_{xy}	.03	.09	.06	.12	.23	.30	.17		26.6
d_{ch}						.54	.46		16.1
d_{p1}					.23	.77			19.7
d_{p2}	.05	.04	.05	.05	.06	.39	.34	.02	23.3

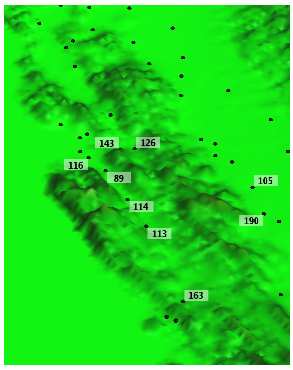
Table 4: Weights used for interpolation at weather station W_{35}. (All new distance functions are bad there.) The last column contains the measured value and the predicted values using the different distance functions at the position of W_{35}.



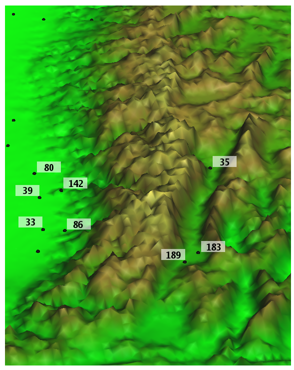
(a) Overview of CIMIS weather stations.



(b) Neighbourhood of weather station W_{88}.



(c) Neighbourhood of weather station W_{113}.



(d) Neighbourhood of weather station W_{35}.

Figure 5: Neighbourhoods of interesting weather stations.

As a first result, we can state that the interpolation quality for selected areas can be increased with the new distance functions. On the other hand, there is no significant improvement in every case. The reason might be the placement of weather stations: The density is high in valleys and low on mountains. Therefore, d_{xy} already prefers stations within the same valley, since the others are too far away, and so the crossvalidation does not show the improvement when using the new distance functions. Presumably, the interpolation result in the mountain regions is better using the new distance functions.

5 Future work

We list a few possibilities for future work:

- The tests can be processed on other data sets. The CIMIS database contains many data sets from other dates, and other weather variables different from relative humidity, e.g., temperature, precipitation, wind speed. Such other tests would also reveal whether W_{35} has just the wrong value, or if it is a local weather phaenomenon.
- z_{exag} must be optimized. $z_{\text{exag}} = 50$ seems to be a good initial value.
- One can find out under what topographic situations what distance function produces the best results.
- Our results should be compared to other interpolation methods, e.g., Hardy's multiquadric, interpolating splines, or kridging.
- The program structure can be improved to allow more efficient processing. Especially the distances and the truncation distances $R(q)$ for every grid point can be calculated in advance. These have to be recalculated every time a weather station is added or eliminated, or when a weather station has a technical problem transmitting values.
- The CIMIS project interpolates the different weather variables and afterwards calculates ET_0 for every point (interpolate first, then calculate: IC). This is rather time- and space-consuming. A more efficient approach calculates ET_0 at the weather stations and interpolates only this (calculate first, then interpolate: CI), see [MKK05], where these two approaches are compared, finding out that the results are similar. At least there is no significant difference between IC and CI.

6 Acknowledgement

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