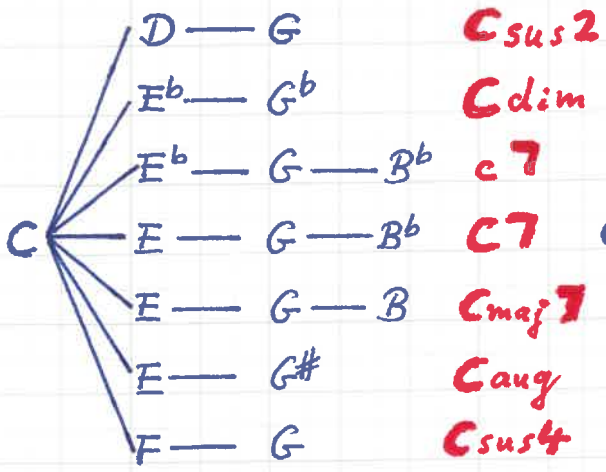
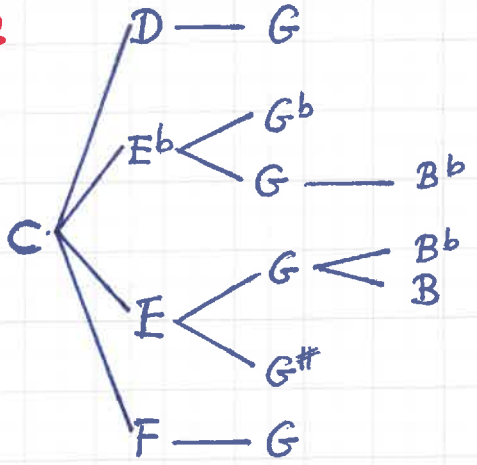
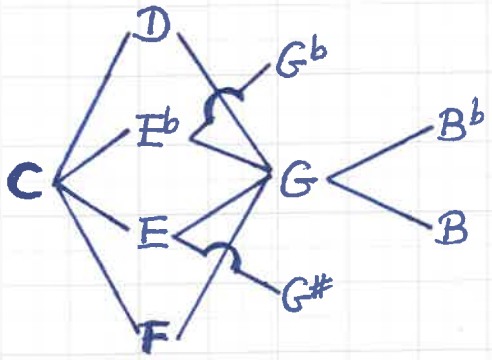
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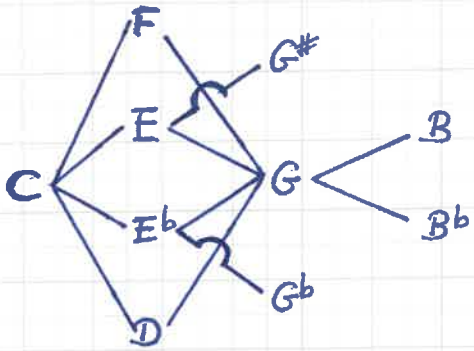
Effective Chord Representation

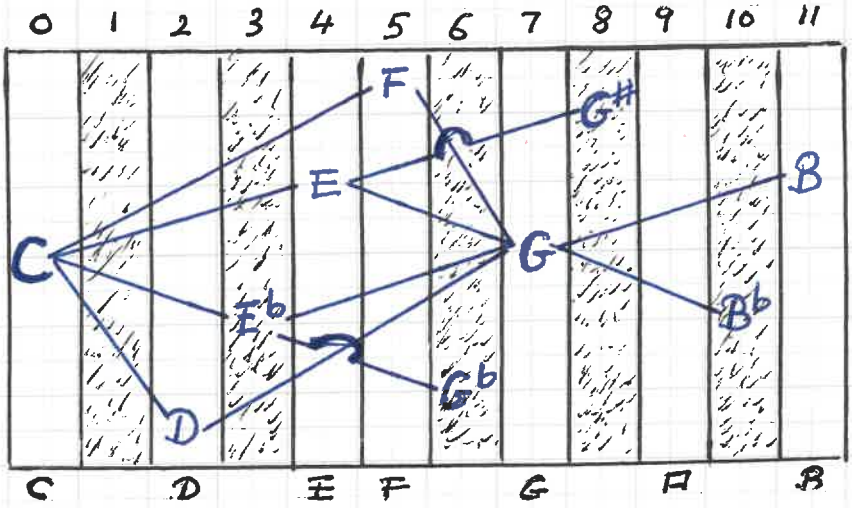
• Example. Representing chords with root note C







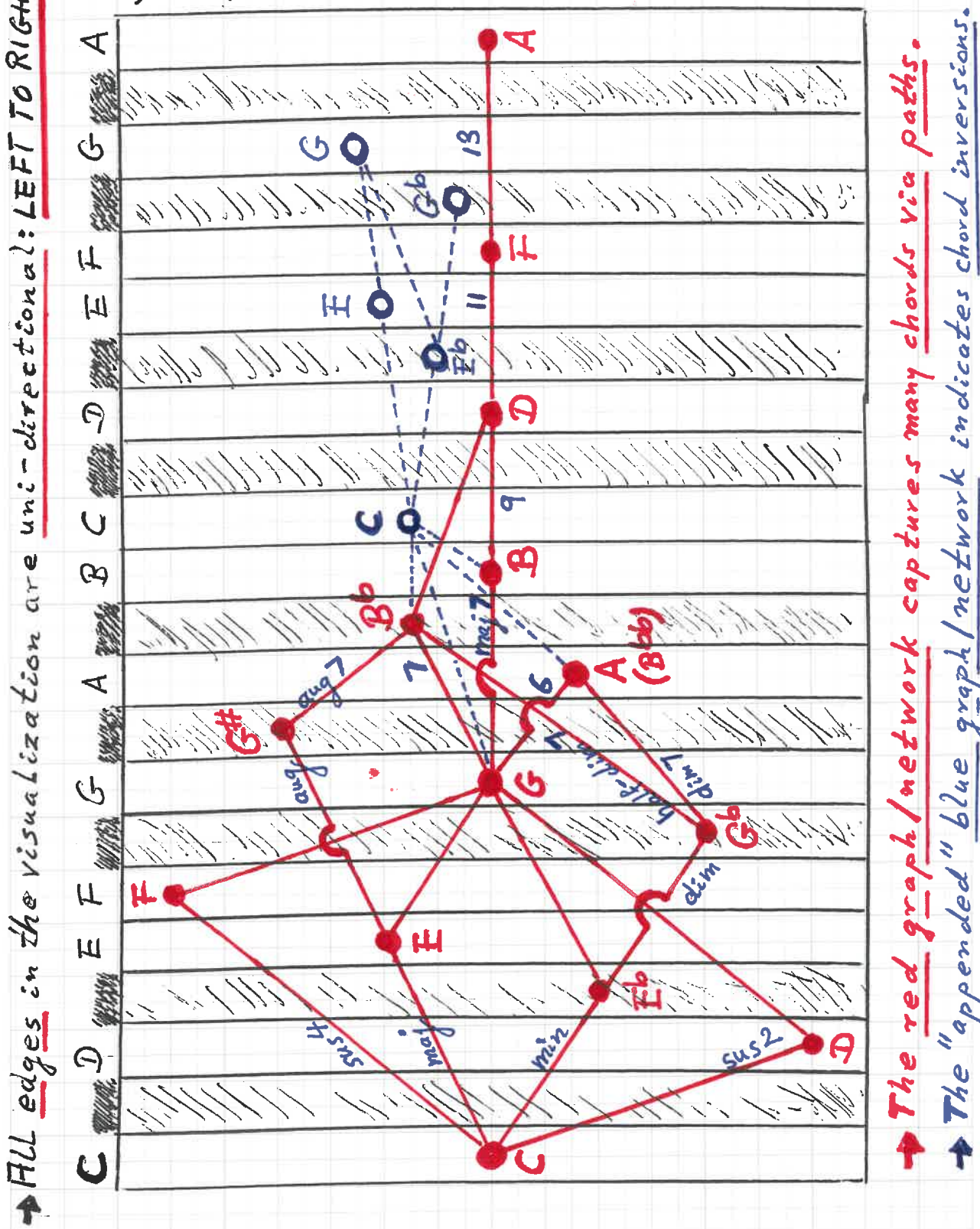




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Effective Chord Representation

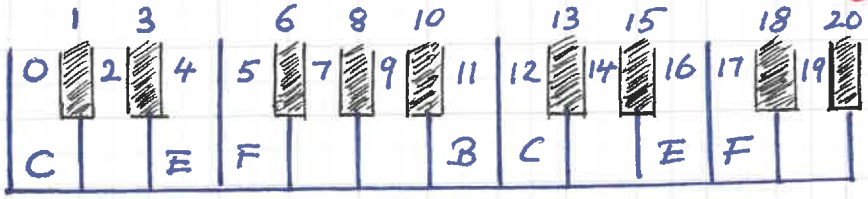
The visualization on this page captures some of the common chords with root note C : triads, 7th, 9th, 11th and 13th chords, major and minor.



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• Effective Chord Representation

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• **GOAL:** "Ideal and simple ways" to memorize "all chords."

• MAJOR Triads:

WWW ○○○
CEG, FAC, GBD

DUAL
WBW ○●○
DF#A, EG#B, AC#E

BWB ●○●
D♭FA♭, E♭GB♭, A♭CE♭

DUAL
WBB ○●●
BD#F#

BWW ●○○
B♭DF

BBB ●●●
F#A#C# (=G♭B♭D♭)

(no WWB, BBW)

• MINOR Triads:

WWW ○○○
DFA, EGB, ACE

DUAL
WBW ○●○
CE♭G, FA♭C, GB♭D

BWB ●○●
C#EG#, F#AC#, G#BD#

DUAL
WWB ○○●
BDF#

BBW ●●○
B♭D♭F

BBB ●●●
D#F#A# (=E♭G♭B♭)

(no WBB, BWW)

DUAL

⊗ DIMINISHED Triads:

○○○ BDF
○○● DFA♭
EGB♭
ACE♭
○●○ FA♭C♭
●○○ C#EG
F#AC
G#BD
○●● CE♭G♭
GB♭D♭
●●○ A#C#E
D#F#A

• DIMINISHED Triads:... ⊗

• Key-inherent Triads – Ex.: A major, F# minor

A B C# D E F# G# A B C# D

M	○ ○ ● ○ ○ ● ● ○	AC#E
m	○ ● ○ ○ ● ● ○ ○	BDF#
m	C#EG#	● ○ ○ ● ● ○ ○ ●
M	DF#A	● ○ ● ● ○ ○ ● ○
M	EG#B	○ ● ● ○ ○ ● ○
m	F#AC#	● ● ○ ○ ● ○
dim	G#BD	● ○ ○ ● ○

3 MAJ (M), 3 MIN (m), 1 DIMINISHED (dim) chords.

F# G# A B C# D E F# G# A B

m	● ● ○ ○ ● ○ ○ ● ● ○ ○	F#AC#
dim	G#BD	● ○ ○ ● ○ ○ ● ○ ○
M	AC#E	○ ○ ● ○ ○ ● ○ ○ ●
m	BDF#	○ ● ○ ○ ● ○ ○ ●
m	C#EG#	● ○ ○ ● ○ ○ ●
M	DF#A	○ ○ ● ○ ○ ●
M	EG#B	○ ● ○ ○ ●

3 MAJ (M), 3 MIN (m), 1 DIMINISHED (dim) chords.

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Effective Chord Representation

Key-inherent Triads of Major and Minor Keys

	C	G	D	A	E	B	F#	C#
M	CEG	GBD	DF#A	AC#E	EG#B	BD#F#	F#A#C#	C#E#G#
m	DFA	ACE	EGB	BDF#	F#AC#	C#EG#	G#BD#	D#F#A#
m	EGB	BDF#	F#AC#	C#EG#	G#BD#	D#F#A#	A#C#E#	E#G#B#
M	FAC	CEG	GBD	DF#A	AC#E	EG#B	BD#F#	F#A#C#
M	GBD	DF#A	AC#E	EG#B	BD#F#	F#A#C#	C#E#G#	G#B#D#
m	ACE	EGB	BDF#	F#AC#	C#EG#	G#BD#	D#F#A#	A#C#E#
dim	BDF	F#AC	C#EG	G#BD	D#F#A	A#C#E	E#G#B	B#D#F#

F# = F#
C# = C#
G# = G#
D# = D#
A# = A#

E# = E# ≙ F
B# = B# ≙ C

Chords:

M - major

m - minor

dim - diminished

	F	Bb	Eb	Ab	Db	Gb	Cb
	FAC	BbDF	EbGB	AbCE	DbFA	GbBbD	CbEbG
	GBD	CEG	FAC	BbDF	EbGB	AbCE	DbFA
	ACE	DFA	GBD	CEG	FAC	BbDF	EbGB
	BDF	EGB	ACE	DFA	GBD	CEG	FAC
	CEG	FAC	BbDF	EbGB	AbCE	DbFA	GbBbD
	DFA	GBD	CEG	FAC	BbDF	EbGB	AbCE
	EGB	ACE	DFA	GBD	CEG	FAC	BbDF

Bb = Bb
Eb = Eb
Ab = Ab
Db = Db
Gb = Gb

Cb = Cb ≙ B
F = Fb ≙ E

Colors:

blue-white keys

black-black keys

'#'- sharp/flat

	a	e	b	f#	c#	g#	d#	a#
m	ACE	EGB	BDF#	F#AC#	C#EG#	G#BD#	D#F#A#	A#C#E#
dim	BDF	F#AC	C#EG	G#BD	D#F#A	A#C#E	E#G#B	B#D#F#
M	CEG	GBD	DF#A	AC#E	EG#B	BD#F#	F#A#C#	C#E#G#
m	DFA	ACE	EGB	BDF#	F#AC#	C#EG#	G#BD#	D#F#A#
m	EGB	BDF#	F#AC#	C#EG#	G#BD#	D#F#A#	A#C#E#	E#G#B#
M	FAC	CEG	GBD	DF#A	AC#E	EG#B	BD#F#	F#A#C#
M	GBD	DF#A	AC#E	EG#B	BD#F#	F#A#C#	C#E#G#	G#B#D#

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• Effective Chord Representation

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• The tables for the natural minor keys must be properly modified for the harmonic and melodic minor keys.

d	g	c	f	b^b	e^b	a^b
DFA	GBD	CEG	FAC	BDF	EGB	ACE
EGB	ACE	DFA	GBD	CEG	FAC	BDF
FAC	BDF	EGB	ACE	DFA	GBD	CEG
GBD	CEG	FAC	BDF	EGB	ACE	DFA
ACE	DFA	GBD	CEG	FAC	BDF	EGB
BDF	EGB	ACE	DFA	GBD	CEG	FAC
CEG	FAC	BDF	EGB	ACE	DFA	GBD

• Note. Only $2^3 = 8$ "key color combinations" are possible for a triad: WWW; WWB, WBW, BWW; WBB, BWB, BBW; or BBB. All triads listed in the triad tables above represent one of the 8 possibilities. CAN ONE USE THIS FACT FOR MEMORIZATION OF TRIAD SHAPES? DO "CLUSTERS OR GROUPS" OF SHAPES EXIST IN THE TABLES?

MEMORIZATION

• Note. KEY-INHERENT TRIADS

1) Given a specific key, major or minor, it has certain #s or bs.
2) The key has a root note and six subsequent notes.
3) For each of the seven notes, one defines a triad on top of it.
4) From an "alphabetical viewpoint," ONLY SEVEN BASIC TRIADS EXIST: CEG, DFA, EGB, FAC, GBD, ACE, BDF.
5) THE SPECIFIC KEY DEFINES WHICH OF THESE TRIADS IS THE FIRST AS KEY-INHERENT TRIAD (KEY=ROOT); THE #s AND b's OF THE KEY "MODIFY" THE BASIC TRIADS.

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• Effective Chord Representation

• Four-note Chords: 7th Chords

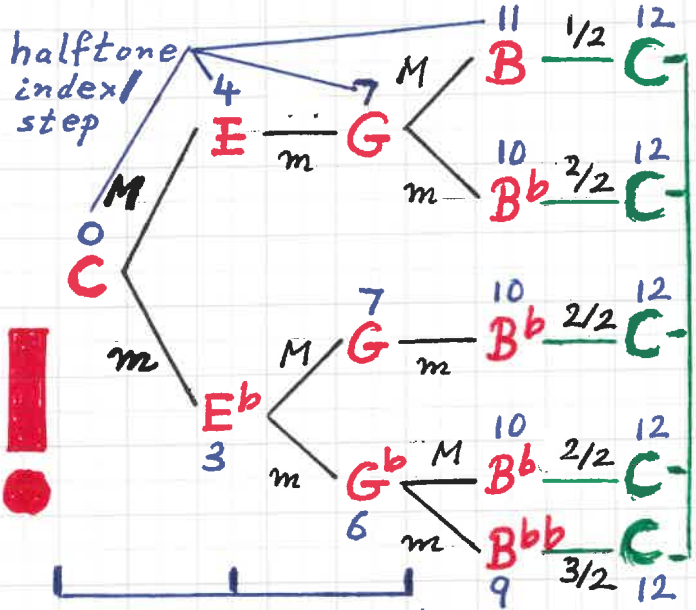
"Stacking major (M) and minor (m) 3rds":

[Example: C = root of 7th chord]

SAME 4 LETTERS: C E G B !

1) C E G B ≙ M m M (7th)
2) C E G B^b ≙ M m m (maj 7th)
3) C E^b G B^b ≙ m M m (min 7th)
4) C E^b G^b B^b ≙ m m M (half-diminished 7th)
5) C E^b G^b B^{bb} ≙ m m m ([fully] diminished 7th)

MEMORIZATION !



"Starting triads":

C E G (C maj.)
C E^b G (C min.)
C E^b G^b (C dimin.)

- The last possible notes of a 7th chord are: B, B^b, B^{bb} (≙A).
- These three notes are 1/2, 2/2, 3/2 HALF-TONES "BELOW C," C being one octave above the root C.
- Note. 1) Given a specific root note and type of 7th chord, determine and play the "starting triad."

2) Depending on the type of 7th chord, play a B, B^b or B^{bb} (≙A) being 1/2, 2/2 or 3/2 steps below the C that is one octave above the chord's root C.

- Note. For 4-note chords $2^4 = 16$ "key color combinations" are possible, from WWWW to BBBB. For example, WWWW, WWWB, WBWB, WBBB, WBBW apply to the C chords.

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- **Effective Chord Representation**

- **But:** Due to the large number of 7th chords when considering all possible root notes/keys and 16 possible white-and-black "key color combinations", it seems that it is not a viable strategy to memorize 7th chord shapes by considering the 16 possibilities.

- **Table of five 7th chords for 12 root notes:**

Key/root	MmM	Mmm	mMm	mmM	mmm
C	C E G B	C E G $^{\bullet}$B	C $^{\bullet}$E G $^{\bullet}$B	C $^{\bullet}$E $^{\bullet}$G B	C $^{\bullet}$E $^{\bullet}$G $^{\bullet\bullet}$B
G	G B D F$^{\bullet}$	G B D F	G $^{\bullet}$B D F	G $^{\bullet}$B $^{\bullet}$D F	G $^{\bullet}$B $^{\bullet}$D $^{\bullet}$F
D	D F$^{\bullet}$ A C$^{\bullet}$	D F$^{\bullet}$ A C	D F A C	D F$^{\bullet}$ A C	D F$^{\bullet}$ A $^{\bullet}$C
A	A C$^{\bullet}$ E G$^{\bullet}$	A C$^{\bullet}$ E G	A C E G	A C $^{\bullet}$E G	A C $^{\bullet}$E $^{\bullet}$G
E	E G$^{\bullet}$ B D$^{\bullet}$	E G$^{\bullet}$ B D	E G B D	E G $^{\bullet}$B D	E G $^{\bullet}$B $^{\bullet}$D
B	B D$^{\bullet}$ F$^{\bullet}$ A$^{\bullet}$	B D$^{\bullet}$ F$^{\bullet}$ A	B D F$^{\bullet}$ A	B D F A	B D F$^{\bullet}$ A
F$^{\#}$	F$^{\bullet}$ A$^{\bullet}$ C$^{\bullet}$ E$^{\bullet}$	F$^{\bullet}$ A$^{\bullet}$ C$^{\bullet}$ E	F$^{\bullet}$ A C$^{\bullet}$ E	F$^{\bullet}$ A C E	F$^{\bullet}$ A C$^{\bullet}$ E
C$^{\#}$	C$^{\bullet}$ E$^{\bullet}$ G$^{\bullet}$ B$^{\bullet}$	C$^{\bullet}$ E$^{\bullet}$ G$^{\bullet}$ B	C$^{\bullet}$ E G$^{\bullet}$ B	C$^{\bullet}$ E G B	C$^{\bullet}$ E G$^{\bullet}$ B
F	F A C E	F A C$^{\bullet}$ E	F$^{\bullet}$ A C$^{\bullet}$ E	F$^{\bullet}$ A C$^{\bullet}$ E	F$^{\bullet}$ A C$^{\bullet}$ E
B$^{\flat}$	$^{\bullet}$B D F A	$^{\bullet}$B D F$^{\bullet}$ A	$^{\bullet}$B D$^{\bullet}$ F A	$^{\bullet}$B D$^{\bullet}$ F A	$^{\bullet}$B D$^{\bullet}$ F$^{\bullet}$ A
E$^{\flat}$	$^{\bullet}$E G$^{\bullet}$ B D	$^{\bullet}$E G$^{\bullet}$ B $^{\bullet}$D	$^{\bullet}$E $^{\bullet}$G B $^{\bullet}$D	$^{\bullet}$E $^{\bullet}$G $^{\bullet\bullet}$B $^{\bullet}$D	$^{\bullet}$E $^{\bullet}$G $^{\bullet}$B $^{\bullet}$D
A$^{\flat}$	$^{\bullet}$A C$^{\bullet}$ E G	$^{\bullet}$A C$^{\bullet}$ E$^{\bullet}$ G	$^{\bullet}$A $^{\bullet}$C $^{\bullet}$E $^{\bullet}$G	$^{\bullet}$A $^{\bullet}$C $^{\bullet\bullet}$E $^{\bullet}$G	$^{\bullet}$A $^{\bullet}$C $^{\bullet\bullet}$E $^{\bullet\bullet}$G
D$^{\flat}$	$^{\bullet}$D F$^{\bullet}$ A C	$^{\bullet}$D F$^{\bullet}$ A $^{\bullet}$C	$^{\bullet}$D $^{\bullet}$F A $^{\bullet}$C	$^{\bullet}$D $^{\bullet}$F $^{\bullet\bullet}$A $^{\bullet}$C	$^{\bullet}$D $^{\bullet}$F $^{\bullet\bullet}$A $^{\bullet\bullet}$C
G$^{\flat}$	$^{\bullet}$G $^{\bullet}$B D F	$^{\bullet}$G $^{\bullet}$B D$^{\bullet}$ F	$^{\bullet}$G $^{\bullet\bullet}$B $^{\bullet}$D F	$^{\bullet}$G $^{\bullet\bullet}$B $^{\bullet\bullet}$D F	$^{\bullet}$G $^{\bullet\bullet}$B $^{\bullet\bullet}$D $^{\bullet\bullet}$F
C$^{\flat}$	$^{\bullet}$C $^{\bullet}$E $^{\bullet}$G $^{\bullet}$B	$^{\bullet}$C $^{\bullet}$E $^{\bullet}$G $^{\bullet\bullet}$B	$^{\bullet}$C $^{\bullet\bullet}$E $^{\bullet}$G $^{\bullet\bullet}$B	$^{\bullet}$C $^{\bullet\bullet}$E $^{\bullet\bullet}$G $^{\bullet\bullet}$B	$^{\bullet}$C $^{\bullet\bullet}$E $^{\bullet\bullet}$G $^{\bullet\bullet\bullet}$B

BLUE: white key

BLACK black key

EQUIVALENT

12 × 5 = 60 7th chords; 3 inversions per chord ⇓ 240 patterns

Notation: F$^{\bullet}$ = F$^{\#}$, $^{\bullet}$B = B$^{\flat}$, $^{\bullet\bullet}$B = B$^{\flat\flat}$, $^{\bullet\bullet\bullet}$B = B$^{\flat\flat\flat}$ etc.

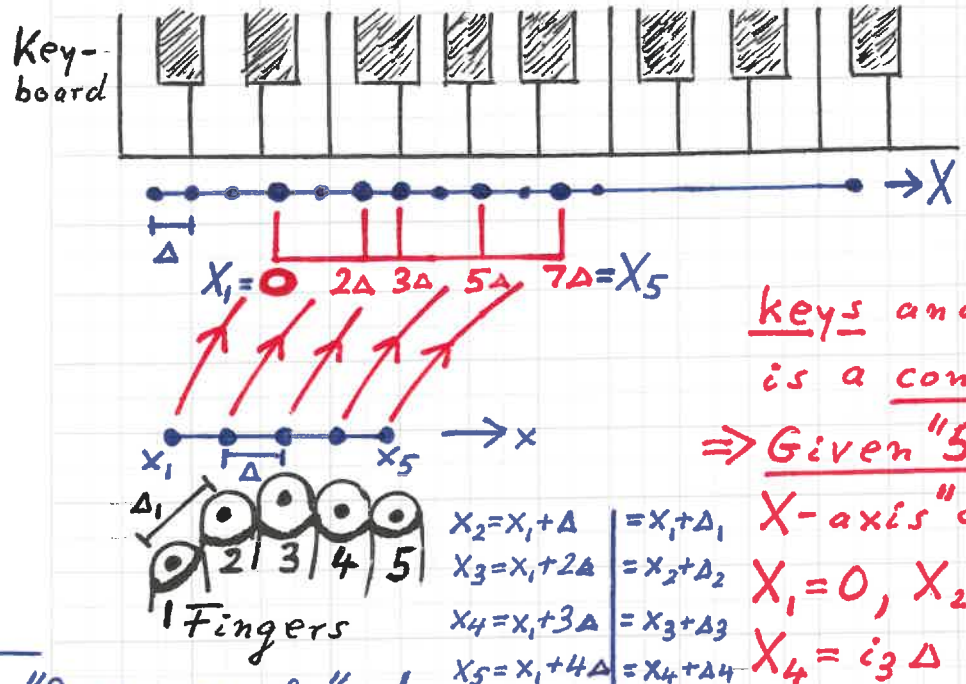
E$^{\#}$ ≙ F, B$^{\#}$ ≙ C, C$^{\flat}$ ≙ B, F$^{\flat}$ ≙ E.

MmM ≙ M3rd + m3rd + M3rd etc.

10/26/2024

• Optimization and Fingering

• Given a chord, a note sequence or a "block of notes," what is a good, near-optimal fingering?



$X_2 = X_1 + \Delta$ $= X_1 + \Delta_1$

$X_3 = X_1 + 2\Delta$ $= X_2 + \Delta_2$

$X_4 = X_1 + 3\Delta$ $= X_3 + \Delta_3$

$X_5 = X_1 + 4\Delta$ $= X_4 + \Delta_4$

"finger-specific" Δ_i!

$X_2 = X_1 + \Delta_1$

$X_3 = X_1 + \Delta_1 + \Delta_2$

$X_4 = X_1 + \Delta_1 + \Delta_2 + \Delta_3$

$X_5 = X_1 + \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4$

Δ_i: "anatomy of fingers"

SIMPLIFICATION: The physical "distance" between halftone keys and between fingers is a constant: Δ.

⇒ Given "5 notes on the X-axis" at X-positions $X_1 = 0$, $X_2 = i_1\Delta_1$, $X_3 = i_2\Delta_1$, $X_4 = i_3\Delta$ and $X_5 = i_4\Delta$) what locations are "ideal" for the fingers? The unknown finger locations on the x-axis — $x_1, x_1+\Delta_1, x_1+2\Delta_1, x_1+3\Delta, x_1+4\Delta$ — must be chosen to **MINIMIZE THE SUM OF THE SQUARED DISTANCES BETWEEN X_i and x_i, $i = 1 \ldots 5$:**

$$D = \sum_{i=1}^{5} (d_i)^2 = (x_1 - 0)^2 + (x_1 + \Delta - i_1\Delta)^2 + (x_1 + 2\Delta - i_2\Delta)^2 + (x_1 + 3\Delta - i_3\Delta)^2 + (x_1 + 4\Delta - i_4\Delta)^2$$

$$= 5x_1^2 + 2\Delta x_1 (10 - (i_1 + i_2 + i_3 + i_4)) + \text{CONST}$$

$$= 5x_1^2 - 2\Delta x_1 (i_1 + i_2 + i_3 + i_4 - 10) + \text{CONST}$$

$$\Rightarrow d/dx_1\, D = 10x_1 - 2\Delta (i_1 + i_2 + i_3 + i_4 - 10) \stackrel{!}{=} 0$$

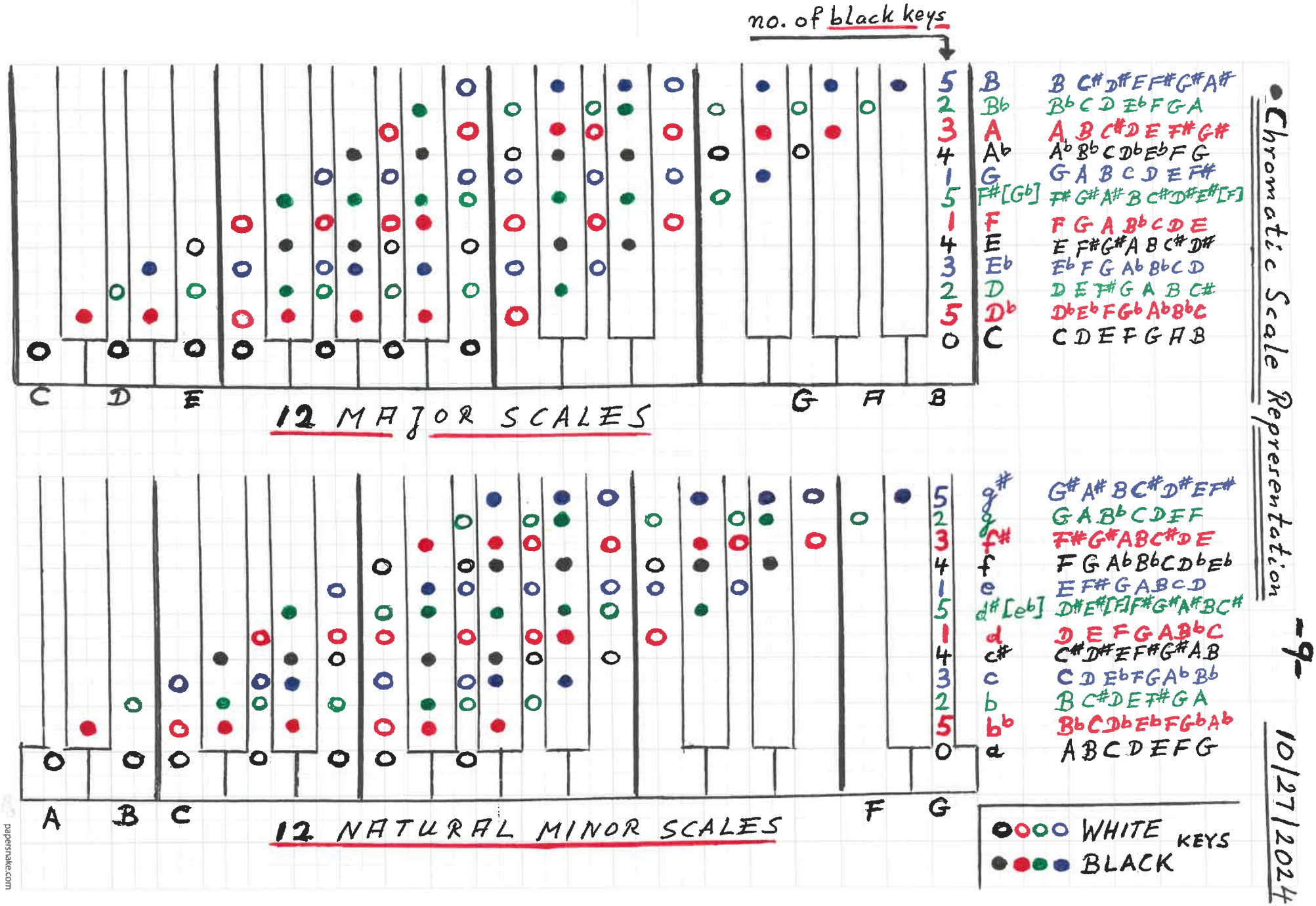
$$\Rightarrow x_1 = \Delta/5 (i_1 + i_2 + i_3 + i_4 - 10)$$

Example: $X_1 = 0$, $X_2 = 1\Delta$, $X_3 = 2\Delta$, $X_4 = 3\Delta$, $X_5 = 4\Delta$

$$\Rightarrow i_1 + i_2 + i_3 + i_4 = 1 + 2 + 3 + 4 = 10$$

$$\Rightarrow x_1 = \Delta/5 (10 - 10) = \Delta/5 \cdot 0 = 0 = X_1 \quad \checkmark$$

Chromatic Scale Representation



10/27/2024

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• Key-inherent Chords & Inversions

10/29/2024

➡ How to memorize these chords?

C	D	E	F	G	A	B	C	D	E	F	G	A	B	C	D	E	F
C		E		G		7th B	C		E		G						
I						root	1st		2nd		3rd	inversions					
	D		F		A		C	D		F		A					
ii		E		G		B		D	E		G		B				
iii		F		A		C			E	F		A		C			
IV		G		B		D				F	G		B		D		
V		A		C		E					G	A		C		E	
vi		B		D		F						A	B		D		F
vii°																	

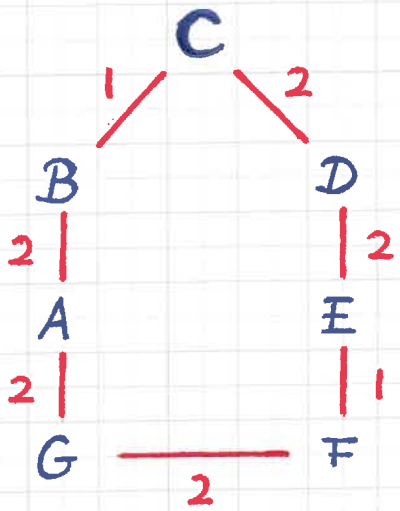
- This example shows all C-major-inherent triads and 7th chords and all inversions!
- One only needs to memorize these SEVEN letter sequences — since the same letter sequences apply to all major and minor keys!
- When a specific key signature is given — i.e., C, G, D, ..., F, B♭, ..., a, ... — one knows that one must apply '#'s and 'b's to certain letters / notes; thus, the seven letters remain the same and are merely modified by the '#' and 'b' symbols!
- Analogously, one can also think of the symbols '#' and 'b' as simple "modifiers of white keys": A '#' implies "playing to the right," a 'b' "playing to the left"!

• Church Modes (Modi) – Kirchentonarten

10/31/2024

• The seven church modes "on the key of C" are:

I	IONIAN = C major	C ^{2} D ^{2} E ^{1} F ^{2} G ^{2} A ^{2} B ^{1} C
ii	DORIAN	D ^{2} E ^{1} F ^{2} G ^{2} A ^{2} B ^{1} C ^{2} D
iii	PHRYGIAN	E ^{1} F ^{2} G ^{2} A ^{2} B ^{1} C ^{2} D ^{2} E
IV	LYDIAN	F ^{2} G ^{2} A ^{2} B ^{1} C ^{2} D ^{2} E ^{1} F
V	MIXOLYDIAN	G ^{2} A ^{2} B ^{1} C ^{2} D ^{2} E ^{1} F ^{2} G
vi	AEOLIAN = a minor	A ^{2} B ^{1} C ^{2} D ^{2} E ^{1} F ^{2} G ^{2} A
vii°	LOCRIAN (HYPOPHRYGIAN)	B ^{1} C ^{2} D ^{2} E ^{1} F ^{2} G ^{2} A ^{2} B



• The seven modes all have 5 whole-tones ('2') and 2 half-tones ('1') between the seven pairs of subsequent notes.

• The sequence of notes CDEFGAB is cyclic, and the half-tone steps are always the steps E-F and B-C.

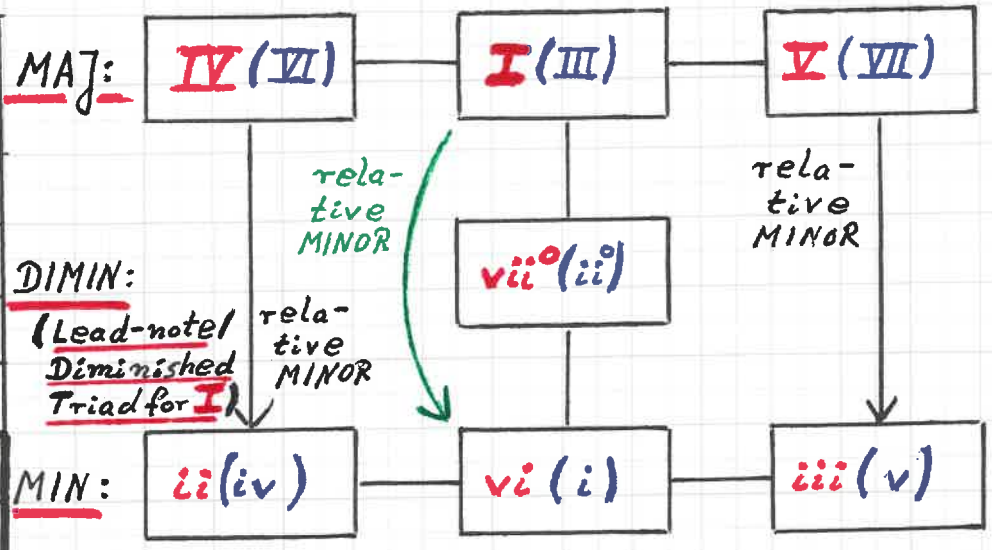
• The note used as first note determines the specific mode and "nature" of an octave.

11/11/2024

The Circle of Fifths and its Relationship with Key-inherent Triads

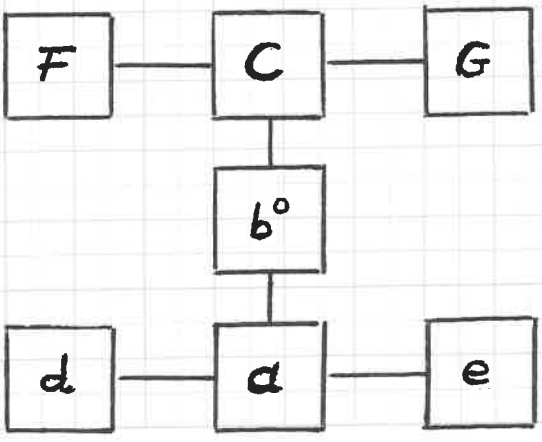
- Every "GROUP" in the circle can be viewed as a group of seven key-inherent triads — representing both MAJOR AND MINOR keys / triads:





Use case: Given a key for I, the resulting sequence of key-inherent triads consists of the triads I maj, ii min, ..., vi min, vii° dimin.

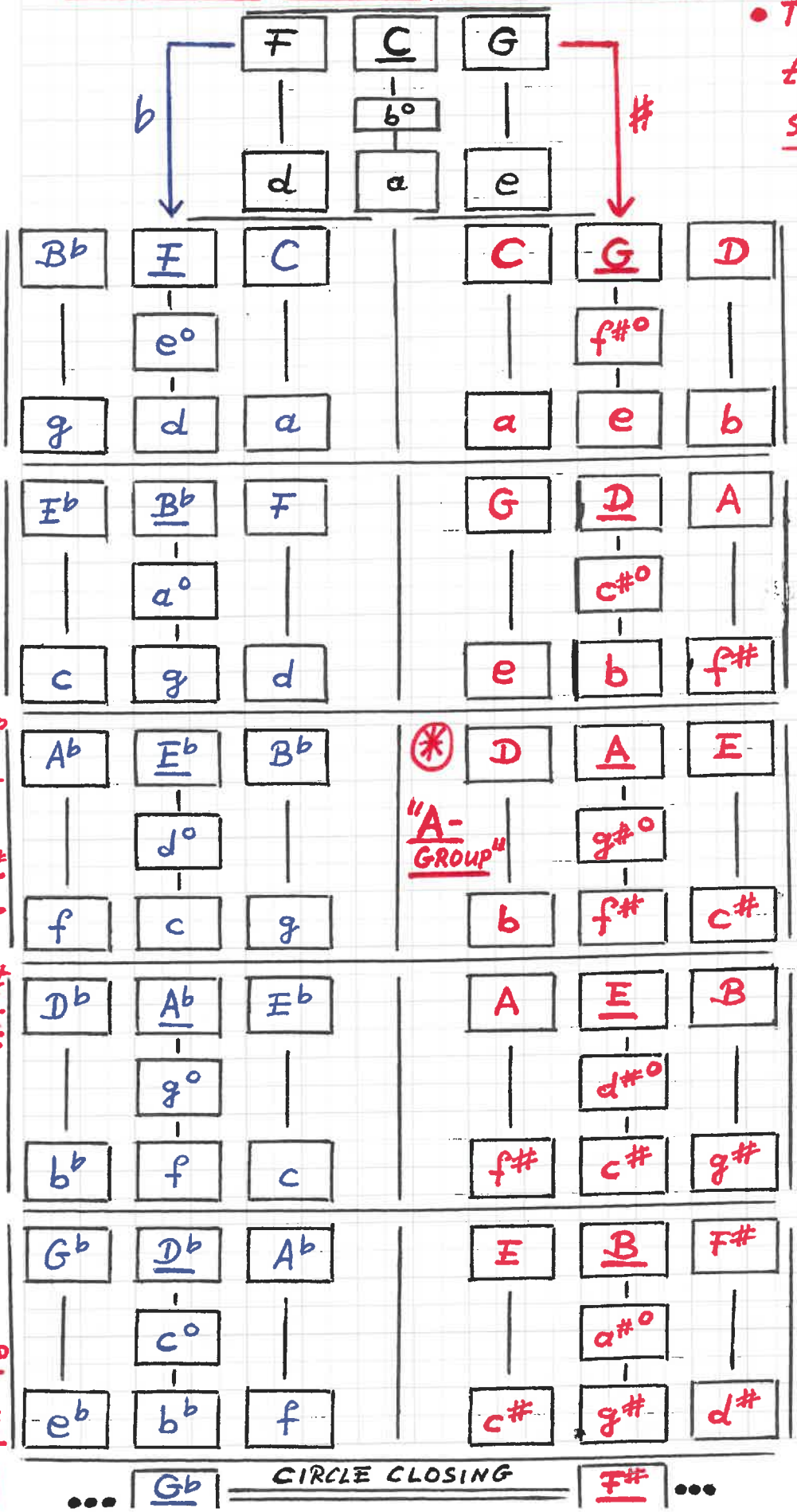
- The seven Roman numerals represent the seven notes C, D, E, F, G, A, B.
- I, IV, V (III, VI, VII): MAJOR key-inherent triad implied
- ii, iii, vi, vii° (i, ii°, iv, v): MINOR key-inherent triad (and one DIMINISHED triad (01) [(01)]) implied.
- The ORDER defined by the seven Roman numerals establishes the key-inherent order of key-inherent triads, i.e., from I (i) to vii° (VII).
- For example, the GROUP FOR I ≙ C leads to:



- The two implied triad sequences are:
CEG, DFA, EGB, FAC, GBD, ACE, BDF;
ACE, BDF, CEG, DFA, EGB, FAC, GBD.
- C maj. and its relative minor, a min., have no sharps or flats. No sharps or flats occur in the seven triads.

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• The Circle of Fifths and its Relationship with Key-inherent Triads



• This figure shows the "GROUPS OF SEVEN" for the keys C, G, D, A, E, B and F, B♭, E♭, A♭, D♭.

• The figure uses the order of the circle of fifths.

• The four groups not shown are these:

B	E#	C#
	e#°	
g#	d#	a#

F#	C#	G#
	b#°	
d#	a#	e#

C♭	G♭	D♭
	f°	
a♭	e♭	b♭

F♭	C♭	G♭
	b♭°	
d♭	a♭	e♭

✱ USE:

1) The group for A maj. has notes A, B, C#, D, E, F#, G#.

2) The resulting A-inherent triad sequence is:
A C# E
B D F#
C# E G#
D F# A
E G# B
F# A C#
G# B D

THE A-GROUP LISTS THE NOTES OF A-INHERENT TRIADS!

"A-GROUP"

• The circle is closing at the bottom, since F# ≙ G♭.

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• THE KEY-INHERENT TRIADS ARE DEFINED BY THE GROUPS OF 7.

→ Circle of 5ths

Group of 7 for C

TP, DP, SP: Tonic, Dominant, Subdominant

PARALLEL (min.)

Major	C^{b}	G^{b}	D^{b}	A^{b}	E^{b}	B^{b}	F	C	G	D	A	E	B	$F^{\#}$	$C^{\#}$	maj. chords
Lead-note	$b^{b\circ}$	f°	c°	g°	d°	a°	e°	b°	$f^{\#\circ}$	$c^{\#\circ}$	$g^{\#\circ}$	$d^{\#\circ}$	$a^{\#\circ}$	$e^{\#\circ}$	$b^{\#\circ}$	dimin. chords
Minor	a^{b}	e^{b}	b^{b}	f	c	g	d	a (TP)	e	b	$f^{\#}$	$c^{\#}$	$g^{\#}$	$d^{\#}$	$a^{\#}$	minor chords
	G^{b}	D^{b}	A^{b}	E^{b}	B^{b}	F	C	G	D	A	E	B	$F^{\#}$	$C^{\#}$	$G^{\#}$	DOMINANT
	F^{b}	C^{b}	G^{b}	D^{b}	A^{b}	E^{b}	B^{b}	F	C	G	D	A	E	B	$F^{\#}$	SUBDOMINANT
	E^{b}	B^{b}	F	C	G	D	A	E	B	$F^{\#}$	$C^{\#}$	$G^{\#}$	$D^{\#}$	$A^{\#}$	$E^{\#}$	DP
	D^{b}	A^{b}	E^{b}	B^{b}	F	C	G	D	A	E	B	$F^{\#}$	$C^{\#}$	$G^{\#}$	$D^{\#}$	SP
	C^{b}	G^{b}	D^{b}	A^{b}	E^{b}	B^{b}	F	C	G	D	A	E	B	$F^{\#}$	$C^{\#}$	TONIC

Notes of keys

No. of half-tone steps between notes: 1, 2, 2, 1, 2, 2, 2

- Major row: Major triads to be constructed on listed notes
- Lead-note row: Diminished triads to be constructed on listed notes
- Minor row: Minor triads to be constructed on listed notes
- Example, group for C: I, ii, iii, IV, V, vi, vii° = CEG, DFA, EGB, FAC, GBD, ACE, BDF
  (C maj. and a min. keys) i, ii°, III, iv, v, VI, VII = ACE, BDF, CEG, DFA, EGB, FAC, GBD

• The Circle of Fifths and its Relationship with Key-inherent Triads

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• CHORD RECOGNITION FROM NOTES

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"How to recognize the chords that (possibly) define the underlying harmonic structure — by considering the given notes?"

• Here: 1, 2, 3, 4, 5, 6, 7 represent the "seven ordered notes of a key, e.g., {1,2,3,4,5,6,7} = {C, D, E, F, G, A, B}.

• We write the seven key-inherent triads and their inversions as follows (maj. and min. triads):

I: 1, 3, 5 ; 3, 5, 8(1) ; 5, 8(1), 10(3)

II: 2, 4, 6 ; 4, 6, 9(2) ; 6, 9(2), 11(4)

III: 3, 5, 7 ; 5, 7, 10(3) ; 7, 10(3), 12(5)

IV: 4, 6, 8(1) ; 6, 8(1), 11(4) ; 8(1), 11(4), 13(6)

V: 5, 7, 9(2) ; 7, 9(2), 12(5) ; 9(2), 12(5), 14(7)

VI: 6, 8(1), 10(3) ; 8(1), 10(3), 13(6) ; 10(3), 13(6), 15(8)=1

VII: 7, 9(2), 11(4) ; 9(2), 11(4), 14(7) ; 11(4), 14(7), 16(9)=2

Ex.: 1,3,5 = CEG ; 2,4,6 = DFA ; 3,5,7 = EGB ; ... ; 7,9,11 = BDF.

• We write the seven key-inherent 7th chords and their inversions as follows (maj. and min 7th chords):

I: 1, 3, 5, 7 ; 3, 5, 7, 8(1) ;
5, 7, 8(1), 10(3) ; 7, 8(1), 10(3), 12(5) ;

II: 2, 4, 6, 8(1) ; 4, 6, 8(1), 9(2) ;
6, 8(1), 9(2), 11(4) ; 8(1), 9(2), 11(4), 13(6) ;

III: 3, 5, 7, 9(2) ; 5, 7, 9(2), 10(3) ;
7, 9(2), 10(3), 12(5) ; 9(2), 10(3), 12(5), 14(7) ;

IV: 4, 6, 8(1), 10(3) ; 6, 8(1), 10(3), 11(4) ;
8(1), 10(3), 11(4), 13(6) ; 10(3), 11(4), 13(6), 15(1) ;

V: 5, 7, 9(2), 11(4) ; 7, 9(2), 11(4), 12(5) ;
9(2), 11(4), 12(5), 14(7) ; 11(4), 12(5), 14(7), 16(2)

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• CHORD RECOGNITION FROM NOTES

VI: 6, 8(1), 10(3), 12(5); 8(1), 10(3), 12(5), 13(6);
10(3), 12(5), 13(6), 15(1); 12(5), 13(6), 15(1), 17(3)

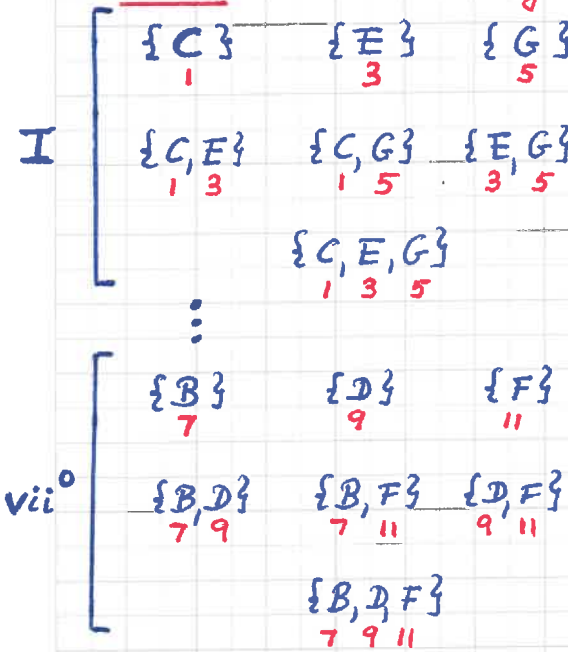
VII: 7, 9(2), 11(4), 13(6); 9(2), 11(4), 13(6), 14(7);
11(4), 13(6), 14(7), 16(2); 13(6), 14(7), 16(2), 18(4).

Ex.: 1, 3, 5, 7 = CEGB; 2, 4, 6, 8(1) = DFAC;
3, 5, 7, 9(2) = EGBD; ...; 7, 9(2), 11(4), 13(6) = BDFA.

• Note. MODULO-ARITHMETIC is used for notes / numbering, i.e., when considering the key of C:
C ≙ 1, 8, 15, 22; D ≙ 2, 9, 16, 23; B ≙ 7, 14, 21, 28.

⇒ ONE CAN "GROUP A SEQUENCE OF NOTES TOGETHER AND ORDER THEM" (FROM LOW TO HIGH) TO ESTABLISH "NOTE SETS" THAT DEFINE POSSIBLE "CHORD SUBSETS", I.E., POTENTIALLY UNDERLYING CHORDS!

Ex.: Considering the key of C major, the seven key-inherent triads CEG, ..., BDF can occur in the form of "incomplete triads," i.e., as "note subsets in a score, see left figure. Mathematically, a set with three elements (notes) has one subset with three notes and three subsets with two notes and one note, respectively. FOR EXAMPLE, THE NOTE SUBSET {C, E} CAN BE A SUBSET OF {C, E, G} OR {A, C, E}.



The seven subsets of possible note subsets defining the seven triads I, ii, iii, IV, V, vi and vii° in C.

• CHORD RECOGNITION FROM NOTES

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⇒ One can systematically determine all triads and/or 7th chords that could potentially reflect an underlying harmonic chord structure/harmony. Thus, one must "map" all selected/extracted note subsets (with ordered notes) to all viable triads and/or 7th chords of which they are (or could be) a part.

Note. By using the concept of "note sets" with the implicit order of notes from Low to high within the sequence C(1)D(2)E(3)F(4)G(5)A(6)B(7), for example, a triad or 7th chord note set defines the chords in root position and their inversions: {C, E, G} defines the triads CEG, EGC and GCE; {C, E, G, B} defines the 7th chords CEGB, EGBC, GBCE and BCEG.

TRIADS

I	II	III	IV	V	VI	VII
CEG	DFA	EGB	FAC	GBD	ACE	BDF
CE, CG, EG	DF, DA, FA	EG, EB, GB	FA, FC, AC	GB, GD, BD	AC, AE, CE	BD, BF, DF
C, E, G	D, F, A	E, G, B	F, A, C	G, B, D	A, C, E	B, D, F

⇒ Each note (C, D, E, F, G, A, B) is part of THREE triads.

The 3rd's (CE, EG, DF, FA, EG, GB, FA, AC, GB, BD, AC, CE, BD, DF) are part of TWO triads.

The 5th's (CG, DA, EB, FC, GD, AE, BF) are part of ONE triad.

7th CHORDS

I	...	VII
CEGB		BDFA
CEG, CEB, CGB, EGB		BDF, BDA, BFA, DFA
CE, CG, CB, EG, EB, GB		BD, BF, BA, DF, DA, FA
C, E, G, B		B, D, F, A

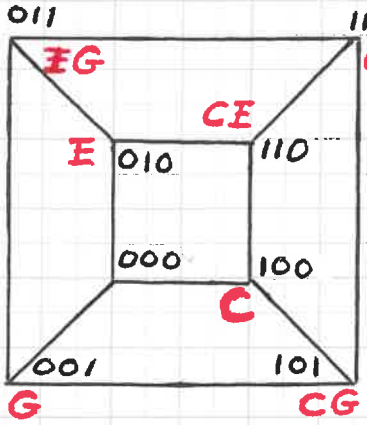
7th chords and their "subset chords" (2-note, 3-note and 1-note chords)

⇐ Notes occur 4 times, 3rd's occur 3 times, 5th's occur 2 times and 7th's occur once in the seven 7th chords. (Notes = C, D, ...; 3rd's = CE, EG, ...; 5th's = CG, EB, ...; 7th's = CB, DC, ...)

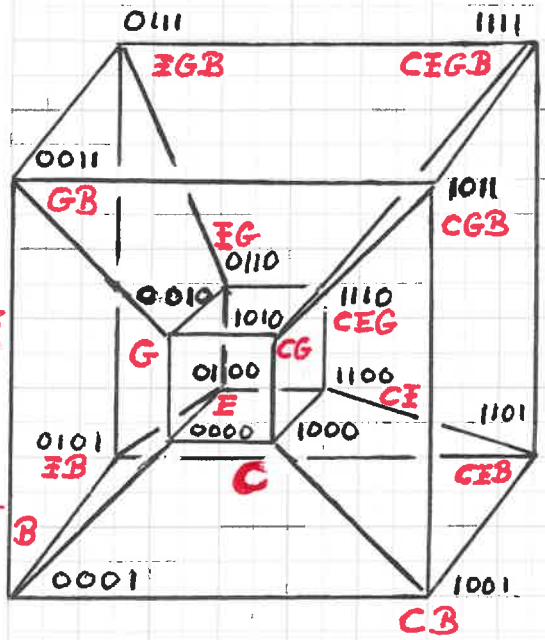
• CHORD RECOGNITION FROM NOTES

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$2^3 = 8$ triad possibilities for $b_1 b_3 b_5$



$2^4 = 16$ 7th chord possibilities for $b_1 b_3 b_5 b_7$



In order to determine a (the) chord(s) possibly implied and played by a sequence of notes, one can think of a bit sequence that determines which notes occur in the considered sequence. Since the seven "base notes" are C, D, E, F, G, A and B, the bit sequence $b_1 b_2 b_3 b_4 b_5 b_6 b_7$ can indicate the notes that exist in a sequence. For example, 1010101 means that the notes C, E, G and B are represented (each of these four notes occurring at least once in the considered sequence). Further, the 3-bit (4-bit) sequence $b_1 b_3 b_5$ ($b_1 b_3 b_5 b_7$) can be used to define the observed note subset that possibly identifies a note subset of the triad CEG (top figure) (of the 7th chord CEGB (bottom figure)). Here, the bit sequence $b_1 b_2 b_3 b_4 b_5 b_6 b_7$ corresponds to the associated key sequence C, D, E, F, G, A, B ($\hat{=}$ I). Of course, the sequence can also be used to represent the other possible key sequences, i.e., D, E, F, G, A, B, C, ..., and B, C, D, E, F, G, A ($\hat{=}$ II to VII). Again, 3-bit (4-bit) bit sequences can be used for triad (7th chord) note subsets.

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• CHORD RECOGNITION FROM NOTES

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Here, it is assumed that the following note-index correspondences apply: C ≙ 1, D ≙ 2, E ≙ 3, F ≙ 4, G ≙ 5, A ≙ 6 and B ≙ 7. For example, one can identify a 4-note 7th chord via 4-bit bit sequences — "over C, D, E, F, G, A and B." Specifically, the seven 7th chords and associated bit sequences are:

CEGB	≙ $b_1 b_3 b_5 b_7$	CEG	≙ $b_1 b_3 b_5$
DFAC ≙ CDFA	≙ $b_1 b_2 b_4 b_6$	DFA	≙ $b_2 b_4 b_6$
EGBD ≙ DEGB	≙ $b_2 b_3 b_5 b_7$	EGB	≙ $b_3 b_5 b_7$
FACE ≙ CEFA	≙ $b_1 b_3 b_4 b_6$	FAC ≙ CFA	≙ $b_1 b_4 b_6$
GBDF ≙ DFGB	≙ $b_2 b_4 b_5 b_7$	GBD ≙ DGB	≙ $b_2 b_5 b_7$
ACEG ≙ CEGA	≙ $b_1 b_3 b_5 b_6$	ACE ≙ CEA	≙ $b_1 b_3 b_6$
BDFA ≙ DFAB	≙ $b_2 b_4 b_6 b_7$	BDF ≙ DFB	≙ $b_2 b_4 b_7$

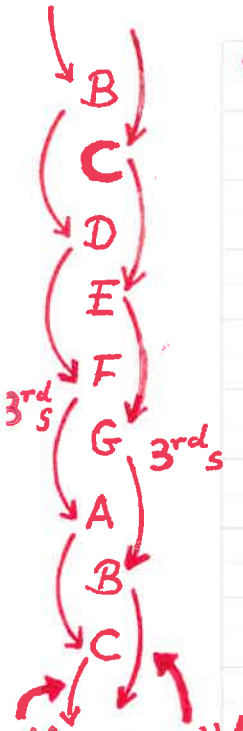
For completeness, the seven triads and their associated bit sequences are listed as well. Since the 3rd's and 5th's constituting the specific triads are also occurring in the corresponding 7th chords, the specific 3-bit sequences also show up as sub-sequences on the left-hand side.

THUS, TO RECOGNIZE A SPECIFIC TRIAD OR 7th CHORD FROM A SEQUENCE / GROUP / SET OF NOTES, ONE CAN USE THE 7-BIT SEQUENCE $b_1 b_2 b_3 b_4 b_5 b_6 b_7$ TO DETERMINE WHETHER THE SUBSET OF "ON-BITS" (= BITS WITH VALUE 1) IS POSSIBLY INDICATING THE EXISTENCE / USE OF A CERTAIN CHORD IN A COMPOSITION.

One must keep in mind that "ornamental," "trill," "transitional" or "chromatic-sequence" notes can arise as well.

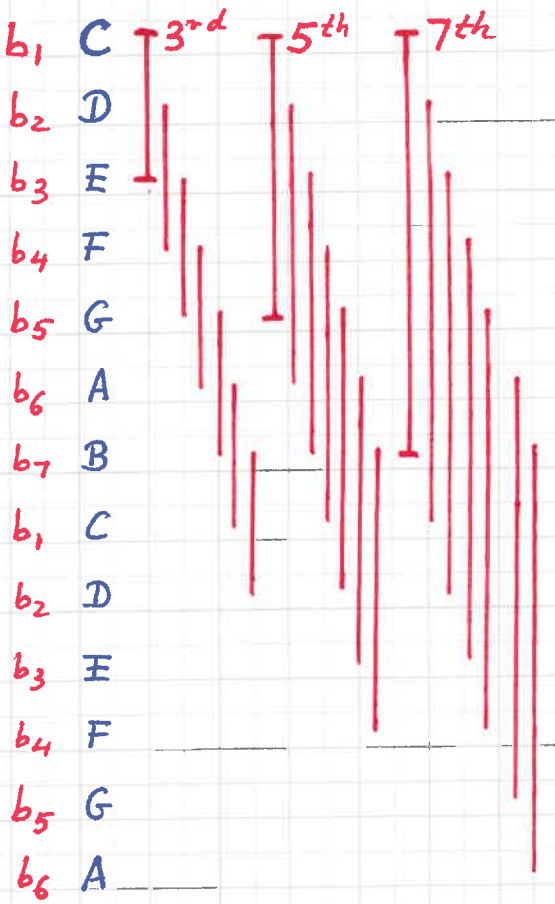
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• CHORD RECOGNITION FROM NOTES



Nearly all/most common chords can be understood or represented by "stacked 3rd's" – when ordering the chord's notes accordingly!

Note. Modifications of the "base notes" via '#' and 'b' are ignored in the discussed method for harmony / chord recognition and analysis. Thus, every "base note" stands as a representative note for its three forms, i.e., $C \hat{=} \{C^b, C, C^\#\}, \ldots, B \hat{=} \{B^b, B, B^\#\}$. Of course, to analyze all major and minor keys and their triads, 7th chords — including all key-inherent ones — properly, one must consider '#' and 'b' modifications.



Since the described method recognizes, for example, a triad by note sets, it does not matter whether the triad is played in root position or in an inversion: CEG, EGC, GCE all have the associated 7-bit "code" $b_1b_2b_3b_4b_5b_6b_7 = 1010100$. The left illustration focuses on the fact that triads and 7th chords "contain" 3rd's, 5th's and 7th's; all possibilities are shown.

Example:

I: CEG

CE CG EG

C E G

∅

⇒ also use II: DFA, ..., VII

Note. (Modifications via '#' and 'b' are not considered.) One must keep in mind the NUMBERS of POSSIBLE CHORDS: THE 3 (4) NOTES OF A TRIAD NOTE SET (7th CHORD NOTE SET) DEFINE 1 3-NOTE, 3 2-NOTE, 3 "1-NOTE" AND 1 "0-NOTE" CHORDS (1 4-NOTE, 4 3-NOTE, 6 2-NOTE, 4 "1-NOTE" AND 1 "0-NOTE CHORDS). THE 3(4) NOTES IMPLY $2^3 = 8$ ($2^4 = 16$) CHORDS.

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• CHORD RECOGNITION FROM NOTES

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It is therefore possible to understand the note sets defining certain chords via "index sets" — where the index value **1** is associated with the "base note". The following examples explain this viewpoint via the seven key-inherent triads and 7th chords for $C \hat{=} 1$:

I	$C_1\ E_3\ G_5$	1 3 5	These seven index
II	$D_2\ F_4\ A_6$	2 4 6	sets represent all
III	$E_3\ G_5\ B_7$	3 5 7	the listed seven
IV	$F\ A\ C_1$	1 4 6	triads (complete
V	$G\ B\ D$	2 5 7	triads) and also
VI	$A\ C\ E$	1 3 6	their two inversions.
VII	$B\ D\ F$	2 4 7	

(+3 between each pair of index rows)

I	$C_1\ E_3\ G_5\ B_7$	1 3 5 7	These seven index
II	$D_2\ F_4\ A_6\ C_1$	1 2 4 6	sets represent all
III	$E\ G\ B\ D$	2 3 5 7	the listed seven
IV	$F\ A\ C\ E$	1 3 4 6	7th chords (complete
V	$G\ B\ D\ F$	2 4 5 7	7th chords) and also
VI	$A\ C\ E\ G$	1 3 5 6	their three in-
VII	$B\ D\ F\ A$	2 4 6 7	versions.

(+1 between each pair of index rows)

- One must keep in mind that such chords can also appear in "incomple representation" — where some not are not used.
- The fact that such chords are constructed by "stacking 3rd's" implies that two consecutive indices (of complete chords) generally have an "index distance" of **2**.
- Analogously, one must consider the notes D, E, F, G, A and B as "base notes" — having the associated index value **1** ($\hat{=}$ **I**).