8/20/2024

Chords and Transitioning between Different Keys —

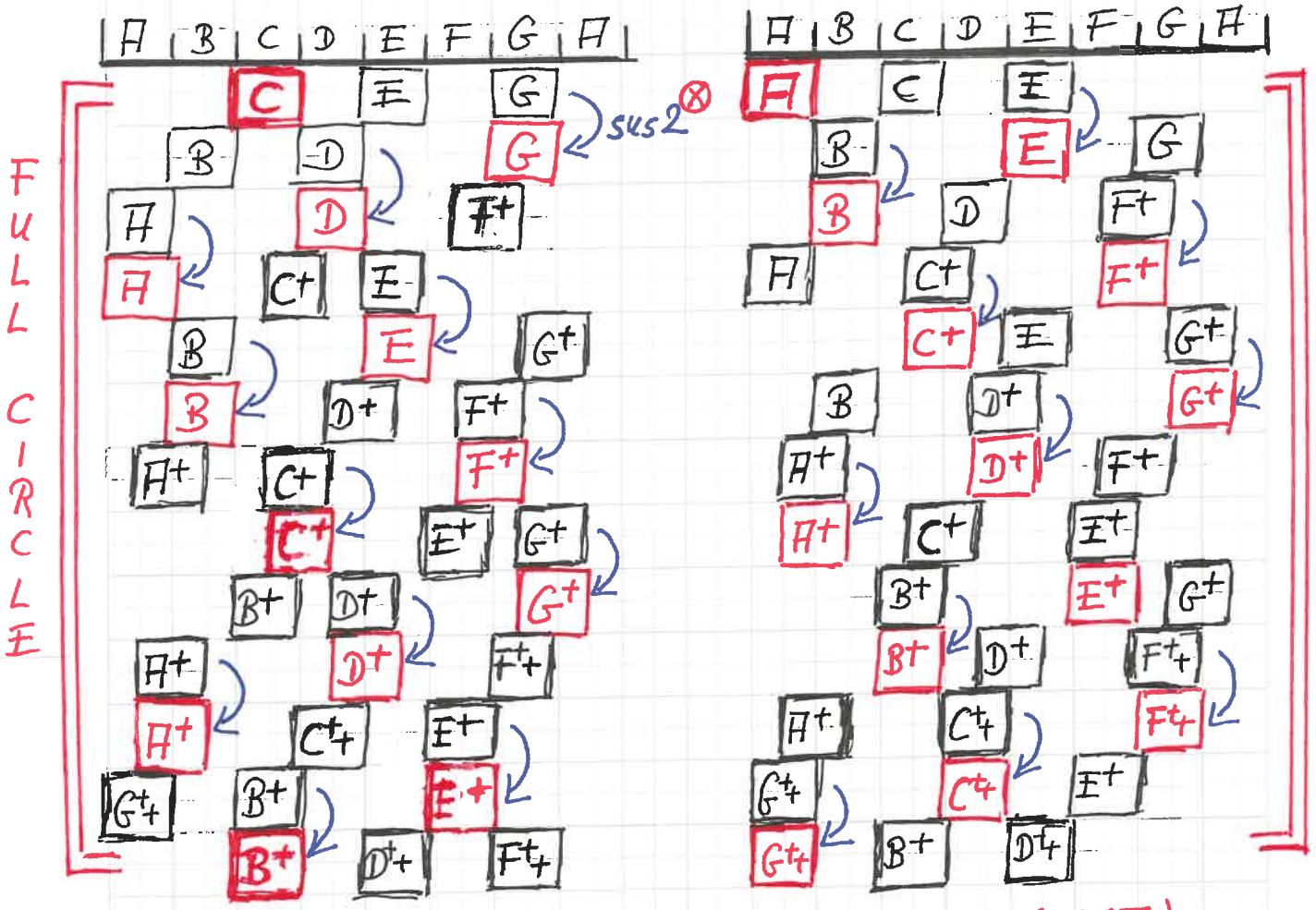
Chords in a Specific Key (Major and Minor Chords) / "Transition Chords" / Circle of 5ths:

- **Key of C (triads):** CEG, DFA, EGB, FAC, GBD, ACE, BDF (+ inversions)
- **Chords with root C:** CEG (maj), CE^bG (min), CDG (sus2), CFG (sus4), CE^bG^b (dim), $CEG^\#$ (aug.)
- **"Transitions":**
  1. CEG → CDG → BDG ; (sus2 → G maj chord)
  2. CEG → CFG → CFA ; (sus4 → F maj chord)
  3. CEG → $CEG^\#$ → CEA ; (aug. → A min chord) ...
- **Circle of 5ths:** ... B^b ↔ F ↔ C ↔ G ↔ D ↔ ... MAJ.
  - IV: GBD → $GBD^\#$ (aug.) → GBE (E min chord) ; (sus4)
  - V: FAC → $FAC^\#$ (aug.) → FAD (D min chord)
  - ... g ↔ d ↔ a ↔ e ↔ b ↔ ... MIN.
  - i: ACE → ABE (sus2) → GBE (E min chord) ; (sus4)
  - ii: ACE → ADE → ADF (D min chord)
  - 3: CEG → $CEG^\#$ (aug.) → CEA (A min chord) ; ...

Tonika – Dominant – Subdominant – Nat. Minor Relationships

⇒ CAN USE "TRANSITION CHORDS" TO "SMOOTLY MOVE" FROM CHORDS OF ANY KEY TO ALL CHORDS OF THE KEYS IN THE "CIRCLE-OF-5TH's NEIGHBORHOOD"!

- **MODULATION – Moving from Chord to Chord / "Harmonic Distance" between Key Signatures & Chords etc.** 8/21/2024



($F^{+} \hat{=} F^{\#}$ etc.; $E^{+} \hat{=} F$, $B^{+} \hat{=} C$; $F^{+}_{+} \hat{=} G$, $C^{+}_{+} \hat{=} D$, $G^{+}_{+} \hat{=} A$, $D^{+}_{+} \hat{=} E$)

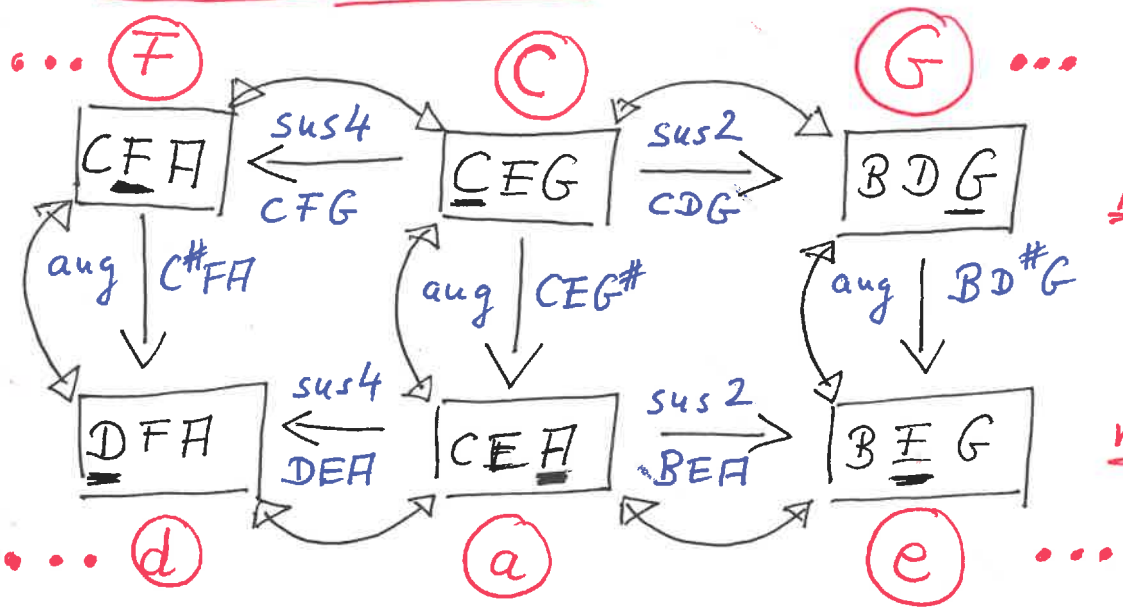
- Tables list the possible 12 major and 12 minor triads using the order of the circle of 5ths, traversed in clockwise order for a full cycle.
- Since the tables show triads only in the interval/range [A, B, C, ..., G^{+}], first and second inversions are included.
- "Harmonically plausible" forward (key) progressions in clockwise circle-of-5ths order are shown via ↘, e.g.,:
⊗ CEG $\xrightarrow[sus2]{}$ CDG → BDG (from C to G major).

MODULATION – cont'd.

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(for "permanent" or "temporary" change of key signature)

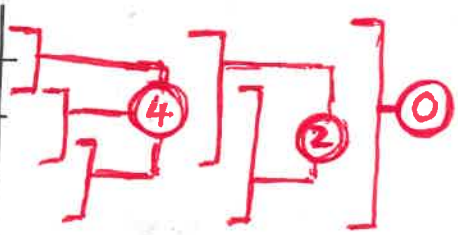
→ Smooth modulation in a "6-key neighborhood" of circle of 5th's:



→ Use sus2 (sus4) chord to move clockwise (counter-clockwise) from MAJOR (minor) to MAJOR (minor); use aug. chord to move from MAJOR to (natural) minor.

• Use common chords (triads) for modulations:

	I	ii	iii	IV	V	vi	DIM
C	CEG	DFA	EGB	FAC	GBD	ACE	BDF
G	GBD	ACE	BDF#	CEG	DF#A	EGB	F#AC
D	DF#A	EGB	F#AC#	GBD	AC#E	BDF#	C#EG
A	AC#E	BDF#	C#EG#	DF#A	EG#B	F#AC#	G#BD
...	...						



⇒ Neighbor keys (in the circle of 5th's) have 4 triads in common; "neighbor-of-neighbor keys" have 2 triads in common.

C	E	G		(135)
C		G	E	(153)
	E	G	C	(351)
(315)	E		C	G
(513)		G	C	E
(531)		G	E	C

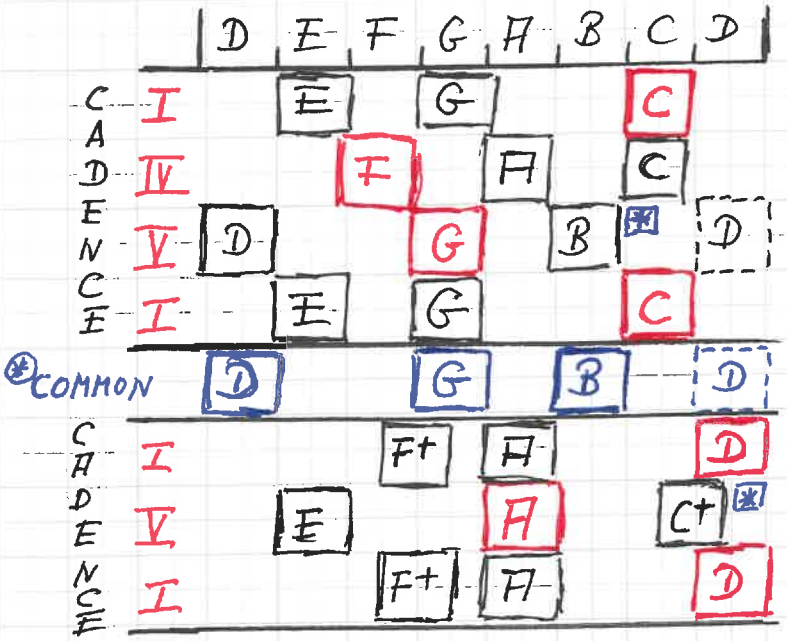
Note. There exist 6 = 3! possible triads of three different notes. Thus, the "true number" of possible (in-) versions is 6 – and one must consider TWO OCTAVES for their "implementation." SEE LEFT FIGURE.

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• MODULATION: Moving from the Current Key (Signature) to a New Key

[Ref.: Franz TITSCHER, pianobeat.de, "Modulationen..."]

- Standard / General Modulation:
  - i) CADENCE for current key, e.g., I-IV-V-I
  - ii) Determine all CHORDS that the current Key and new key have IN COMMON (triads).
    - Key of C (current): CEG, DFA, EGB, FAC, GBD, ACE
    - Key of D: DF#A, EGB, F#AC#, GBD, AC#E, C#EG
    - → Common triads: EGB, GBD
  - iii) Pick one of the common chords to modulate! (transition / modulation chord)
  - iv) CADENCE for new key, e.g., I-IV-V-I or I-V-I



- Example: from C to D (from EGC to F#AD):
  - Cadence for C: EGC → FAC → DGB(D) → EGC
  - Transition / common chord: DGB(D)
  - Cadence for D: F#AD → EAC# → F#AD
- Other transitions / transition chords & cadences possible!

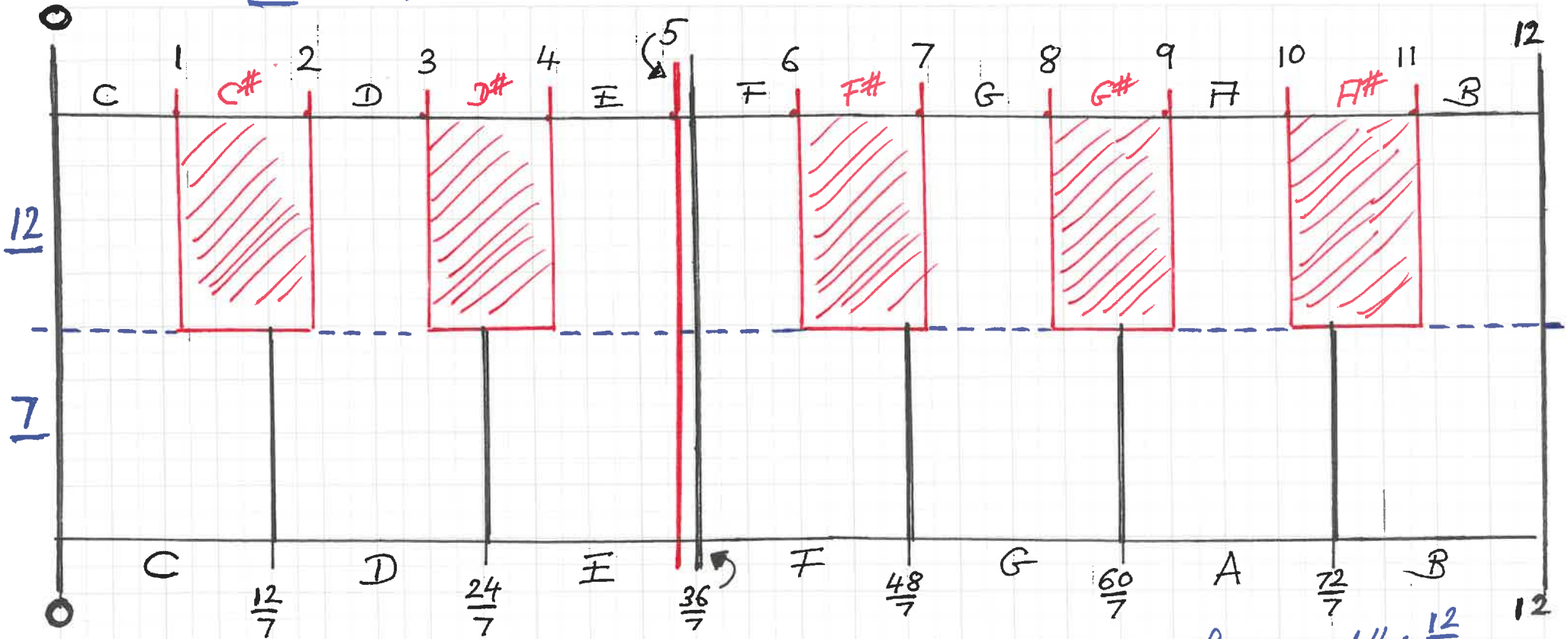
* Common chord = "Bindeakkord" ("connecting" / "glue chord")

* B = Lead note to C, C# = Lead note to D ⇒ USE CHORDS WITH Lead notes effectively: moving up by ½ note!

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LAYOUT OF AN OCTAVE

12 half-tones — uniform spacing ⇒ uniform width: 1



7 tones/notes of C — uniform spacing ⇒ uniform width: 12/7

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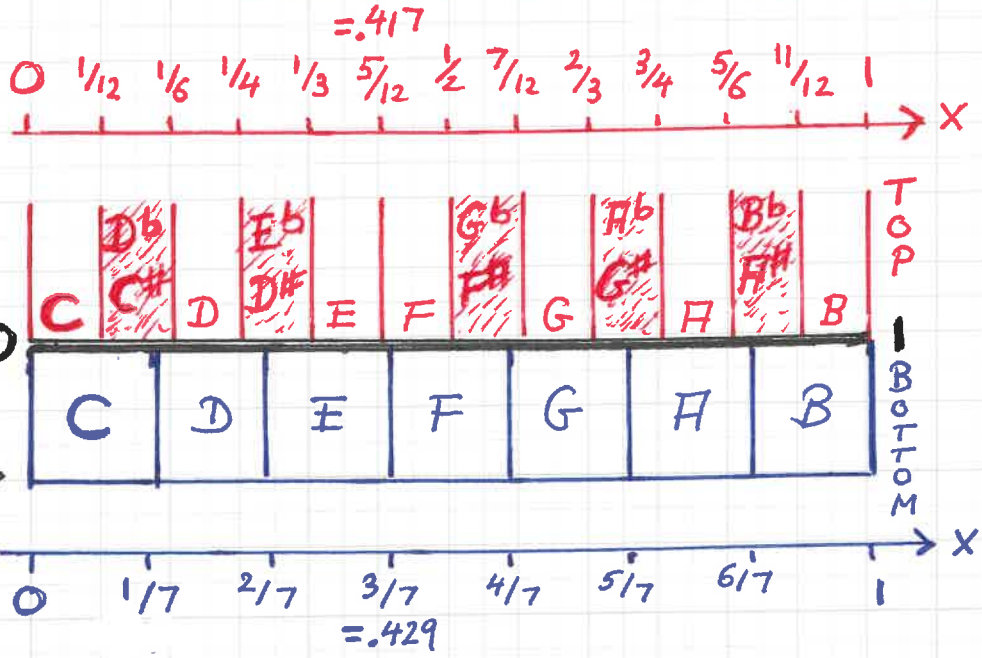
• Note. The physical placement (and approximate width) of keys should "ideally" be based on a UNIFORM spacing and width of the physical keys. Why? Chords and scales must be "remembered" i.e. "shapes" should be "invariant" under a change of key signature. Of course, "shapes" should be "memorized". Ideally, "shapes" should be "invariant" under a change of key signature. Of course an "OPTIMAL LAYOUT" is (still) an "unsolved problem." ~

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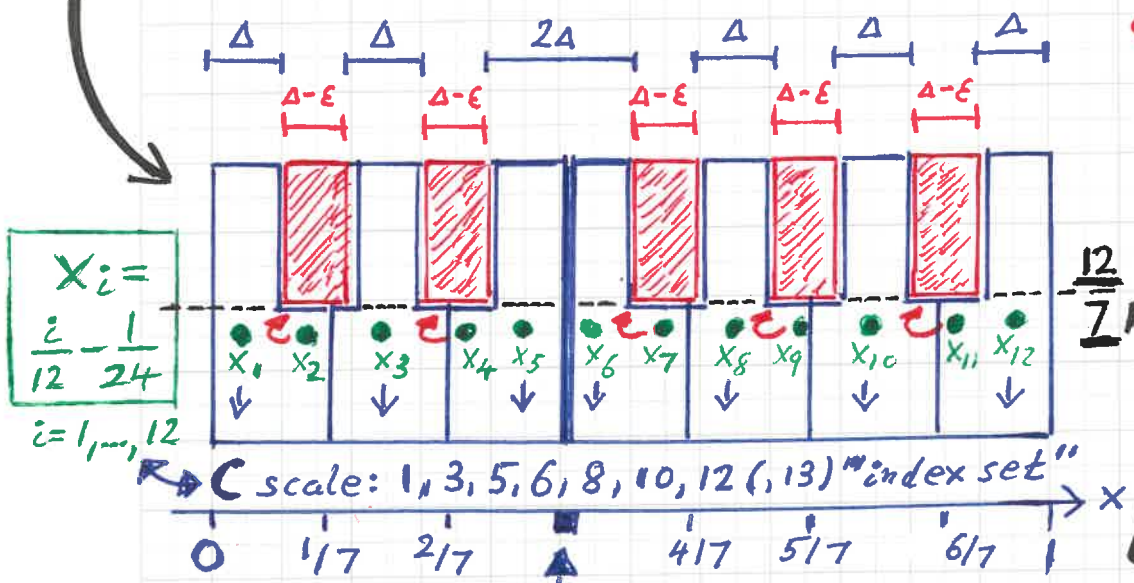
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Note. Physical Layout of Piano Keyboard – Key Placement & Width

8/25/2024



- The keyboard has a "TOP" & "BOTTOM" part.
  "TOP": 12-note chromatic scale;
  "BOTTOM": 7 notes of "natural octave"



- FINGERS MUST FIT BETWEEN BLACK KEYS: SPECIFY ε!

$\Rightarrow 7\Delta + 5(\Delta - \varepsilon) = 1$

$\Rightarrow \Delta = \frac{1}{12}(1 + 5\varepsilon)$

(e.g., $\varepsilon = 1/100 \Rightarrow \Delta = 7/80$)

- At the "INTERFACE" between the 12-key & 7-key parts, SHAPES OF ALL CHORDS can be understood and memorized as SAME SHAPES REGARDLESS OF KEY SIGNATURE! This fact is a consequence of the EQUIDISTANT SPACING BETWEEN THE LOCATIONS/POINTS "•", $X_1 \ldots X_{12}$.

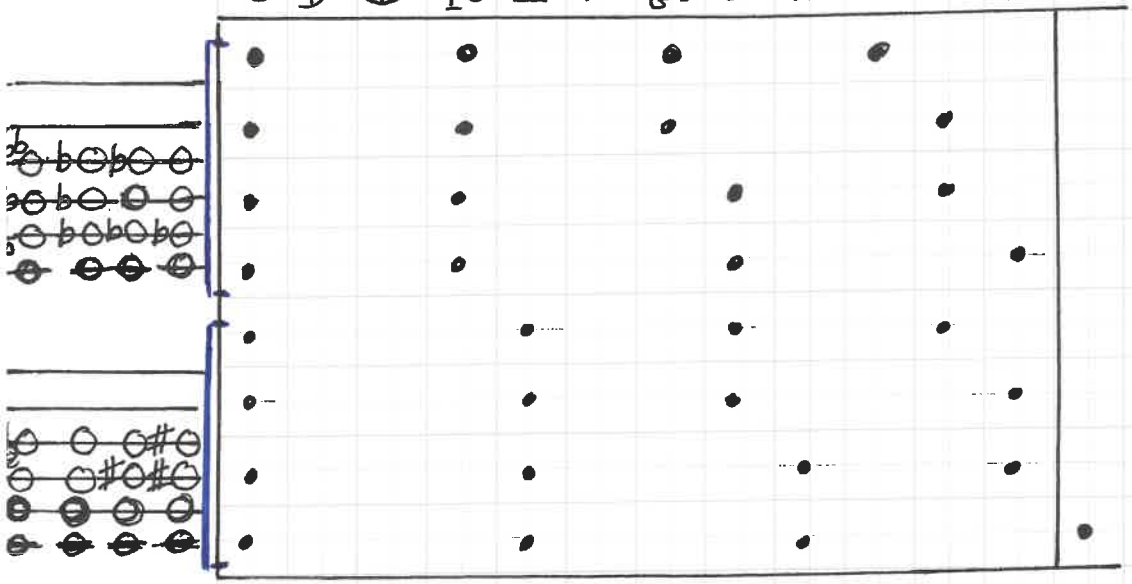
INDICES all maj. (min.) SCALES "starting on X_i":
$\{i, i+2, i+4, i+5, i+7, i+9, i+11 (, i+12), \{i, i+2, i+3, i+5, i+7, i+8, i+10 (, i+12)\}\}$.

• STACKING 3rds

8/26/2024

(1) —— half-tones —— (12)

C Db D Eb E F Gb G Ab A Bb B C



C Eb Gb A (=Bbb)
C Eb Gb Bb
C Eb G Bb
C Eb G B
C E G Bb
C E G B (=G#)
C E Ab B
C E Ab C (=B#)

8 = 2³ possibilities to stack 3 minor and major 3rds on top of each other; root = C

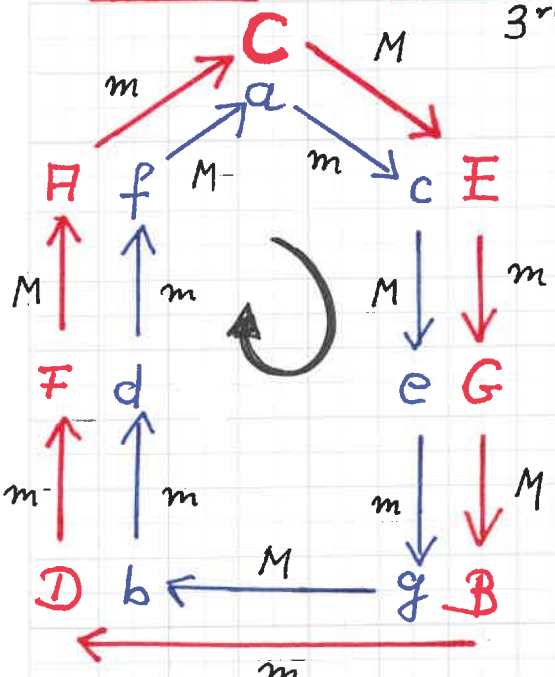
(PLUS: INVERSIONS!)

→ The specific chords resulting from stacking 3 3rds exhibit varying degrees of "consonance." (For example, CEAbC is "unpleasant.")

→ The circle of 3rds is not commonly known/used. It illustrates how one can stack minor (m) and major (M) 3rds — i.e., stacking 7 3rds to cover two octaves, for major and minor key signatures (CEGBDFA(c), acegbdf(a)).

• "CIRCLE OF 3rds"

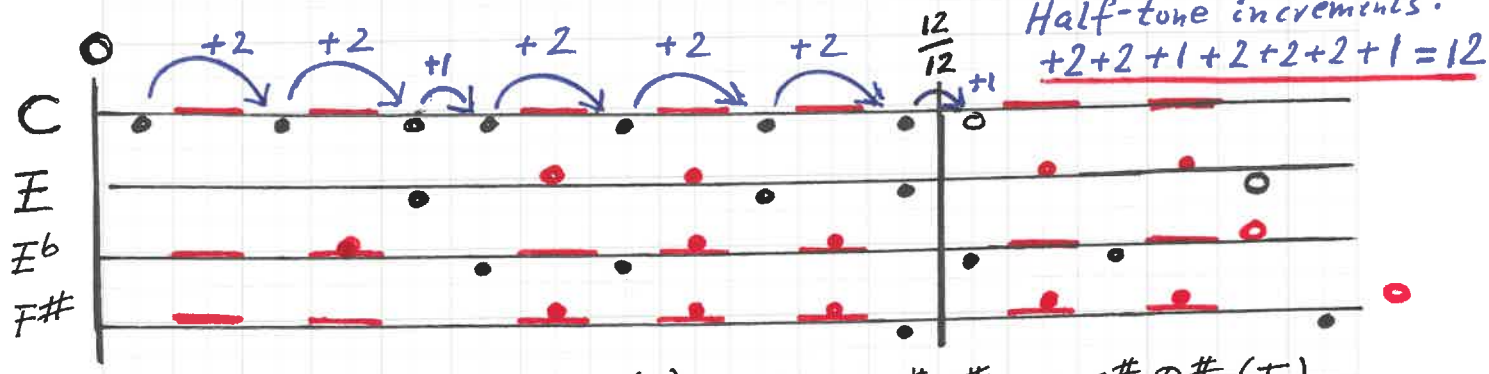
(m = minor 3rd, M = major 3rd)



CEGBDFA(C) major & acegbdf(a) minor cirles by stacking 3rds

"Key Signature & Chromatic Notation" 8/26/2024

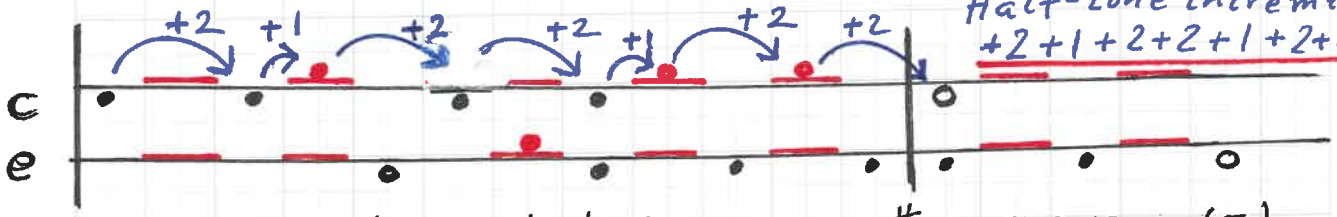
C C# D D# E F F# G G# A A# B C C# D D# E F F#



Half-tone increments: +2+2+1+2+2+2+1 = 12

(MAJ.) C: C D E F G A B (C) E: E F# G# A B C# D# (E)

E♭: E♭ F G A♭ B♭ C D (E♭) F#: F# G# A# B C# D# E# (F#)

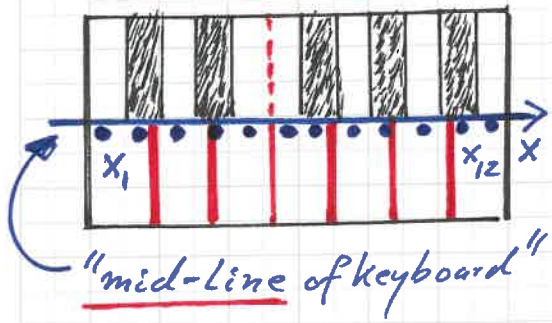


Half-tone increments: +2+1+2+2+1+2+2 = 12

(MIN.) c: C D E♭ F G A♭ B♭ (C) e: E F# G A B C D (E)

→ Examples of major & minor scales shown in the context of chromatic notation / a "chromatic keyboard" / the "top part of the piano keyboard — where half-tones have the SAME, UNIFORM EQUIDISTANT SPACING/POSITIONING!

→ IMPLICATION for "SHAPES & FINGERING":



{•} = {x1, ..., x12} = set of equidistant midpoints of the "top chromatic keys of same width"

→ TIPS OF FINGERS: HOLD THEM AS CLOSELY AS POSSIBLE TO THE xi-VALUES ON THE x-LINE; "MOVE THE HAND INWARDS / UPWARDS — WITH THE TIP OF THE THUMB AS CLOSE AS POSSIBLE TO THE "MID-LINE"!

!!! ⇒ CHORD SHAPES & FINGERINGS EQUAL FOR ALL KEYS!!!

• More on Chords — Chord Distances and Their Visualization

8/27/2024

→ Motivating examples for distance definition:
(using integers after note names to indicate octave on keyboard, e.g., C1 ... C8)

dist(C1, C1) = 0 ; dist(C3, C4) = ? small ? 0 ? ;

dist(C3, C4) ≤ dist(C3, C5) ;

dist((C,E,G), (C,E,G)) = 0 ; dist((C,E,G), (E,G,C)) = ? ;

dist((C1,E1,G1), (C2,E2,G2)) ≤ dist((C1,E1,G1), (C3,E3,G3)) ;

dist((C,G), (C,E,G)) = ? 0 ? small ?? ;

dist((C1,G1), (C2,E2,A2)) = ???

VIS. ⇓

= combine(dist(C1, (C2,E2,A2)),
dist(G1, (C2,E2,A2)),
dist(C2, (C1,G1)),
dist(E2, (C1,G1)),
dist(A2, (C1,G1)) ??? ;

- (n1, n2, n3 (n4)) require 3 (4) D vis. grid ; (Δ12, Δ23 (, Δ34)) require only 2 (3) D vis. grid.
- CLOSE CHORDS OR Δ-TUPLES SHOULD BE CLOSE IN VIS. LAYOUT.

dist(c1, c2) = no of Common Notes / (len(c1) + len(c2)),
where len( ) = no of Notes in Chord( ) ;

Δ(C4, E4, A4) = combine(Δ12, Δ13, Δ23) (Δ13 = Δ12 + Δ23)
? = combine(dist(C4,E4), dist(C4,A4), dist(E4,A4))
? = combine(dist(C4,E4), dist(E4,A4))
= Δ123 = Δ13 (C4, E4, A4) = ... ???
= (Δ12, Δ23) ⇒ from triple (C4,E4,A4) to (Δ12, Δ23)

Δ(C4,E4,G4,A4) = (Δ12, Δ23, Δ34) ⇒ DIMENSION REDUCTION BY 1 .

• More on Chords — Indexing, Numerical Values, Distances and Visualization

8/27/2024

→ How should one index notes / keys "meaningfully"?

→ How should one define numerical values, to be able to define and calcute "meaningful chord distances," when associating a number with a note? "Meaningful" note-to-note distances must also lead to "meaningful" chord-to-chord distances! (Special cases: distance def. for chord inversions / permutations)

	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A
half-tone steps	0	1	2	3	4	5	6	7	8	9
integer index+1	1	2	3	4	5	6	7	8	9	10
real nos. for steps	0	1/2	2	3/2	2	5/2	3	7/2	4	9/2
chord numbers	1	...	2	...	3	4	...	5	...	6
Roman numerals	I		ii		iii	IV		V		vi
...	...									:

Possible "index sets" of C.

→ ISSUES: — Notes are close in terms of lexicographical distance of notes' names (dist(C,D) < dist(C,G)), but are "harmonically very distant" (i.e., dissonant), and vice versa.

— Chords (n1, n2, n3) & (n1, n2, n3) should / must have a distance that is zero (?) or very small when they represent inversions or permutations: dist((C,E,G),(G,C,E)) ≙ "small"; dist((C1,E1,G1),(C2,E2,G2))!!

— One must define a distance for 2 chords consisting of different numbers of notes: dist((n1,n2),(n1,n2,n3)) = ?

— Distance for the notes defining ONE chord to be defined: dist(n1,n2,n3) ≙ combination(dist(n1,n2), dist(n1,n3), dist(n2,n3))

⇒ dist(c1, c2) = combination(dist(allNotes of c1 to allNotes of c2), (" c2 to " c1))
(chord, chord)

• More on Chords — "Distance"

8/27/2024

→ One can define a chord "key-specific" by providing its notes or, more generally, by providing the "distances," i.e., the number of half-tones," between pairs of two consecutive notes (in any key). For example, the specific-note triad $n1n2n3$ is more generally defined — key independently — by the expression $\Delta_{12}\Delta_{23}$; here Δ_{12} (Δ_{23}) represents the distance between $n1$ and $n2$ ($n2$ and $n3$).

Example: $n1n2n3 = CEG \Rightarrow \Delta_{12} = dist(C,E)$, $\Delta_{23} = dist(E,G)$; i.e., $\Delta_{12}\Delta_{23}$ = "combined distance expression for a general "stack of major and minor 3rd's".

$\Rightarrow$ A 3-note (4-note) chord representation $n1n2n3$ ($n1n2n3n4$) has a general 2-delta (3-delta) expression $\Delta_{12}\Delta_{23}$ ($\Delta_{12}\Delta_{23}\Delta_{34}$). THUS ONE OBTAINS A "DIMENSION REDUCTION BY 1."

→ How does this "Δ-concept" relate to the definition of distance of two chords?

Problem: chord 1 $= (n1, n2, n3) = c_1$; $c_2 = (\overline{n1}, \overline{n2}, \overline{n3})$ = chord 2

$\Rightarrow dist(c_1, c_2) = dist((n1,n2,n3), (\overline{n1}, \overline{n2}, \overline{n3})) = ?$

Problem: A def. of distance must consider that 2 chords that are inversions "are the same chord" (really?) and should have dist. zero (really?)

• More on Chords: "Chord Distances", "Chord Similarities," "Chord Inversions & Permutations," "Chord Visualization" 8/27/2024

→ **GOAL:** Definition of "chord distance" — allowing one to identify close chords, for "smooth transitions," and to visualize them

→ **Chord Inversions & Permutations:**

A 3-note (4-note) chord — e.g., triads and 7th chords — can be written as a tuple $(n1, n2, n3)$ $((n1, n2, n3, n4))$, with $n1, n2, n3$ $(n4)$ being notes of the chord. There are $6 = 3!$ permutations of a 3-note chord: $(n1,n2,n3)$, $(n1,n3,n2)$, $(n2,n1,n3)$, $(n2,n3,n1)$, $(n3,n1,n2)$, $(n3,n2,n1)$ — e.g., (CEG)*, (CGE), (ECG), (EGC)*, (GCE)*, (GEC); using 2 octaves, one can play:



* standard chord inversions.

(Analogously, there are $24 = 4!$ 4-note chord permutations: $(n1,n2,n3,n4), \ldots, (n4,n3,n2,n1)$.)

→ IF ONE DEFINES THE (numerical?) DISTANCE BETWEEN 2 NOTES $n1$ AND $\overline{n1}$ "WELL," THEN ONE WILL BE ABLE TO DEFINE AND CALCULATE CHORD DISTANCE OF 2 CHORDS $(n1, n2, n3\,(, n4))$ AND $(\overline{n1}, \overline{n2}, \overline{n3}\,(, \overline{n4}))$.

More on Chords — 7th Chords etc.

8/27/2024

→ Ref: https://musikanalyse.net/tutorials/septakkorde (in German)

→ The seven "inherent" 7th chords associated with a specific key (signature) can be defined similarly to the triads of a key.
Example: 7 7th chords of the C major key:
CEGB, DFAC, EGBD, FACE, GBDF, ACEG, BDFA.
When considering the nature of the first 3rd of these seven 7th chords, one could once again use the Roman numerals I, ii, iii, IV, V, vi, dim.
Each of these 7th chords has 3 inversions, e.g.,
CEGB (root pos.) → EGBC (1st) → GBCE (2nd) → BCEG (3rd).
(There are 7th chords for all major and minor keys, of course.)

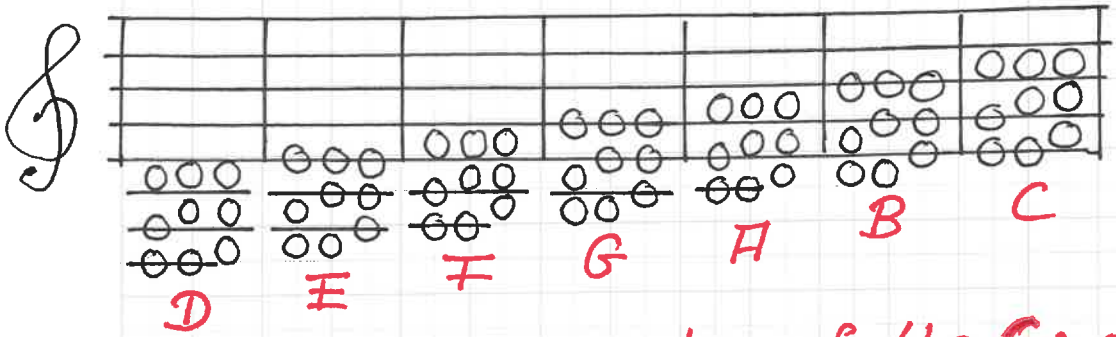
→ Purpose/use of 7th chords? Transitions, i.e., moving from one chord to another; providing tension via dissonant nature; providing a means to modulate, i.e., establish a new key signature etc. (A 7th chord used as a transition chord must/should have a note (notes) in common with a chord/key one is coming from and the chord/key one is moving to.)

→ Stacking more than 3 3rds leads to even more complicated chords/harmonies: 9th, 11th, 13th chords.

• More on CHORDS & Inversions: Triads & 4-Tones (7th Chords)

8/27/2024

→ "The highest note in a chord (played with the right hand) is the note 'being heard,' defining the 'melody'."



• Example: Three triads (in the key of C major) for the seven notes of the C major scale are shown. These chords are major, minor and diminished chords, using various inversions (1st, 2nd).

C	F A C	= F
	E A C	= Am 2nd
	E G C	= C 1st
B	E G B	= Em
	D G B	= G 2nd
	D F B	= B dim 1st
A	D F A	= Dm
	C F A	= F 2nd
	C E A	= Am 1st
G	C E G	= C
	B E G	= Em 2nd
	B D G	= G 1st
F	B D F	= B dim
	A D F	= Dm 2nd
	A C F	= F 1st
E	A C E	= Am
	G C E	= C 2nd
	G B E	= Em 1st
D	G B D	= G
	F B D	= B dim 2nd
	F A D	= Dm 1st

Triads for C maj. key

The left table shows all the possible $21 = 7 \cdot 3$ triads for the seven tones of the C maj. key (C, B, A, G, F, E, D). Every "key-inherent triad" occurs three times, in its three inversions: root position, 1st inversion and 2nd inversion.

→ 7th chords (= special 4-note chords) - SEPTAKKORDE - result by stacking minor and major 3rds. The "C-major-key-inherent" 7th chords are shown in the right figure.

