• Visualizing Most Common Chords

11/20/2024

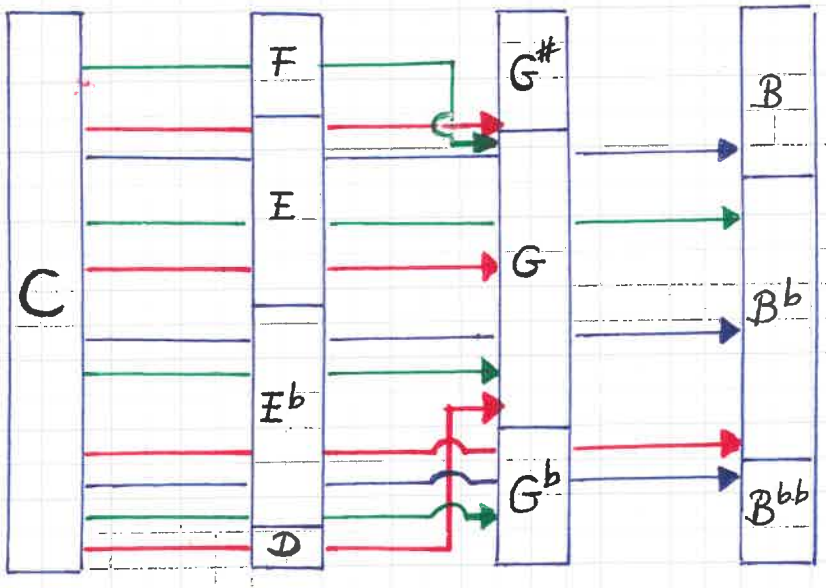
Common triads and 7th chords (over C)* are:

Name	Notes of chord, with notes of inversions		Notes of "general chords" represented via ordinal numbers relative to root note ≙ 1 (e.g., C≙1, D≙2, E≙3, F≙4, G≙5, B≙7)	
C sus4	C F G	C F	1 4 5	1 4
C aug	C E $G^{\#}$	C E	1 3 5^{+}	1 3
C maj 7	C E G B	C E G	1 3 5 7	1 3 5
C 7	C E G B^{b}	C E G	1 3 5 $^{-}7$	1 3 5
C maj	C E G	C E	1 3 5	1 3
C min 7	C E^{b} G B^{b}	C E^{b} G	1 $^{-}3$ 5 $^{-}7$	1 $^{-}3$ 5
C min	C E^{b} G	C E^{b}	1 $^{-}3$ 5	1 $^{-}3$
C halfdim 7	C E^{b} G^{b} B^{b}	C E^{b} G^{b}	1 $^{-}3$ $^{-}5$ $^{-}7$	1 $^{-}3$ $^{-}5$
C fulldim 7	C E^{b} G^{b} B^{bb}	C E^{b} G^{b}	1 $^{-}3$ $^{-}5$ $^{--}7$	1 $^{-}3$ $^{-}5$
C dim	C E G^{b}	C E	1 3 $^{-}5$	1 3
C sus2	C D G	C D	1 2 5	1 2

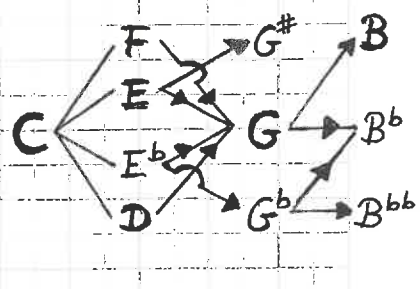
* Keep also in mind:
- Neapolitan
- Sixte Ajoutée chords

(-) ≙ 'b'
(+) ≙ '#'
(--) ≙ "bb"

⇒ Visualization:



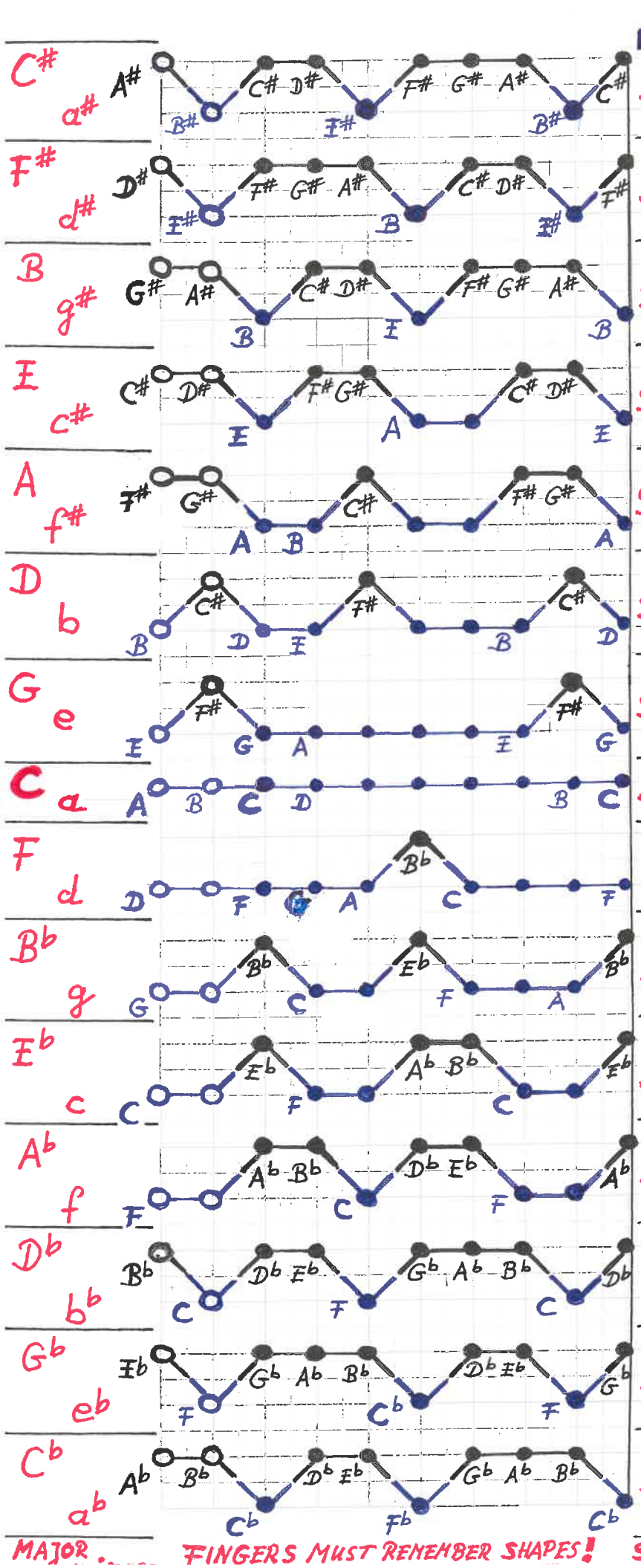
"Compact" simplification:



Last note of chord indicated by ▶, ▶, ▶.

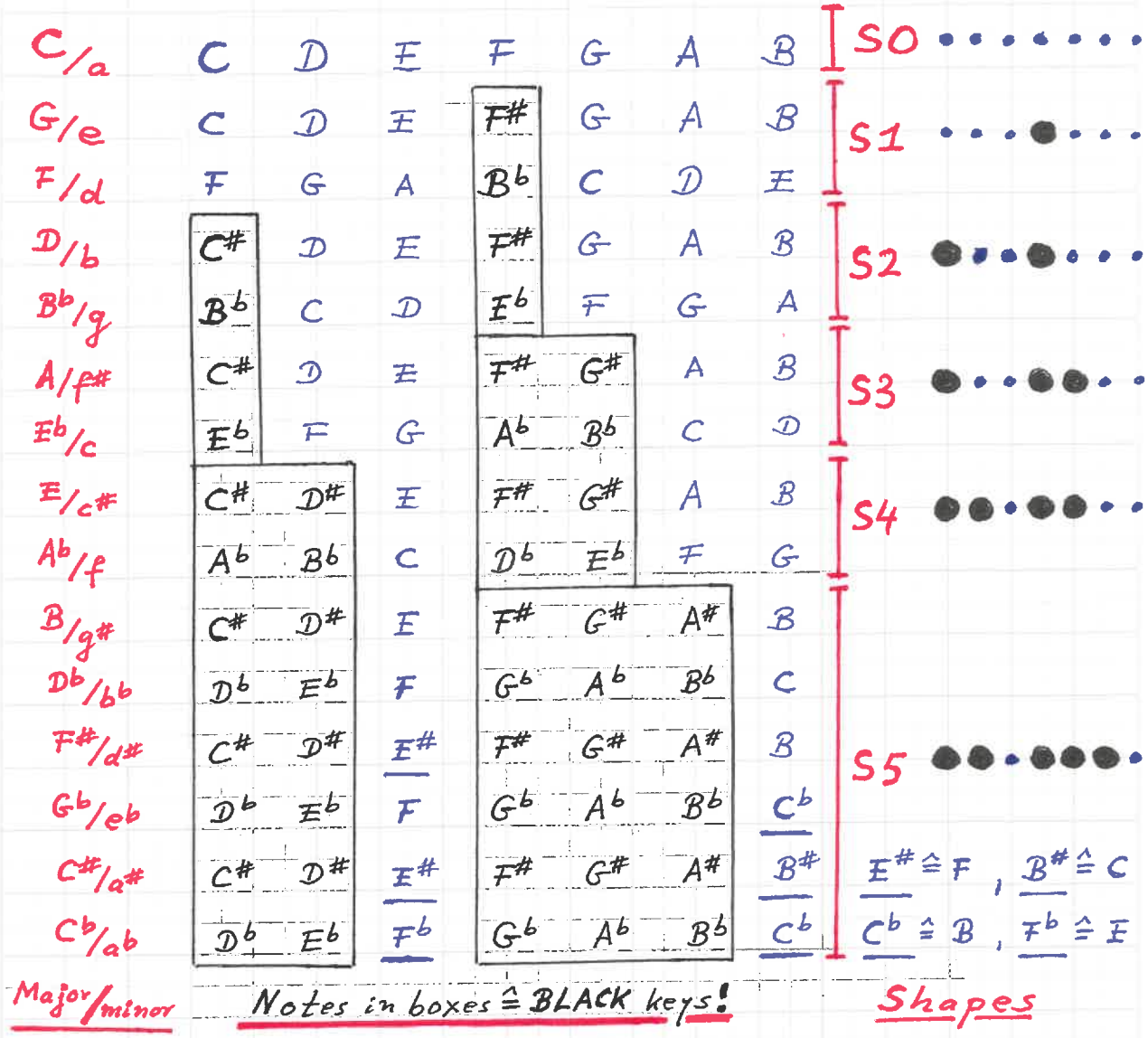
This representation applies to all 12 half-tones serving as root 1:

"$1 \in \{C, C^{\#}(D^{b}), D, D^{\#}(E^{b}), E, F, F^{\#}(G^{b}), G, G^{\#}(A^{b}), A, A^{\#}(B^{b}), B\}$."


Scale Shapes -I- 11/21/2024
C# a#
A# C# D# F# G# A# C# B# F# B#
F# d#
D# F# G# A# C# D# F# E# B E#
B g#
G# A# C# D# F# G# A# B E B
E c#
C# D# F# G# C# D# E A E
A f#
F# G# C# F# G# A B A
D b
C# F# C# B D E B D
G e
F# G A F# E G
C a
A B C D B C
F d
D F G A Bb C F
Bb g
G Bb C Eb F A Bb
Eb c
C Eb F Ab Bb C Eb
Ab f
F Ab Bb C Db Eb F Ab
Db bb
Bb C Db Eb F Gb Ab Bb C Db
Gb eb
Eb F Gb Ab Bb Cb Db Eb F Gb
Cb ab
Ab Bb Cb Db Eb Fb Gb Ab Bb Cb
S7#
S6#
S5#
S4#
S3#
S2#
S1#
S0
S1b
S2b
S3b
S4b
S5b
S6b
S7b
=
=
=
EQUAL
MAJOR
FINGERS MUST REMEMBER SHAPES!
Shape
Legend
WHITE keys are shown as 'o', BLACK keys are shown as '•'.
(The additional keys required for minor key signatures are shown as 'o' and 'o'.)
Scale shapes are called based on their numbers of '#' or 'b', i.e., S0, S1#, ..., S7#, S1b, ..., S7b. Since C#maj ≙ Dbmaj, F#maj ≙ Gbmaj and Bmaj ≙ Cbmaj, their shapes are equal: S7#=S5b, S6#=S6b, S5#=S7b.
Key-inherent chords, e.g., triads and 7th chords, are also defined by the shape of a specific key – by considering / playing the respective chord "subshapes".
The shapes S1b, ..., S7b are "shifted versions" of the shapes S1#, ..., S7#, respectively – "polygon shifts."
Shapes change gradually, "smoothly," from S0 to S7# and from S0 to S7b.

11/22/2024

Scale Shapes



■ By SHIFTING the notes of all the major and minor keys — such that the notes with associated black keys appear in the same columns, as much as possible — one sees the SIX SHAPES that define the six possible black-and-white-key sequences: S0 (no black keys), S1 (one black key), ..., S5 (five black key). For a given key, its associated scale shape repeats in all octaves of the 88-key keyboard.

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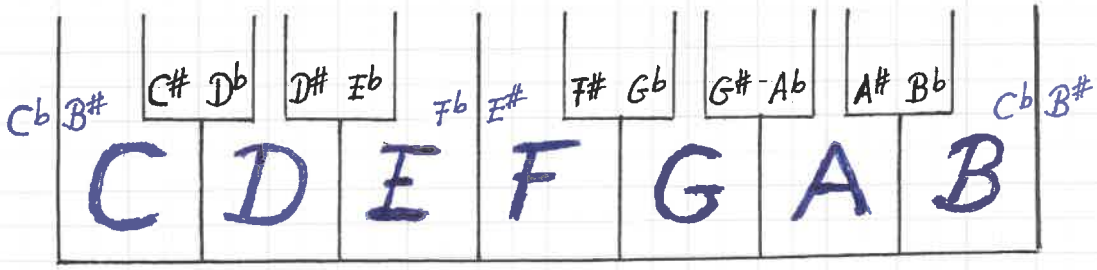
11/23/2024

Scale Shapes

An alternative way to organize the notes to represent scale shapes is the following table:

Major/minor								
C/a	F	G	A	B	C	D	E	S0
G/e	F#	G	A	B	C	D	E	S1
F/d	Bb	C	D	E	F	G	A	
D/b	F#	G	A	B	C#	D	E	S2
Bb/g	Eb	F	G	A	Bb	C	D	
A/f#	F#	G#	A	B	C#	D	E	S3
Eb/c	Ab	Bb	C	D	Eb	F	G	
E/c#	F#	G#	A	B	C#	D#	E	S4
Ab/f	Db	Eb	F	G	Ab	Bb	C	
B/g#	F#	G#	A#	B	C#	D#	E	
Db/bb	Gb	Ab	Bb	C	Db	Eb	F	
F#/d#	F#	G#	A#	B	C#	D#	E#	S5
Gb/eb	Gb	Ab	Bb	Cb	Db	Eb	F	
C#/a#	F#	G#	A#	B#	C#	D#	E#	
Cb/ab	Gb	Ab	Bb	Cb	Db	Eb	Fb	

Notes in boxes ≙ BLACK keys! Shapes

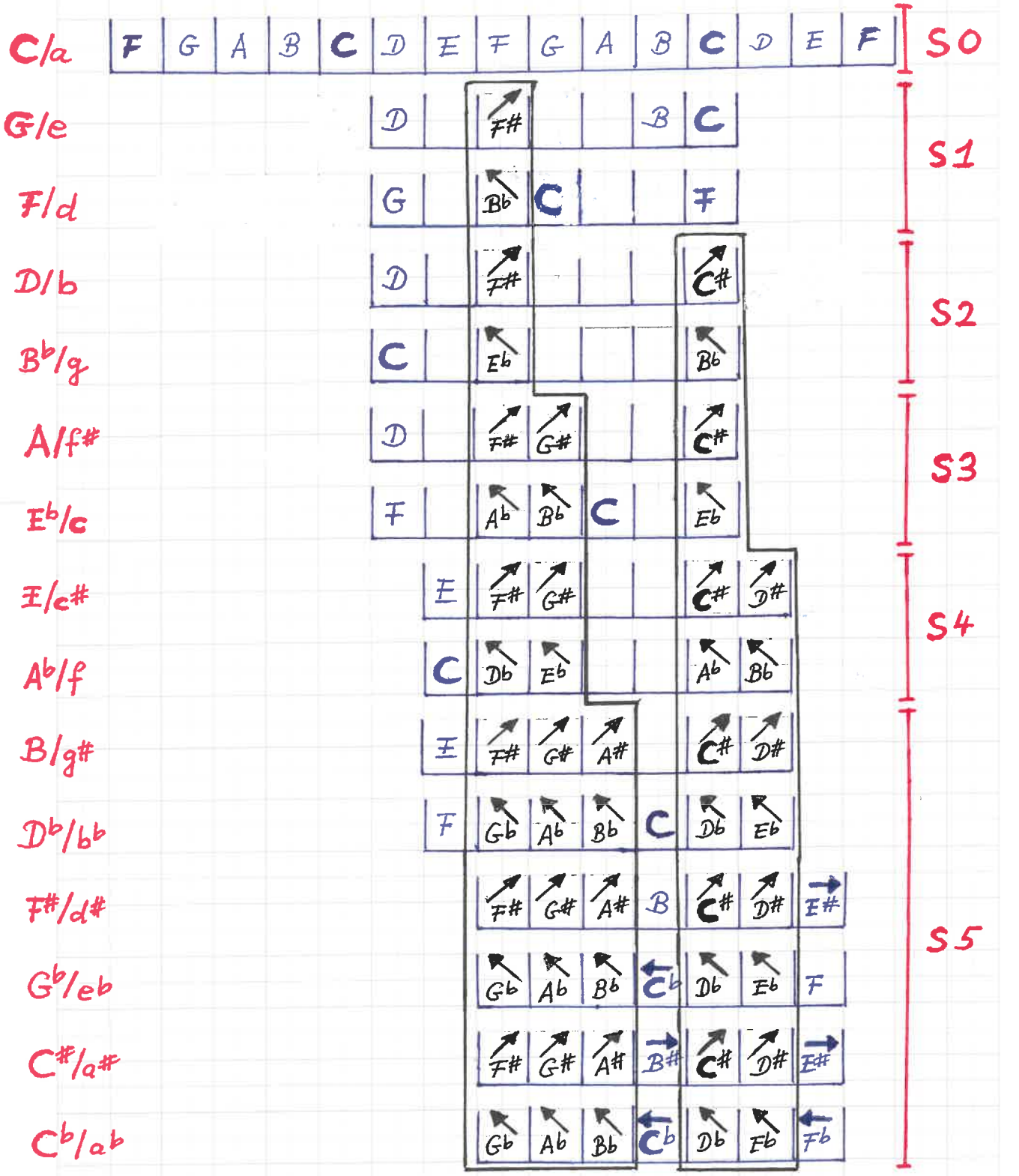


"One can always think of only the 'basic notes' C, D, E, F, G, A, B — modified by the key-specific '#' and 'b'.

11/24/2024

Scale Shapes

One can also visualize the notion of understanding all scales / scale shapes consisting only of the "seven basic notes (and keys)" as follows:

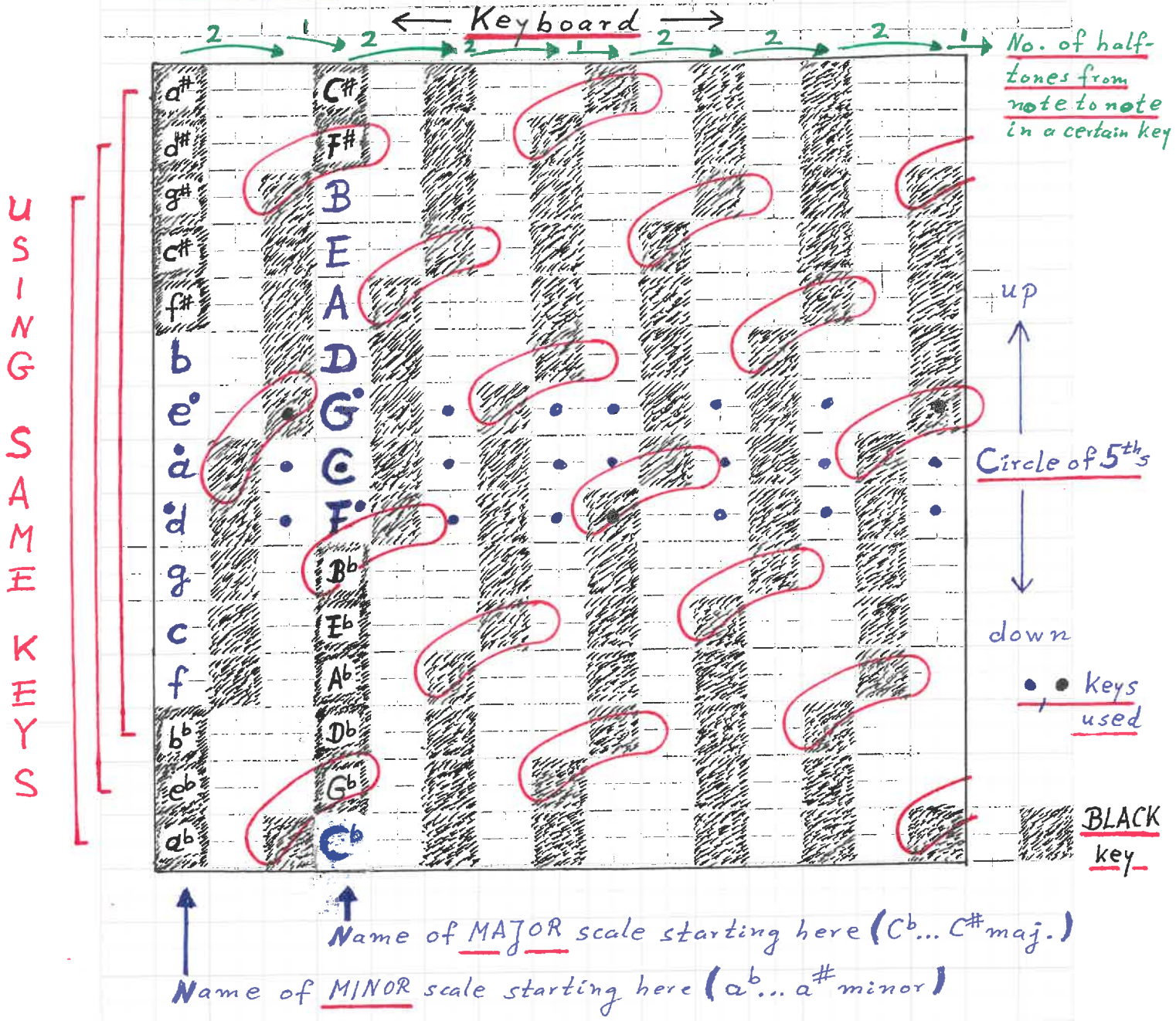


Here, the "seven-note keyboard" is shifted, leaving notes modified by '#' and 'b' in BLACK boxes.

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11/26/2024

Scale Shapes



- This visualization shows that "key movement one-by-one, following the circle of 5ths order," is the "smoothest possible movement from key to key: Only the key-pairs with a ( ) -high-lighted pair of notes/keys permit minimal change. For example, key modulation following the circle of 5ths order requires one to only change the use of black keys minimally."

• KEYS, SCALES, CIRCLES

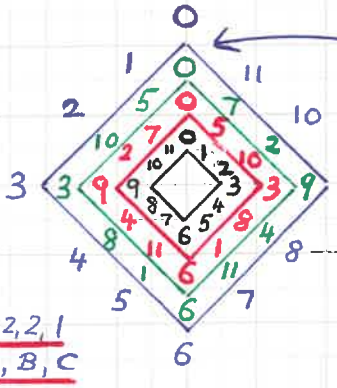
12/2/2024

One can analyze key signatures, implied scales and "circles," e.g., the circle of 5ths, using a viewpoint from combinatorics. The following analysis considers half-tone steps as basis for indexing and numbering. Further, the index of a root note is 0, and (most) calculations use "modulo-12 arithmetic." We do not use '#' or 'b' notation; instead, for example, we write ^{F}G to represent $F^{\#}$ and G^{b}. The next sequences of notes are obtained when starting with the root note C (=0) and ending with the note C after "moving up" from note to note by a constant number of half-tone steps, called s:

s=1	0	1	2	3	4	5	6	7	8	9	10	11	0
	C	^{C}D	D	^{D}E	E	F	^{F}G	G	^{G}A	A	^{A}B	B	C
s=2	0	2	4	6	8	10	0						
	C	D	E	^{F}G	^{G}A	^{A}B	C						
s=3	0	3	6	9	0								
	C	^{D}E	^{F}G	A	C								
s=4	0	4	8	0									
	C	E	^{G}A	C									
s=5 ('b')	0	5	10	3	8	1	6	11	4	9	2	7	0
	C	F	^{A}B	^{D}E	^{G}A	^{C}D	^{F}G	B	E	A	D	G	C
s=6	0	6	0										
	C	^{F}G	C										
s=7 ('#')	0	7	2	9	4	11	6	1	8	3	10	5	0
	C	G	D	A	E	B	^{F}G	^{C}D	^{G}A	^{D}E	^{A}B	F	C
s=8	0	8	4	0									
	C	^{G}A	E	C									

(number of half-tone steps = $\frac{1}{2}\cdot s$)

Sequence of numbers of half-tone steps for C maj.: 2,2,1,2,2,2,1
⇒ C, D, E, F, G, A, B, C



Four circles for number of half-tone steps s=1, s=5, s=7, s=11, MODULO 12.

MOVING FORWARD / MOVING BACKWARD → IN CIRCLE OF FIFTHS

s=5 ('b'): C, F, B^{b}, E^{b}, A^{b}, D^{b}, G^{b}, C^{b}

s=7 ('#'): C, G, D, A, E, B, $F^{\#}$, $C^{\#}$

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• KEYS, SCALES, CIRCLES

s=9	0	9	6	3	0								
	C	A	F_G	D_E	C								
s=10	0	10	8	6	4	2	0						
	C	A_B	G_A	F_G	E	D	C						
s=11	0	11	10	9	8	7	6	5	4	3	2	1	0
	C	B	A_B	A	G_A	G	F_G	F	E	D_E	D	C_D	C
s=0 (s=12)	0	0											
	C	C											

• **Note.** Instead of using positive integer values for s, with $0 \leq s \leq 11$, one can also use NEGATIVE integer values for s, with $-11 \leq s \leq 0$. These are "corresponding positive and negative values" for s: $s=1 \hat{=} s=-11$; $s=2 \hat{=} s=-10$; $s=3 \hat{=} s=-9$; $s=4 \hat{=} s=-8$; $s=5 \hat{=} s=-7$; $s=6 \hat{=} s=-6$; $s=7 \hat{=} s=-5$; $s=8 \hat{=} s=-4$; $s=9 \hat{=} s=-3$; $s=10 \hat{=} s=-2$; $s=11 \hat{=} s=-1$. Thus, a specific positive s-value has the corresponding negative s-value of $s-12$: $s \hat{=} s-12$, where $s \in \{0, 1, \ldots, 11\}$.

(Example: Using $s=-5$, one obtains the index/number sequence $0, -5, -10, -15, -20, -25, \ldots, -60$ that is equivalent to $0, -5 \bmod 12, -10 \bmod 12, -15 \bmod 12, \ldots, -60 \bmod 12$, which becomes **0, 7, 2, 9, 4, 11, 6, 1, 8, 3, 10, 5, 0** — which is the sequence one also obtains for $s = -5+12 = 7$. In other words, "moving from C by -5 half-tone steps again and again corresponds to / is equivalent to moving from C by +7 half-tone steps again and again.") As a consequence, one sees that the sequences for s=1 and s=-11 are 0, 1, 2, ..., 0; for s=11 and s=-1 are 0, 11, 10, ..., 0; for s=5 and s=-7 are 0, 5, 10, ..., 0; and for s=7 and s=-5 are 0, 7, 2, ..., 0.

• KEYS, SCALES, CIRCLES

12/3/2024

	S=1: C	C_D	D	D_E	E	F	F_G	G	G_A	A	A_B	B	C
s=0	0 <												12(0)
s=1	0 <	1 <	2 <	3 <	4 <	5 <	6 <	7 <	8 <	9 <	10 <	11 <	12(0)
s=2	0 <		2 <		4 <		6 <		8 <		10 <		12(0)
s=3	0 <			3 <			6 <			9 <			12(0)
s=4	0 <				4 <				8 <				12(0)
s=5	12(0) >	5 <	10 >	3 <	8 >	1 <	6 <	11 >	4 <	9 >	2 <	7 >	0(12)
s=6	0(12) ≶						6 ≶						12(0)
s=7	0(12) <	7 >	2 <	9 >	4 <	11 >	6 >	1 <	8 >	3 <	10 >	5 <	12(0)
s=8	12(0) >				8 >				4 >				0(12)
s=9	12(0) >			9 >			6 >			3 >			0(12)
s=10	12(0) >		10 >		8 >		6 >		4 >		2 >		0(12)
s=11	12(0) >	11 >	10 >	9 >	8 >	7 >	6 >	5 >	4 >	3 >	2 >	1 >	0(12)
s=12 (0)	12(0) >												0(12)

specific chromatic note sequence and index sequence

EQUIVALENT

6 '>', 6 '<'

'6' = tritonus

6 '<', 6 '>'

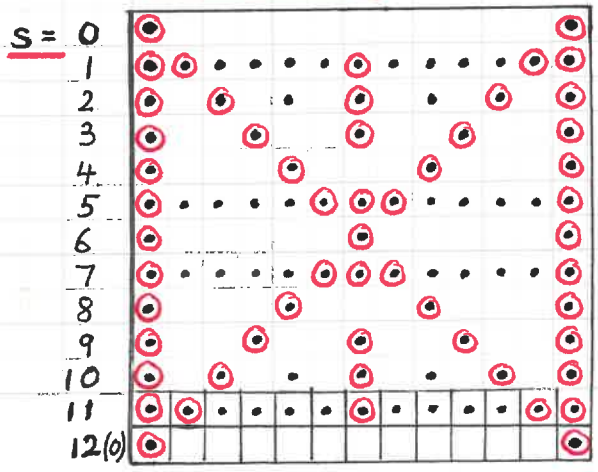
s = 0, 1, 2, 3, 4 : only '<' ⇒ moving up in octave

s = 8, 9, 10, 11, 12 : only '>' ⇒ moving down in octave

s = 5, 7 : '>' and '<' ⇒ moving up/down in octave

s = 6 : only '<' or only '>' ⇒ moving up or down in octave

(All "movements" from note to note are restricted to only one octave as a consequence of modulo-12 arithmetic, allowing only 0, 1, 2, ..., 12 half-tone steps from C.)

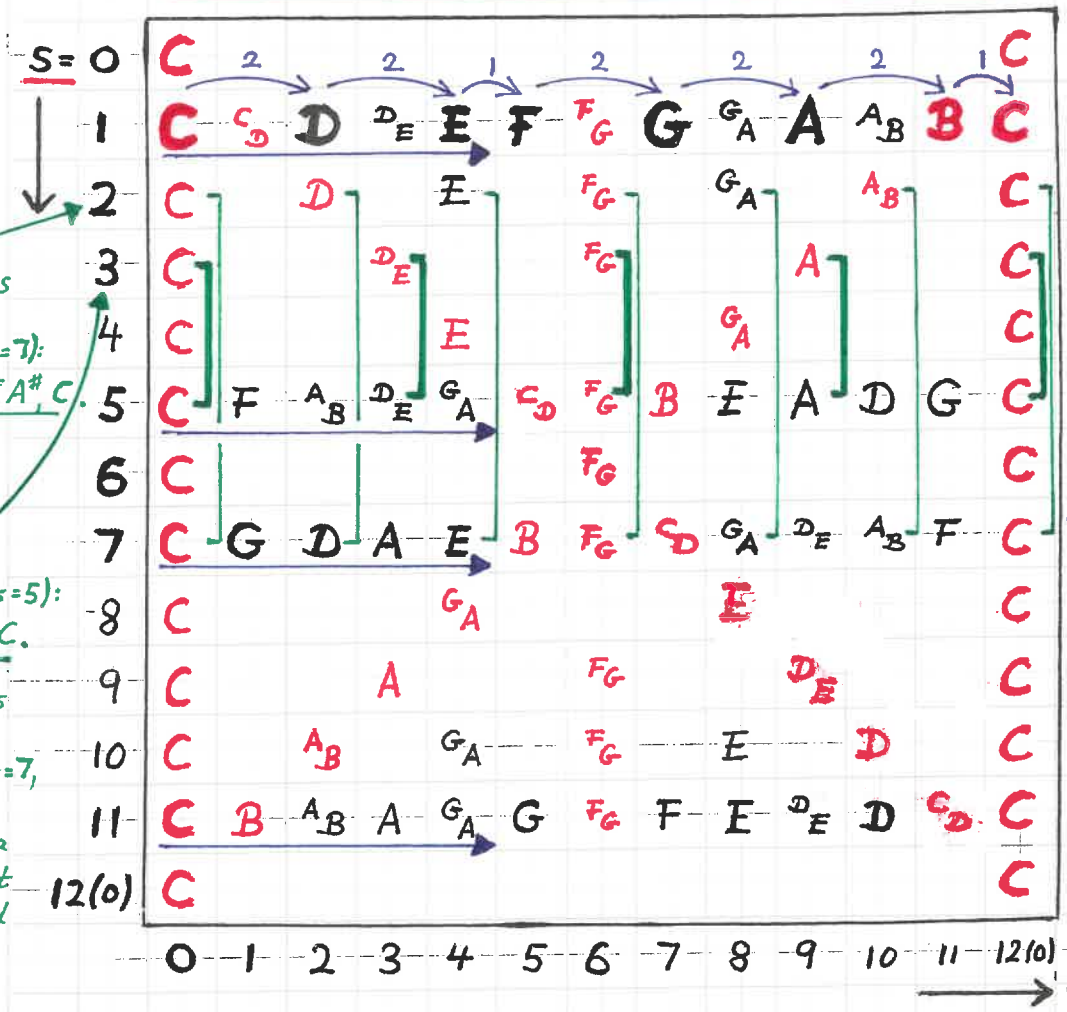


The left figure shows the abstract "number-of-half-tone-steps" pattern defined by the above list of the 13 (s=0...12) index and implied note sequences. The figure of the "bi-periodic pattern" shows vertical, diagonal and horizontal (c-c) symmetry axes.

• KEYS, SCALES, CIRCLES

12/3/2024

TABLE OF THE 13 CIRCLES WITH ROOT C



Chromatic scale "up": C, C♯, D, D♯, E, F, ...

Circle of 2nd's

Octave = 4 minor 3rd's

Octave = 3 major 3rd's

Circle of 5th's "backward": C, F, B♭, E♭, A♭, D♭, ...

Octave = 2 tritones

Circle of 5th's "forward": C, G, D, A, E, B, ...

Octave = 3 major 3rd's

Octave = 4 minor 3rd's

Circle of 2nd's (reverse)

Chromatic scale "down": C, B, B♭, A, A♭, G, ...

- s=2 leads to a note subset (of s=7): C, D, E, F♯, G♯, A♯, C.
- s=3 leads to a note subset (of s=5): C, E♭, G♭, A, C.
- s=4 leads to a note subset of s=7, and s=6 leads to a note subset of s=5 and s=7.

⇒ THIS TABLE OF NOTES — THE NOTES OF THE 13 CIRCLES WITH ROOT C — REPRESENTS ALL NOTES IN A TWO-DIMENSIONAL TABLE $T_{s,n}$, $s = 0 \dots 12$, $n = 0 \dots 12$. THE TABLE ILLUSTRATES AN AMAZING SYMMETRY DEFINED BY AND CHARACTERIZING THE CIRCLES:

$$T_{s,n} = T_{12-s,\,12-n} \quad , \quad s, n = 0 \dots 12 \, .$$

(Examples: $T_{1,1} = C_D = T_{11,11}$; $T_{5,3} = D_E = T_{7,9}$ ..)

Thus, this "system of notes," this table of circles, has POINT SYMMETRY with respect to the "note center" F_G — the tritonus for root C.

12/4/2024

• KEYS, SCALES, CIRCLES

The table of the 13 circles with root a — using small letters to indicate MINOR keys — can be constructed analogously:

S=														
0	a	2		1		2		2		1		2	2	a
1	a	a_b	b	c	c_d	d	d_e	e	f	f_g	g	g_a	a	Chromatic-up
2	a		b		c_d		d_e		f		g		a	
3	a			c			d_e			f_g			a	
4	a				c_d				f				a	
5	a	d	g	c	f	a_b	d_e	g_a	c_d	f_g	b	e	a	Circle of 5ths – backward
6	a						d_e						a	
7	a	e	b	f_g	c_d	g_a	d_e	a_b	f	c	g	d	a	Circle of 5ths – forward
8	a				f				c_d				a	
9	a			f_g			d_e			c			a	
10	a		g		f		d_e		c_d		b		a	
11	a	g_a	g	f_g	f	e	d_e	d	c_d	c	b	a_b	a	Chromatic-down
12(0)	a												a	

$T_{s,n}$ 0 1 2 3 4 5 6 7 8 9 10 11 12(0) → note (n)

⇒ This table also exhibits symmetry:

$$T_{s,n} = T_{12-s,\,12-n}\,, \quad s,n = 0 \ldots 12\,.$$

⇒ This table can be constructed for any root note, for all major and minor keys: ..., B^b, F, C, G, D, ...; ..., g, d, a, e, b, ...

• The circle of 5ths and the note-associated indices/numbers (in modulo-12 arithmetic) are especially important:

C	G	D	A	E	B	F_G	C_D	G_A	D_E	A_B	F	C
0	7	2	9	4	11	6	1	8	3	10	5	0

"Moving from C (root) with index 0 by 7 half-tone steps – again and again"

• KEYS, SCALES, CIRCLES 12/5/2024

It is possible to "analyze and understand" the relationship between the circle of 5ths and scales by considering the note-assigned indices from the table on the previous page (bottom table) and note-assigned indices following the scale-implied order of notes – using modulo-12 arithmetic.

In the following, we use the following indexing / numbering of the notes of the chromatic sale on C:

C=0(12), $^{C}_{D}$=1, D=2, $^{D}_{E}$=3, E=4, F=5, $^{F}_{G}$=6, G=7, $^{G}_{A}$=8, A=9, $^{A}_{B}$=10, B=11, C=12(0). We can thus use these numbers to list the notes of the scales of all major keys, for example:

C	0	2	4	5	7	9	11	0	C=B#
G	7	9	11	0	2	4	6	7	
D	2	4	6	7	9	11	1	2	
A	9	11	1	2	4	6	8	9	
E	4	6	8	9	11	1	3	4	E=Fb
B	11	1	3	4	6	8	10	11	B=Cb
$^{F}_{G}$	6	8	10	11	1	3	5	6	F#=Gb
$^{C}_{D}$	1	3	5	6	8	10	0	1	C#=Db
$^{G}_{A}$	8	10	0	1	3	5	7	8	G#=Ab
$^{D}_{E}$	3	5	7	8	10	0	2	3	D#=Eb
$^{A}_{B}$	10	0	2	3	5	7	9	10	A#=Bb
F	5	7	9	10	0	2	4	5	F=E#
+		2	2	1	2	2	2	1	

The left table shows the scale indices for all 12 major scales. Analogously, one can construct the table for all 12 minor scales:

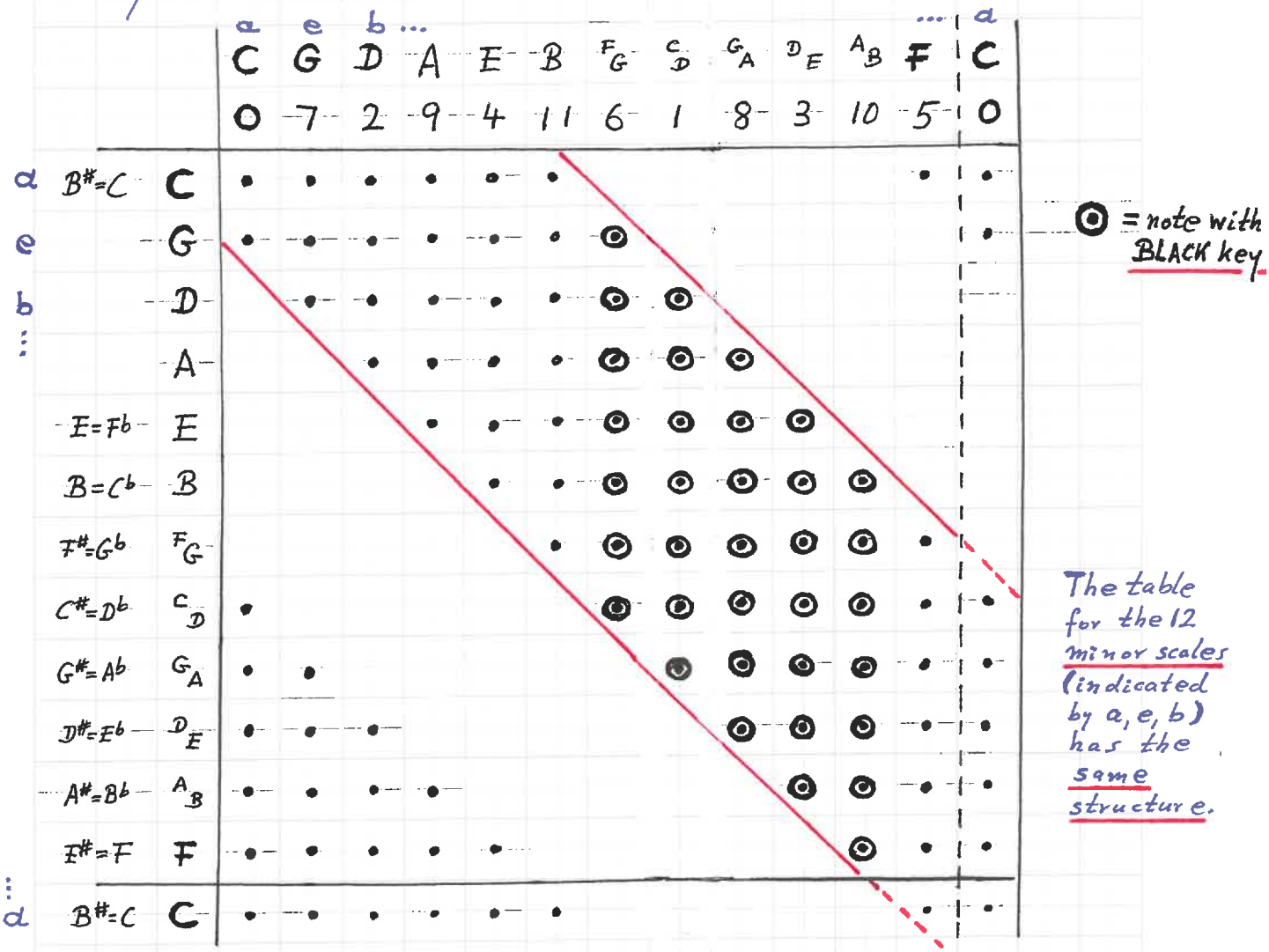
a	9	11	0	2	4	5	7	9
e	4	6	7	9	11	0	2	4
b	11	1	2	4	6	7	9	11
f#	6	8	9	11	1	2	4	6
c#	1	3	4	6	8	9	11	1
g#	8	10	11	1	3	4	6	8
d#	3	5	6	8	10	11	1	3
a#	10	0	1	3	5	6	8	10
f	5	7	8	10	0	1	3	5
c	0	2	3	5	7	8	10	0
g	7	9	10	0	2	3	5	7
d	2	4	5	7	9	10	0	2
+		2	1	2	2	1	2	2

Based on these indices and index tables, one can construct a table that nicely captures the relationship between the circle of 5ths and the notes of all 12 keys, see next page.

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• Keys, Scales, Circles 12/5/2024

In the following table we place a mark in a specific place when the associated note belongs to the scale of the key being considered. The order of the rows follows the circle-of-5ths-implied order of keys; the order/indices of the columns are defined by the note indices of the circle-of-5ths notes:



This table perfectly shows the periodic/cyclic nature of the notes belonging to the scales of the 12 keys — when using the circle of 5ths as basis for "ordering." Further, the visible diagonal "band structure" of the table shows that moving from C to G, from G to D, from D to A etc. is "optimal" — since only one note changes each time.

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• Keys, Scales, Circles 12/6/2024

A main goal for analyzing the relationships between keys and scales is a "simple understanding" of the numbers and keyboard locations of the BLACK keys to be played. A combinatorics-based viewpoint combined with a modulo-12 arithmetic could be helpful for this purpose. The table on p. 6 (12/5/2024) can be used to define the implied pattern of white and black keys to be used for playing (○=white, ●=black):

	NOTE 1st	2nd	3rd	4th	5th	6th	7th	WHITE(○): 0,2,4,5,7,9,11 BLACK(●): 1,3,6,8,10							NO.'●'
C	0	2	4	5	7	9	11	○	○	○	○	○	○	○	0
G	7	9	11	0	2	4	6	○	○	○	○	○	○	●	1
D	2	4	6	7	9	11	1	○	○	●	○	○	○	●	2
A	9	11	1	2	4	6	8	○	○	●	○	○	●	●	3
E	4	6	8	9	11	1	3	○	●	●	○	○	●	●	4
B	11	1	3	4	6	8	10	○	●	●	○	●	●	●	5
F_G	6	8	10	11	1	3	5	●	●	●	○	●	●	○	5
C_D	1	3	5	6	8	10	0	●	●	○	●	●	●	○	5
G_A	8	10	0	1	3	5	7	●	●	○	●	●	○	○	4
D_E	3	5	7	8	10	0	2	●	○	○	●	●	○	○	3
A_B	10	0	2	3	5	7	9	●	○	○	●	○	○	○	2
(E#) F	5	7	9	10	0	2	4	○	○	○	●	○	○	○	1
(B#) C	0	2	4	5	7	9	11	○	○	○	○	○	○	○	0

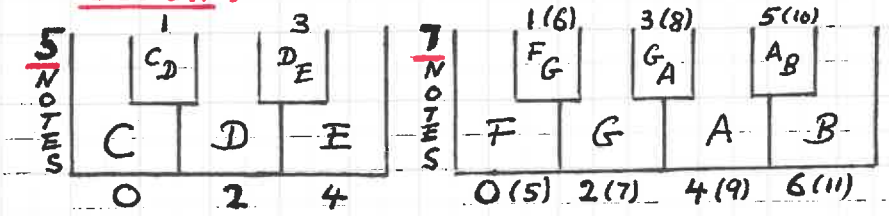
The right part of this table identifies the black keys to be played for the 12 major keys (and the 12 parallel minor keys). Considering the total of 24 major and (parallel) minor keys, 2 keys (C and a) do not use black keys; 4 keys use 1, 4 keys use 2, 4 keys use 3, 4 keys use 4, and 6 keys use 5 black keys.

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• Keys, Scales, Circles 12/6/2024

Notes. Many other concepts and approaches can be considered to study and analyze keys, scales, circles and the relationship to "meaningful and helpful rules" for memorizing the specific white and black keyboard keys to be used for playing. A few potential viewpoints are summarized next.

• The octave C-C consists of a 5-note and 7-note "block": C-$^{C}_{D}$-D-$^{D}_{E}$-E and F-$^{F}_{G}$-G-$^{G}_{A}$-A-$^{A}_{B}$-B.



The two blocks are shown in the left figure. Each block uses its own "local indexing," 0-4 and 0-6, respectively. Thus, viewing the blocks independently, the first is defined by modulo-5 and the second by modulo-7 arithmetic — when adopting a periodic viewpoint. When joining the blocks, the modulo-12 character results, as expected for the octave. Is this helpful for memorizing patterns?

• On page 4 (12/3/2024), we show that one obtains the circle of 5th for s=7 (forward) and s=5 (backward). This fact must be related to the two blocks described above.

• The keys with 0 (1; 2; 3; 4; 5) BLACK keyboard keys are C and a (G, F, e and d; D, $^{A}_{B}$, b and g; A, $^{D}_{E}$, $^{f}_{g}$ and c; E, $^{G}_{A}$, $^{c}_{d}$ and f; B, $^{F}_{G}$, $^{C}_{D}$, $^{g}_{a}$, $^{d}_{e}$, $^{a}_{b}$).

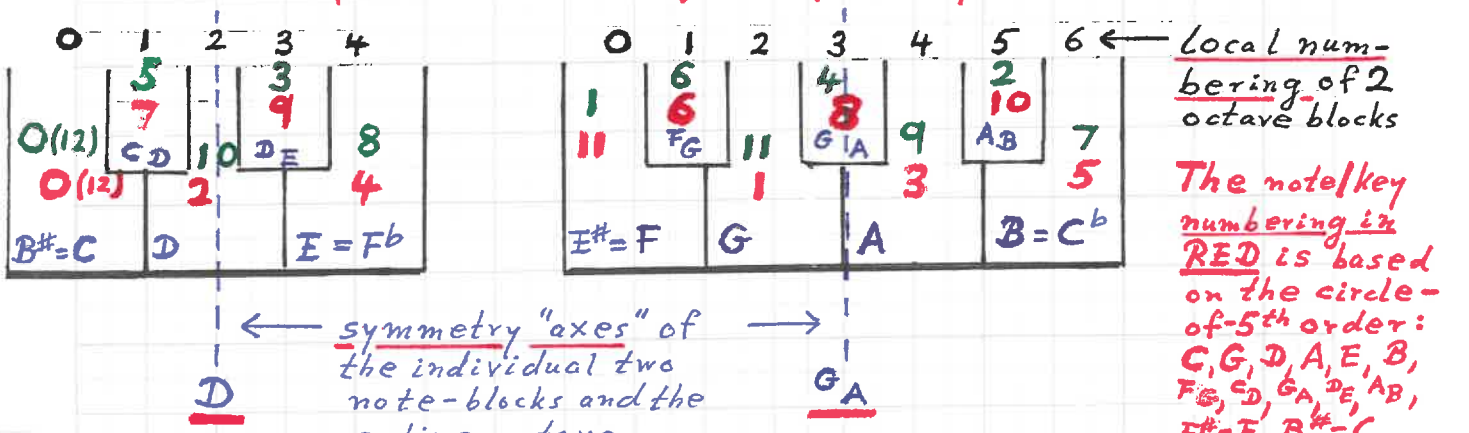
• The keyboard has 2 "symmetry axes": the key for D and the key for $^{G}_{A}$ (see above figure). Is this helpful for memorizing?

• TRANSLATING/TRANSPOSING, SCALING, REFLECTING are transformations potentially applicable to notes, scales, chords, keys, keyboard keys, note indices etc. — Ex.: SCALE (C, G, D, A, E, ...) = C, D, E, ... ≙ circle of 5ths ↦ circle of 9ths. Also: "Krebs" ⇒ forward ⇔ backward. Helpful?

Keys, Scales, Circles

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- One can use the partition of the octave C–C into a 5-note block $C-{}^{C}D-D-{}^{D}E-E$ and 7-note block $F-{}^{F}G-G-{}^{G}A-A-{}^{A}B-B$ to describe the sequential, one-by-one introduction of '#' and 'b' notes — and thus the sequences of introduced black keyboard keys — when using the circle of 5ths to define the order of introduction ($s=7$: forward, $s=5$ backward).



← local numbering of 2 octave blocks

The note/key numbering in RED is based on the circle-of-5th order: $C, G, D, A, E, B, {}^{F}G, {}^{C}D, {}^{G}A, {}^{D}E, {}^{A}B, E^{\#}=F, B^{\#}=C$

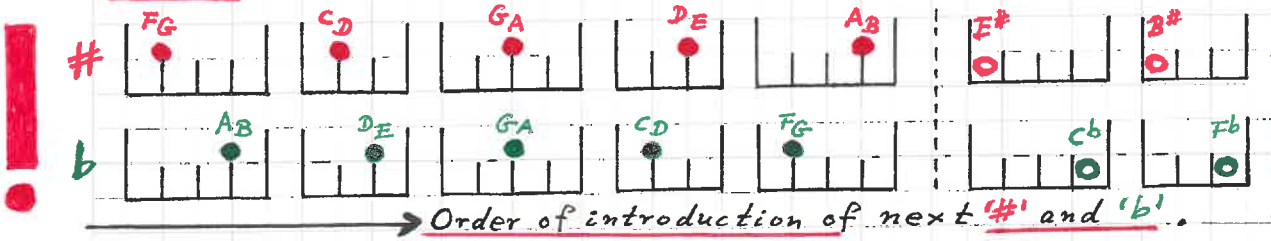
The numbering in GREEN is based on the order $C, F, {}^{A}B, {}^{D}E, {}^{G}A, {}^{C}D, {}^{F}G, C^{b}=B, F^{b}=E, B^{bb}=A, E^{bb}=D, A^{bb}=G$

⇒ Circle of 5ths forward/backward ≙ RED/GREEN nos.

From a GEOMETRICAL keyboard viewpoint, the keys with RED indices 7, 9, 12(0) are mapped to the keys with GREEN indices 3, 5, 8 via reflection by D.

From a GEOMETRICAL keyboard viewpoint, the keys with indices 6, 8, 10, 11 are mapped to the keys with GREEN indices 2, 4, 6, 7 via reflection of the RED-index keys 6, 8, 10, 11 by "axis" G_A.

⇒ ONE CAN UNDERSTAND AND MEMORIZE THE INTRODUCTION OF '#' AND 'b' — AND THUS OF BLACK KEYS — BY APPLYING '#' AND 'b' IN ALTERNATING FASHION TO THE 7- AND 5-NOTE BLOCKS:



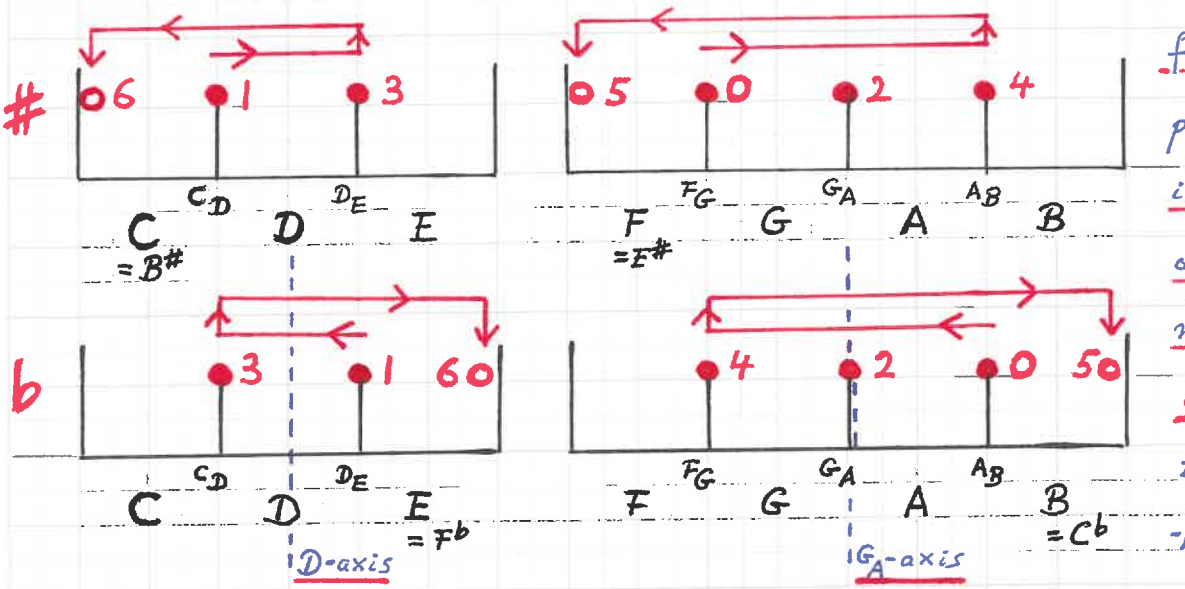
Note. The minor key scales/keys are a "by-product."

"Introducing '#'s ('b's) introduces BLACK KEYS from right to left (left to right), relative to the 7- and 5-note blocks.

• Keys, Scales, Circles 12/8/2024

"The main goal" when analyzing keys and the introduction of '#' and 'b' notes is the understanding of the implied, resulting patterns and sequences of keys on the keyboard – the "physical geometry." Ideally, one can identify a minimal number of patterns and sequences to be memorized.

Again, we consider the sequences of '#'- and 'b'-note introductions by the key sequences C-G-D-A-F-B-F#C# and C-F-B♭-E♭-A♭-D♭-G♭-C♭. These are the patterns:



The left figures represent the introductions of '#'- and 'b'-notes via RED cycles above the 5- and 7-key blocks.

The '#' cycles have counter-clockwise orientation, starting with a left-to-right keyboard movement.

The 'b' cycles have clockwise orientation, starting with a right-to-left keyboard movement.

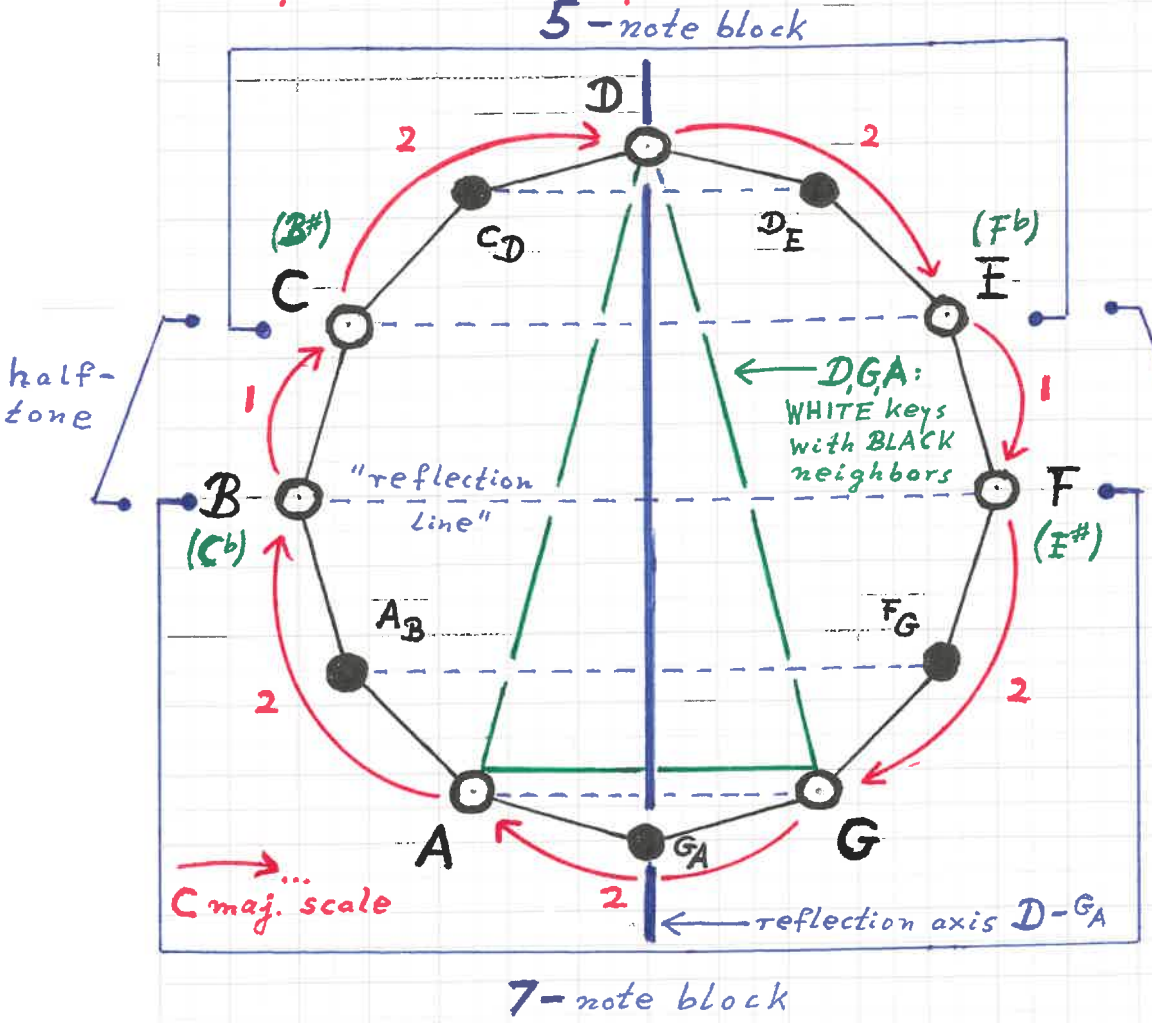
⇒ The two relevant sequences to memorize are $F_G - C_D - G_A - D_E - A_B - E^{\#}(F) - B^{\#}(C)$ for the introductions of '#'s and $A_B - D_E - G_A - C_D - F_G - C^{b}(B) - F^{b}(E)$ for the introductions of 'b's.

The seven indices used for both sequences are identical: 0, 1, 2, 3, 4, 5, 6. One can view the 'b' cycles as the result of "reflecting the '#' cycles against the D- and G_A-axis." Thus, one only needs to memorize the '#' or 'b' cycles.

• Keys, Scales, Circles

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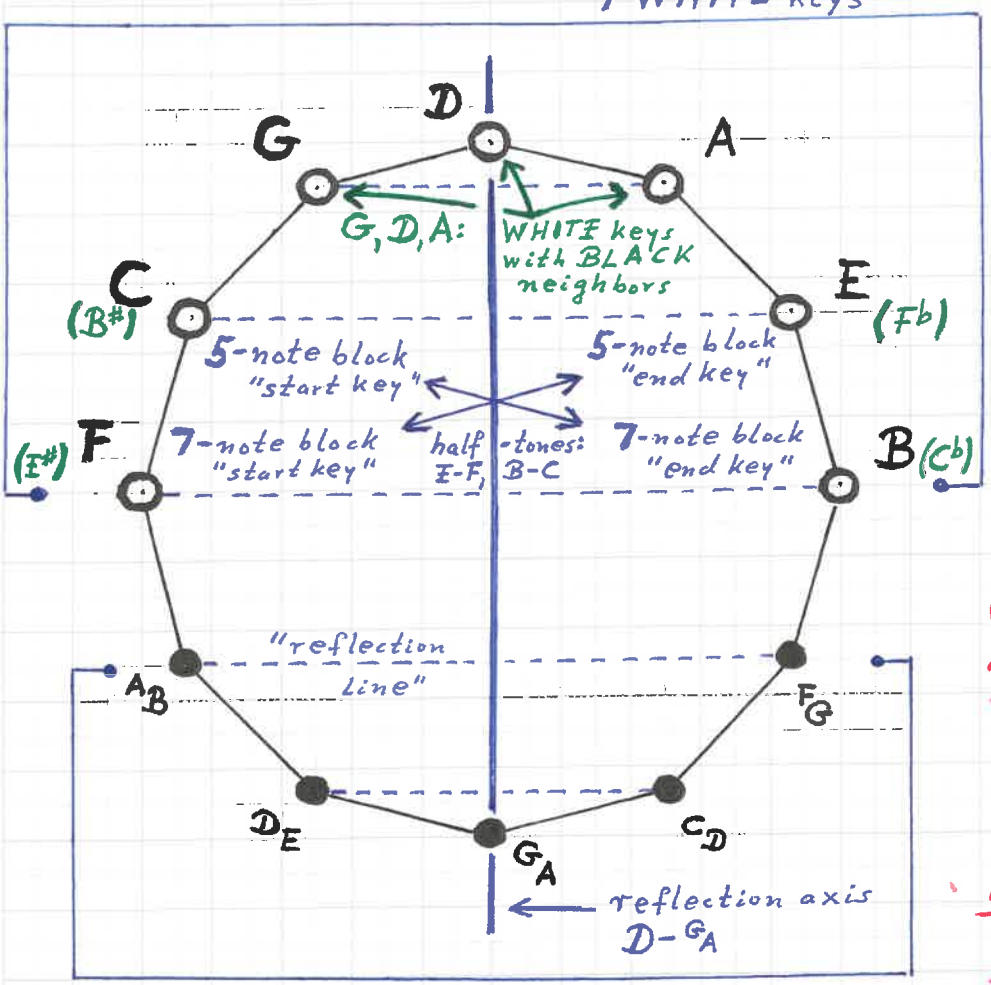
A circlar visualization of 12 halftones/notes defining the movement from C (0) to C (12) by "incrementing" by either 12 halftone steps (chromatic scale) or by 12 steps involving 7 halftone "jumps" at each step (circle of 5ths) makes it possible to clearly see SYMMETRIES. The goal must therefore be to relate these visible "geometrical symmetries" to the physical linear layout of the keyboard and to use them for memorization.



The left figure represents the linear keyboard octave C-C in a circular layout. It uses the 12 halftone steps to move from C to C. WHITE keys are visualized by 'O' and BLACK keys by '●'. The figure shows that the WHITE-BLACK key sequences exhibit symmetry relative to the "reflection axis" D-G_A, with D being the "center key" of the 5-note block and G_A being the "center key" of the 7-note block. HOW CAN ONE USE THIS VISUALIZATION TO MEMORIZE PATTERNS?

• Keys, Scales, Circles

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The left figure shows the circle of 5th's. The "scales with root notes having an associated WHITE key" are next to each other (top 7 notes); the "scales with root notes having an associated BLACK key" are also next to each other (bottom 5 notes [≙ 5 '#', 5 'b']).

This visualization also shows SYMMETRY, relative to the reflection axis D-G_A. As before, a note with an associated WHITE key (BLACK key) is reflected to a note that also has an associated WHITE key (BLACK key).

HOW CAN ONE USE THIS VISUALIZATION WITH ITS "SYMMETRY AND STRUCTURE" TO UNDERSTAND AND MORE EASILY MEMORIZE "GEOMETRY, SHAPES AND PATTERNS" OF SCALES, CHORDS ETC. ON THE LINEAR KEYBOARD?

One can analyze keys, scales and cyclic "behavior and geometry" using many other approaches — e.g., determining the "complementary nature" of pairs of scales (one having '#'s, one having 'b's) or the "opposite directions" of '#' and 'b' introductions (left-to-right, right-to-left).

• Keys, Scales, Circles 12/11/2024

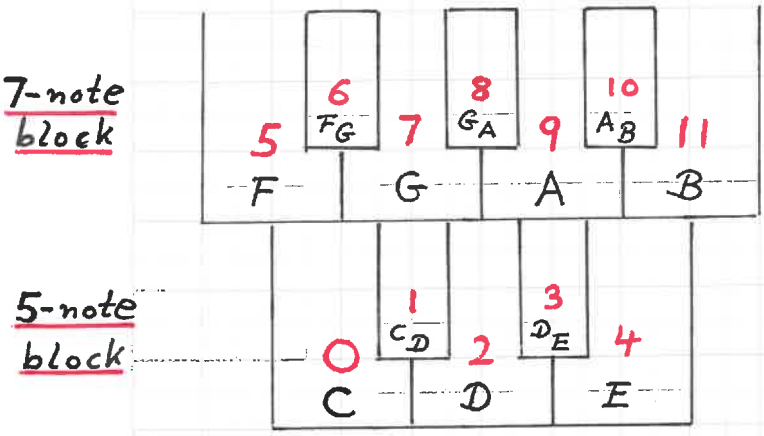
In the following, keys, scales, key-specific '#'s and 'b', and the order / direction of '#' and 'b' introductions are reviewed — when using the circle-of-5th progression order. Further, one can view specific keys and scales — and their corresponding keys on the keyboard — as being "complementary" to each other. Again, the goal of such analyses and characterizations of keys and scaled is the development of "simple rules" one can easily memorize for playing.

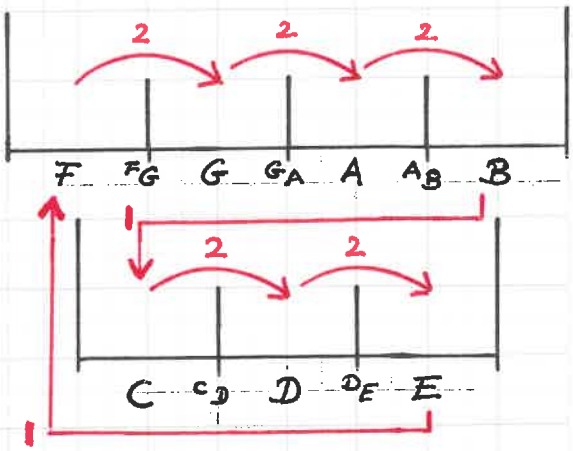
! ON THE PHYSICAL KEYBOARD, THE 5-NOTE AND 7-NOTE BLOCKS — C, ^{C}D, D, ^{D}E, E AND F, ^{F}G, G, ^{G}A, A, ^{A}B, B — ARE NEXT TO EACH OTHER, ARRANGED IN A "LINEAR FASHION." ALTERNATIVELY — AT LEAST CONCEPTUALLY — ONE CAN ARRANGE THE TWO BLOCKS "ON TOP EACH OTHER": THE 5-NOTE BLOCK CAN BE PLACED DIRECTLY UNDER THE 7-NOTE BLOCK — OR VICE VERSA — ANALOGOUSLY TO PLACING SEVERAL ORGAN MANUALS ON TOP OF EACH OTHER. THIS CONCEPT IS EXTREMELY HELPFUL FOR AN "EFFECTIVE VISUALIZATION AND UNDERSTANDING" OF THE RELATIONSHIPS BETWEEN KEYS, SCALES, CIRCLES AND THE PHYSICAL KEYS ON THE KEYBOARD. SPECIFICALLY, THIS CONCEPT GREATLY SIMPLIFIES THE STUDY OF THESE CHARACTERISTICS: ORDER OF INTRODUCTION OF '#'S AND 'b'S AND IMPLIED DIRECTION / ORIENTATION OF PHYSICAL KEY SEQUENCES; "COMPLEMENTARY NATURE" OF CERTAIN SCALES AND IMPLIED "PHYSICAL KEY GEOMETRY"; VIEWING CERTAIN "PHYSICAL KEY GEOMETRIES" AS RESULT OF REFLECTION AGAINST THE AXIS D-^{G}A.

• Keys, Scales, Circles

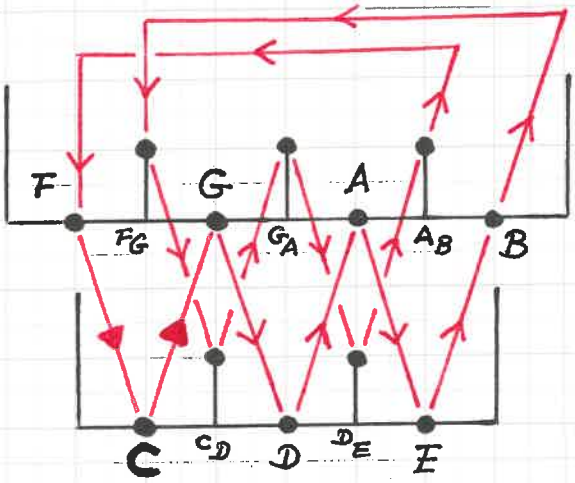
12/11/2024

• Abstract representation:



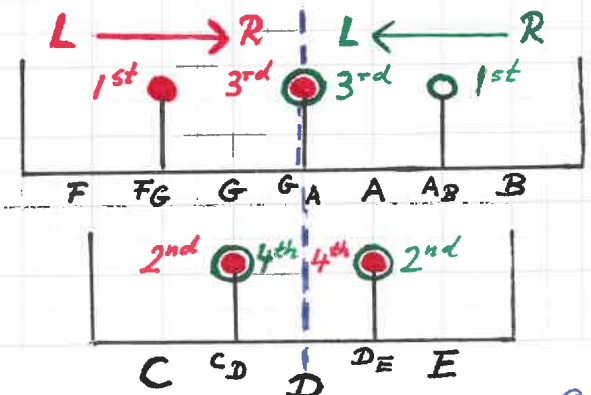


The figures above show the concept of the alternative block arrangement. Left: Visualization of physical key layout with note names and possible indexing / numbering. Right: More abstract visualization of layout, emphasizing the 2212221 halftone-jump sequence of C major.



Circle of 5ths.

The left figure represents the circle-of-5th order of notes (C-G-...$^{F}_{G}$-$^{C}_{D}$-...-F-C), shown by the red polygon with directed edges. The positions (•) for notes F, C, G, D, A, E, B "move from left to right"; the same holds for $^{F}_{G}$, $^{C}_{D}$, $^{G}_{A}$, $^{D}_{E}$, $^{A}_{B}$.



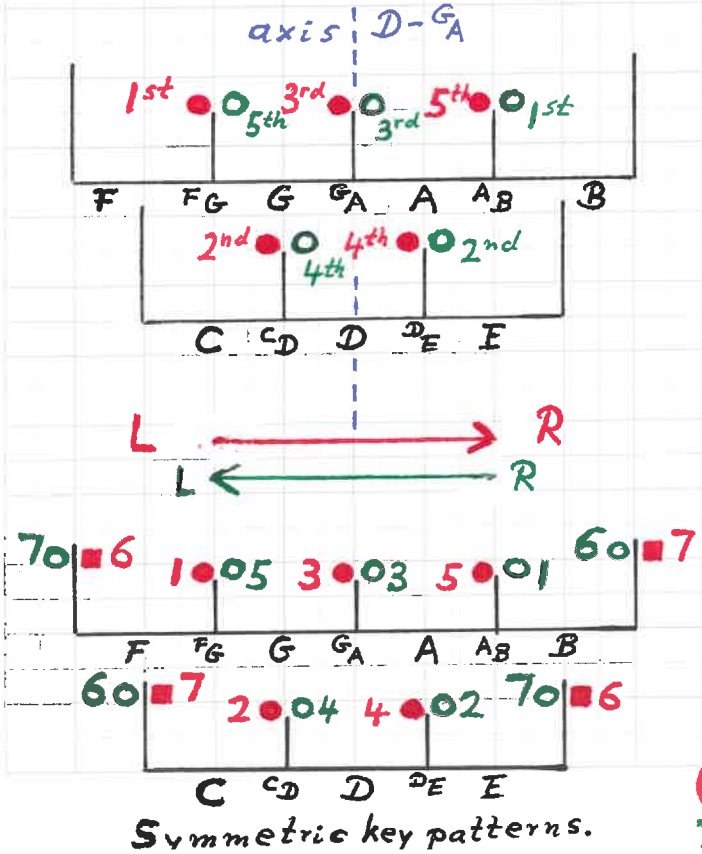
Symmetry axis of patterns: D-$^{G}_{A}$.

The left figure visualizes the order of introduction of '#'s (•) and 'b's (○), as implied by the key sequences C-G-D-A-E and C-F-B^{b}-E^{b}-A^{b}, respectively. The associated "directions" are left-to-right (L→R) and right-to-left (L←R).

• Keys, Scales, Circles 12/12/2024

Thus, the most important facts about this visualization are: (i) The L→R introduction of '#'s (and the corresponding physical keys) is "complemented" by the L←R introduction of 'b's (and the corresponding physical keys). (ii) The physical key patterns/shapes resulting after the introduction of 1 (2,3,4,5) '#' and 1 (2,3,4,5) 'b', respectively, are axis-symmetric relative to the D-G_A axis. THESE FACTS ARE VERY HELPFUL FOR MEMORIZING SCALE PROPERTIES ON THE PHYSICAL KEYBOARD.

First, we consider the "key patterns" that result from the introduction of 1 '#' and 1 'b'; 2 '#'s and 2 'b's; 3 '#'s and 3 'b's; 4 '#'s and 4 'b's; and 5 '#'s and 5 'b's. In other words, we are interested in the sequence of patterns resulting from the circle-of-5th keys G→D→A→E→B and F→B^b→E^b→A^b→D^b. The patterns are symmetric:



Symmetric key patterns.

The left figure visualizes the symmetry of the "black-key patterns" — being symmetric with respect to the D-G_A axis and evolving from left to right (L→R) and from right to left (L←R) when introducing '#'s and 'b's, respectively. The left figure captures the symmetric keyboard patterns for the introductions of 7 '#'s (F_G, C_D, G_A, D_E, A_B, $E^{\#}=F$, $B^{\#}=C$) and 7 'b's (A_B, D_E, G_A, C_D, F_G, $C^b=B$, $F^b=E$).

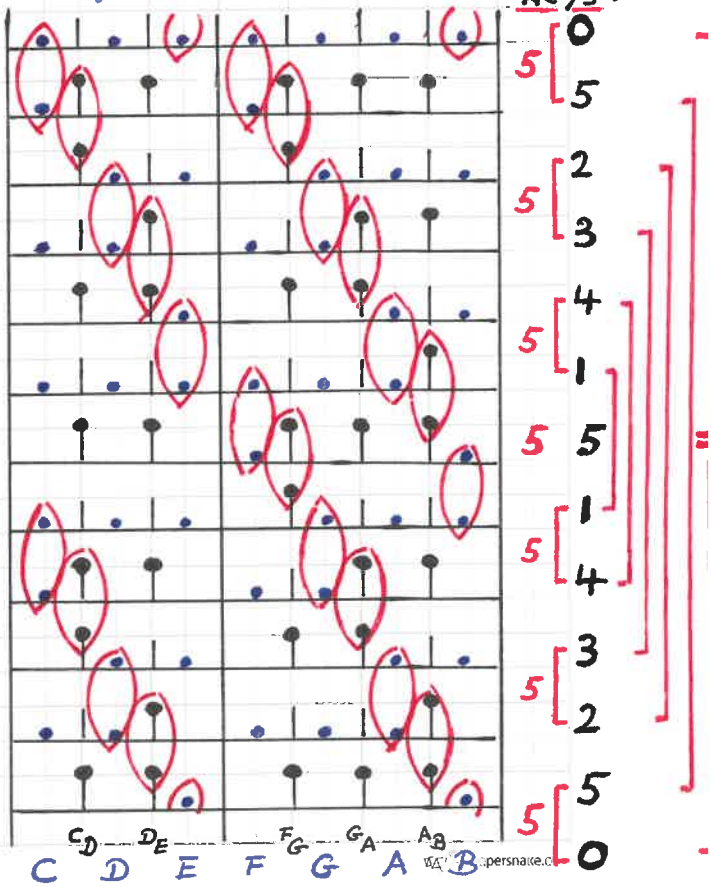
• Keys, Scales, Circles 12/12/2024

Second, we consider the concept of "complementary-keys, scales and keyboard patterns." IN ORDER TO DETERMINE POTENTIALLY MEANINGFUL AND HELPFUL "COMPLEMENTARY PATTERNS," IT SEEMS REASONABLE TO CONSIDER THE "BEHAVIOR OF ALL POSSIBLE PAIRS," E. G., PAIRS OF KEYS OR SCALES. Again, the objective is the identification of patterns useful for memorizing and playing. It is not obvious how to define two "complementary keys" or two (associated) "complementary keyboard patterns" (of the associated scale!). For example, one can consider the 7 notes of a specific key/scale as a subset of the 12 notes of the chromatic scale; thus, one can use a key/scale with notes that include the 5 "missing notes" — with respect to the "complete 12-note chromatic note set" — a complementary key/scale.

COMPLEMENTARY SCALE PAIRS

C	C	D	E	F	G	A	B
$D^b = C^\#$	C_D	D_E	F	F_G	G_A	A_B	C
D	D	E	F_G	G	A	B	C_D
$E^b = D^\#$	D_E	F	G	G_A	A_B	C	D
$F^b = E$	E	F_G	G_A	A	B	C_D	D_E
F	F	G	A	A_B	C	D	E
$G^b = F^\#$	F_G	G_A	A_B	B	C_D	D_E	F
G	G	A	B	C	D	E	F_G
$A^b = G^\#$	G_A	A_B	C	C_D	D_E	F	G
A	A	B	C_D	D	E	F_G	G_A
$B^b = A^\#$	A_B	C	D	D_E	F	G	A
$C^b = B$	B	C_D	D_E	E	F_G	G_A	A_B

Twelve (major) scales.



Scale keyboard patterns.

• Keys, Scales, Circles

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THE BOTTOM-RIGHT FIGURE ON THE PREVIOUS PAGE IS AN ABSTRACT VISUALIZATION OF THE KEYBOARD PATTERNS OF THE 12 (24!) SCALES OF INTEREST. PAIRS OF NEIGHBOR PATTERNS (IN TWO CONSECUTIVE ROWS) BELONG TO TWO COMPLEMENTARY KEY/SCALE PAIRS (BOTTOM LEFT). THE KEYBOARD PATTERNS MUST BE "TRANSLATED" INTO "PATTERN SEQUENCE RULES" THAT CAN BE MEMORIZED FOR PLAYING.

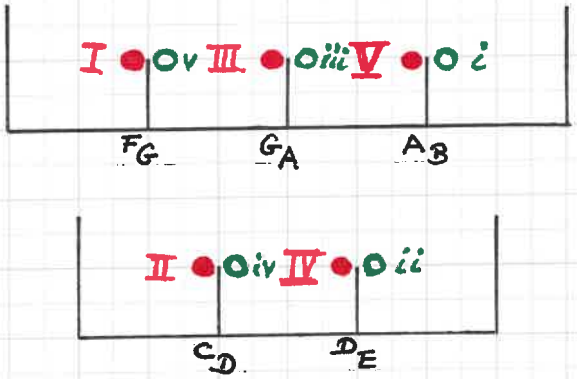
Instead of using the key/scale order induced by the chromatic scale, one can also use the circle-of-5th order to define complementary keys/scales. For example, one can consider pairs of keys and their numbers of associated '#'s and 'b's, written as tuples (N#, Nb) in a table:

b \ #	$_0$C	$_1$G	$_2$D	$_3$A	$_4$E	$_5$B	$_5$F#	$_5$C#
$_0$C	0,0	1,0	2,0	3,0	4,0	5,0	6,0	7,0
$_1$F	0,1	1,1	2,1	3,1	4,1	5,1	6,1	
$_2$B^b	0,2	1,2	2,2	3,2	4,2	5,2		
$_3$E^b	0,3	1,3	2,3	3,3	4,3			
$_4$A^b	0,4	1,4	2,4	3,4	4,4			
$_5$D^b	0,5	1,5	2,5			5,5		
$_5$G^b	0,6	1,6					...	
$_5$C^b	0,7							

The '#' keys are listed in the columns and have between 0 and 7 '#'s, with between 0 and 5 implied BLACK keys (indicated by the lower-left index). The 'b' keys are listed in the rows and have between 0 and 7 'b's, with between 0 and 5 implied BLACK keys (lower-left index). The table's diagonal shows tuples for which N# = Nb. These were already viewed as belonging to complementary keys — since their associated '#' and 'b' keyboard patterns are axis-symmetric relative to the D-G^A axis. DO TUPLES REPRESENT COMPLEMENTARY PATTERNS?

• Keys, Scales, Circles 12/14/2024

• A MORE GENERAL WAY TO VIEW "COMPLEMENTARY" — I.E., NOT ONLY CONSIDERING KEYS WITH AXIS-SYMMETRIC KEYBOARD PATTERNS — IS BASED ON KEYS(PAIRS!) THAT SATISFY THE CONDITION N#+Nb= const. For example, the key tuples satisfying N#+Nb=5 are underlined in the table on the previous page. Six key pairs satisfy this condition, where their combined total number of '#'s and 'b's (5) implies complementary BLACK key keyboard patterns that involve all 5 BLACK keys. The specific key pairs are: (B, C), (E, F), (A, B^b), (D, E^b), (G, A^b), (C, D^b). The left figure shows, in a combined manner, the resulting six BLACK key combinations that all involve all black keys: (I, II, III, IV, V), (I, II, III, IV, i), (I, II, III, i, ii), (I, II, i, ii, iii), (I, i, ii, iii, iv), (i, ii, iii, iv, v). Analogously, one can also use the key conditions N# + Nb = const, with const = 1, 2, 3, 4, for the other tuples in the table that imply the use of only a subset of the five black keys when combining the '#'s and 'b' of a key pair. Similarly, one can also consider the conditions N# + Nb = const, with const = 6, 7. In these cases, certain BLACK keys are "used twice" — since they are implied by both keys of a key pair.

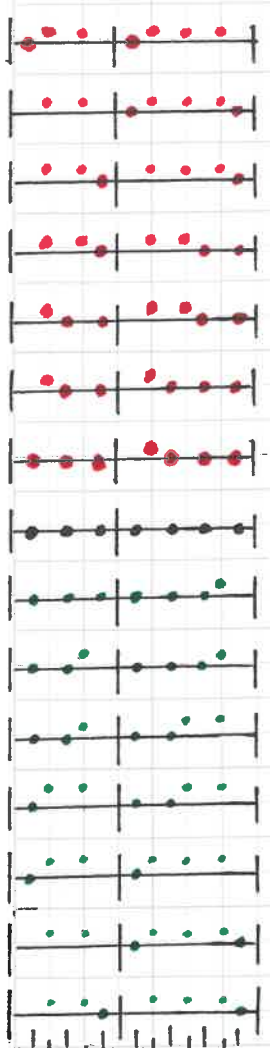


• Yet another possibility to define "complementary keys/scales" can use the number of identical or different keyboard keys of pairs of keys as basis (degree of similarity/dissimilarity).

• Keys, Scales, Circles

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			Cb	Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#
a#	C#	C#	5(2)	6(1)	7(0)	6(1)	5(4)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)
d#	F#	F#	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)
g#	B	B	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)
c#	E	E	6(1)	5(4)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)
f#	A	A	5(4)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)
b	D	D	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)
e	G	G	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)
a	C	C	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)
d	F	F	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)
g	Bb	Bb	2(5)	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)
c	Eb	Eb	3(4)	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(4)
f	Ab	Ab	4(3)	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(2)	6(1)
bb	Db	Db	5(2)	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(4)	6(1)	7(0)
eb	Gb	Gb	6(1)	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(4)	6(1)	7(0)	6(1)
ab	Cb	Cb	7(0)	6(1)	5(2)	4(3)	3(4)	2(5)	2(5)	2(5)	3(4)	4(3)	5(4)	6(1)	7(0)	6(1)	5(2)



Parallel minor keys

Major keys — keys played on keyboard (shown in 5-note and 7-note blocks)

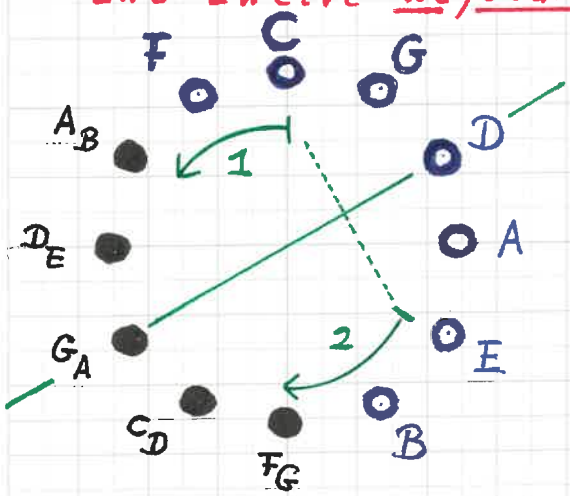
Table of pairs of major keys, listing the numbers of identical and different keys used for playing the two scales on the keyboard. The entry m(n) means that the two key pair implies m identical keys on the keyboard, while n = 7-m keys on the keyboard are different.

This symmetric table applies to all major and (parallel) minor keys. Underlined entries for key pairs high-light key pairs "very close to each other on the keyboard" (7(0), 6(1)) and pairs "far away from each other on the keyboard" (2(5)). THUS, ONE COULD VIEW THE "2(5)-pairs" AS KEY PAIRS THAT ARE "MAXIMALLY COMPLEMENTARY" TO EACH OTHER. For example, the pairs C-B, C-F# and C-C# have maximally different patterns, i.e., are complementary.

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Based on this perspective, one could view 2(5)-pairs as pairs for which a "nearly complementary pattern of keyboard keys" must be memorized: ONE ONLY MUST MEMORIZE THE 2 KEYBOARD KEYS THAT ARE COMMON AND 5 KEYBOARD KEYS (OF ONE OF THE TWO SCALES) THAT ARE NOT IN COMMON (SINCE THE 5 KEYBOARD KEYS OF "THE SECOND SCALE" ARE COMPLEMENTARY).

• Of course, one can characterize "complementary" or "symmetric" behavior of keys, scales and keyboard patterns/shapes in several other possible ways. Generally, one can use set theory to define "complementary" behavior; and a geometrical viewpoint can be employed to define "symmetric" behavior. For example, one can observe these characteristics when using the circle-of-5th order to represent the twelve keyboard keys. The left figure does NOT represent the keys of the circle of 5ths; it simply uses the same circle arrangement to visualize the 12 keyboard keys of one octave, where the white keys (F, C, G, D, A, E, B) define the 7-note sequence and the black keys (F_G, C_D, G_A, D_E, A_B) define the 5-note sequence. One can use this layout of keyboard keys to "see" two "complementary and symmetric note sequences":



1: C F A_B D_E G_A C_D F_G B E A D G
2: E B F_G C_D G_A D_E A_B F C G D A

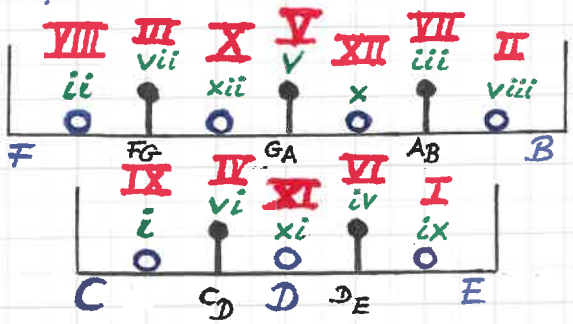
Both sequences start with 2 white keys, followed by 5 black keys, followed by 5 white keys.

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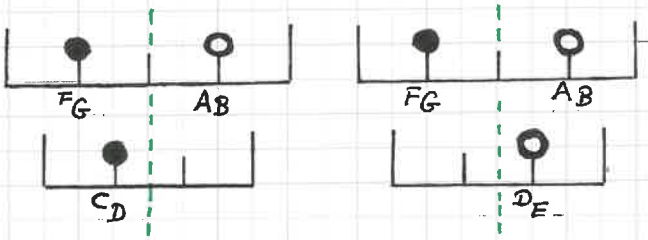
The "complementary and symmetric" nature of these two sequences of keyboard keys becomes evident when using the combined 5-key-and-7-key layout of the 12 notes/keys of the octave:



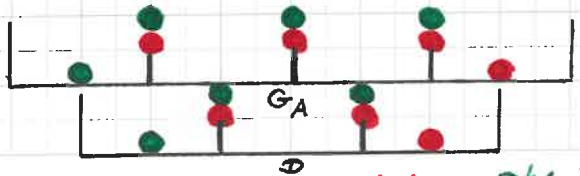
Two 12-tone key sequences that are symmetric relative to the axis D-GA.

This visualization that playing the first k ($1 \le k \le 12$) keys of the two sequences produces two symmetric "key pattern sequences": i, ii, iii, ..., k and I, II, III, ..., K.

• Specifically, one can consider the case of two keys (key signatures), where the first key has m associated '#'s and the second key has n associated 'b's. By defining $m+n=\text{const}=c$, one can also establish a "complementary key pair," where the first key has n '#'s and the second has m 'b's. For example, given the initial key pair D maj. and F maj. ($m=2$, $n=1$), the second "complementary pair" is G maj. and B^b maj. Thus, these pairs satisfy $m+n=3$. The pairs' symmetric keyboard patterns can be visualized via the 5-note and 7-note block representation:



Black keys of D(•), F(○). Black keys of G(•), B^b(○).



All 7 keys used by B(•) and D^b(•).

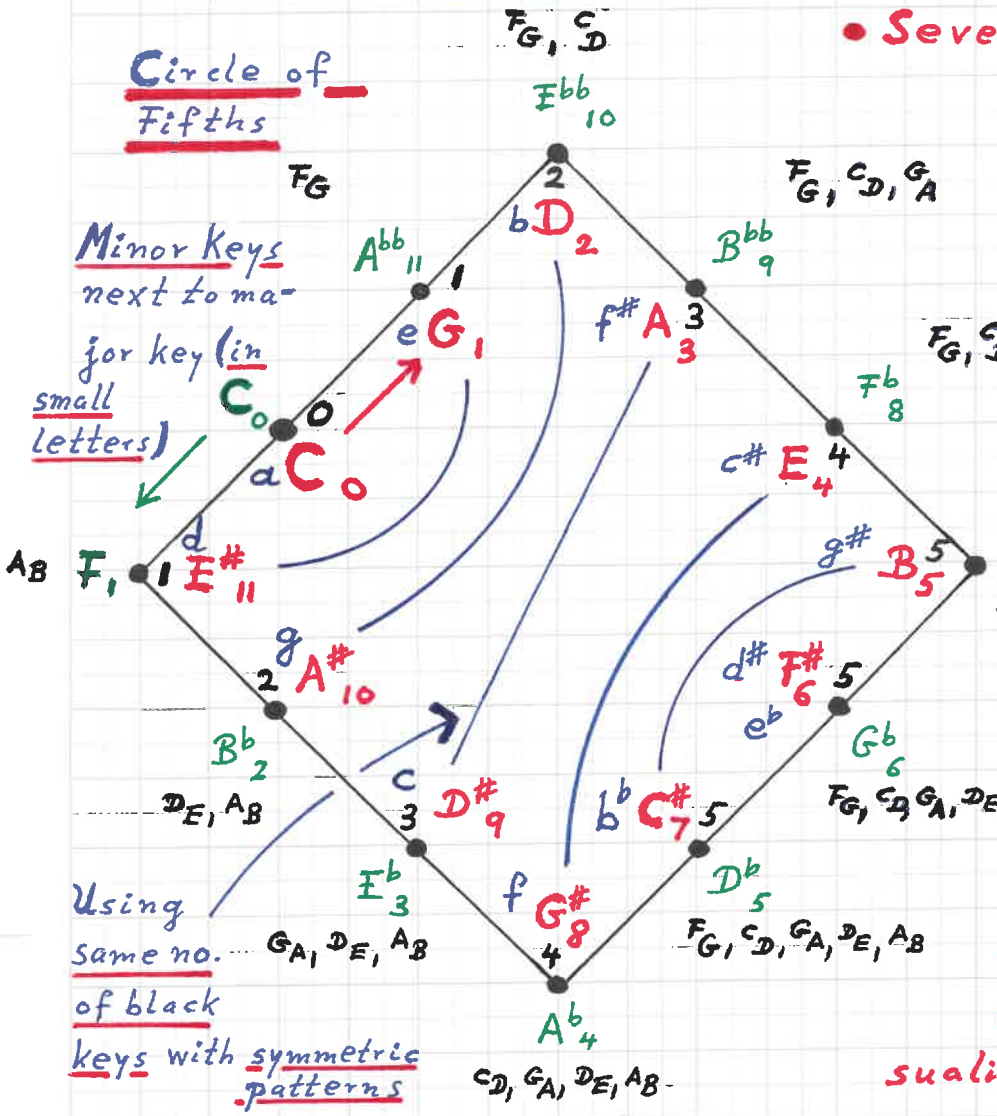
The left figure shows the black keys to be used for the pairs (D, F) and (G, B^b). The resulting keyboard patterns are symmetric relative to axis D-GA.

The left figure shows the 7-key keyboard key-set patterns for B maj. and D^b maj. They are symmetric.

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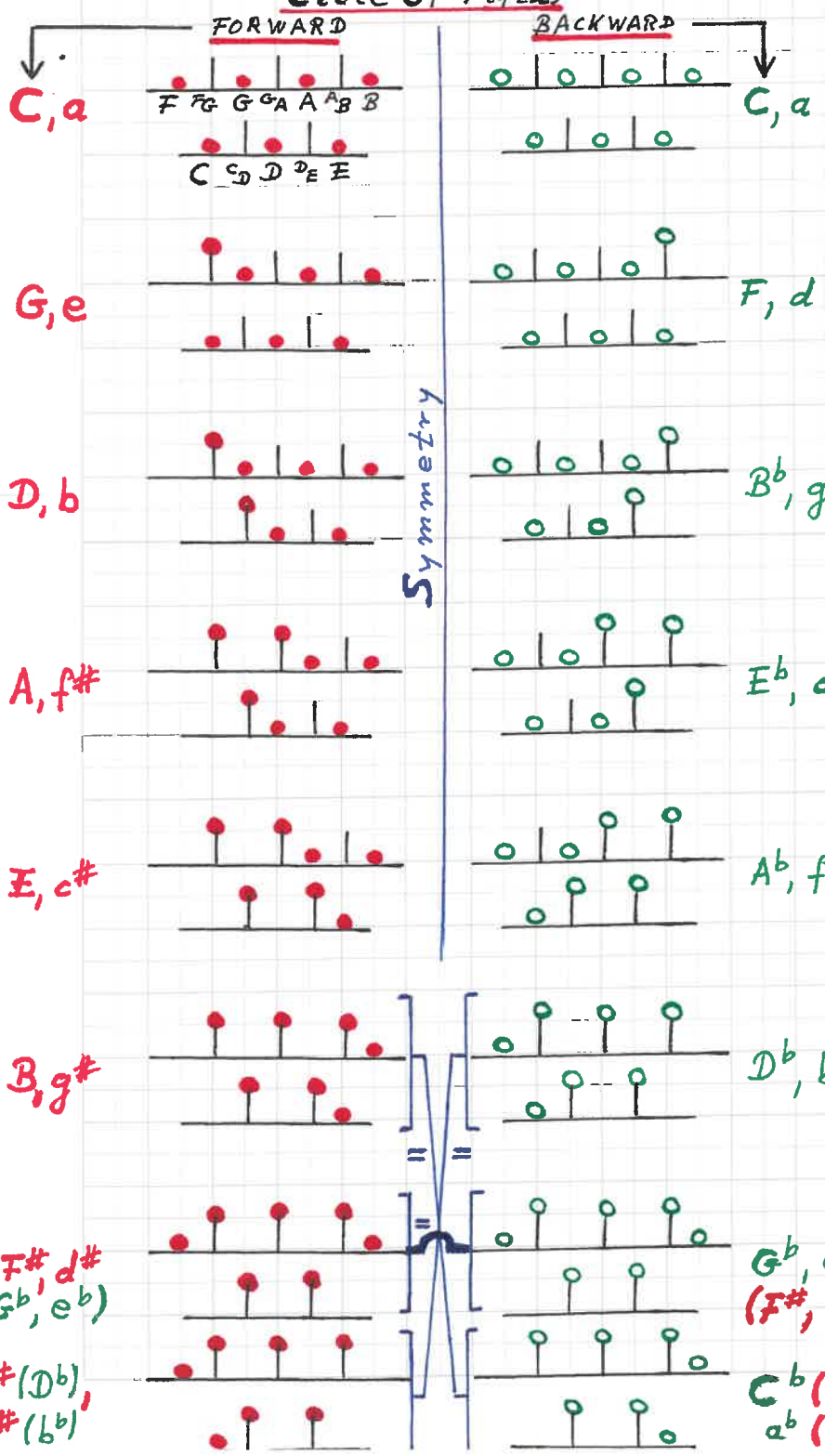
• Several facts that are relevant for the identification of relationships between keys (key signatures) and resulting symmetric and complementary characteristics can be visualized by the circle of 5th's.

The figure above includes all 12 keys obtained by moving forward by 5th's and all 12 keys obtained by moving backward by 5th's. Next to a '#-key' the number of '#'s is shown; next to a 'b-key' the number of 'b's is shown. At every position '•', these two numbers always add up to 12 (or 0). Next to each '•', the number of black keyboard keys implied by the associated pair of keys (key signatures) is given; this number is between 0 and 5, of course. The blue lines in the interior of the square connect keys (key signatures) with keyboard key patterns that are symmetric — when considering their entire scales or only their implied black keys.

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The visualization of the 12 halftones from C to B via a 5-note and 7-note block and considering the described concepts of "complementary and symmetric" patterns of scales (note sets) played on the keyboard makes it possible to group the 12 major and 12 minor keys:



The figure on the left shows the 7 keys on the keyboard that must be played for the 12 major and 12 minor scales. The IMPORTANT INSIGHT IS THE FACT THAT THE PAIRS OF LEFT ("FORWARD") AND RIGHT ("BACKWARD") PATTERNS OF 7 KEYBOARD KEYS ARE SYMMETRIC. THUS, ONE MUST MEMORIZE ONLY THE SETS / KEYS FOR MAJOR KEYS C, G, D, A, E, B, F♯.

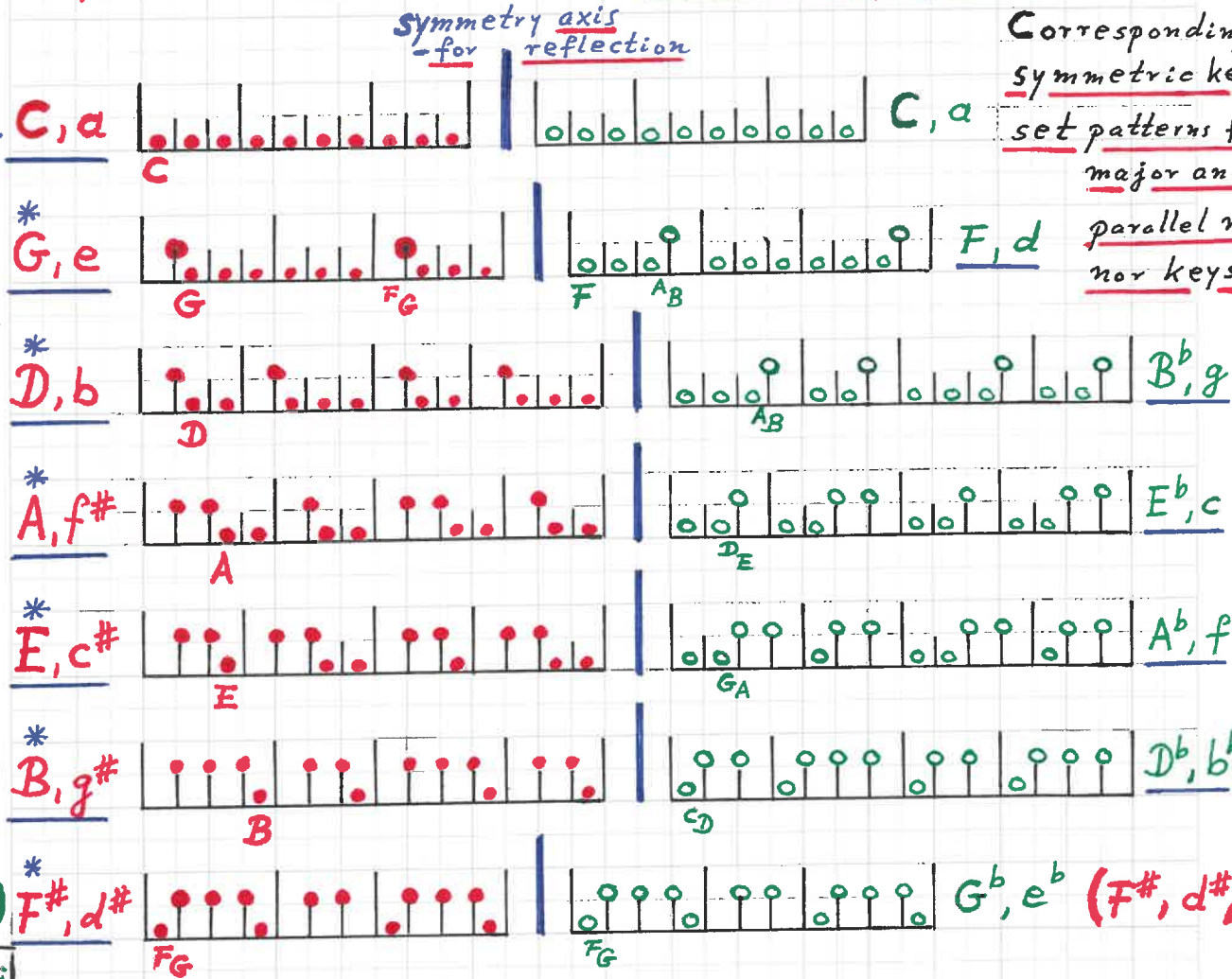
C MAJOR INVOLVES WHITE KEYS; THUS, SIX SETS MUST BE MEMORIZED. FURTHER, A MAJOR KEY AND ITS PARALLEL MINOR KEY SHARE THE SAME SET OF 7 KEYBOARD KEYS

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Instead of using the visualization with the 5-note and 7-note blocks to represent symmetric patterns of note/key sets, one can also see the symmetric patterns on the actual "linear keyboard":

The 24 keys are underlined. The SIX patterns to be memorized are marked with an '*'.



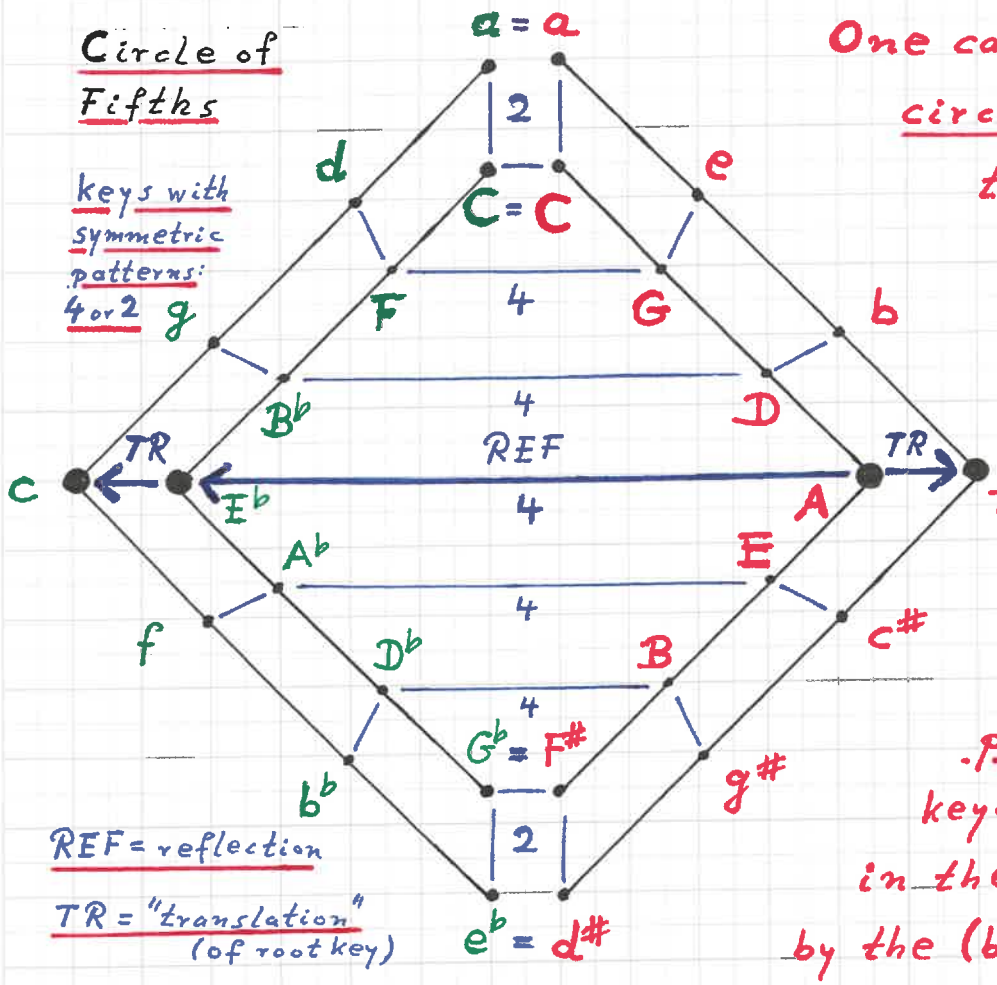
maj.	min.	refl. maj.	refl. min.
C	a	C	a
G	e	F	d
D	b	B♭	g
A	f♯	E♭	c
E	c♯	A♭	f
B	g♯	D♭	b♭
F♯	d♯	G♭	e♭

Table with rows listing keyswith symme-

These visualizations of pairs of note/key sets show symmetry: The sets on the right-hand side are obtained by "reflecting" the sets on the left-hand side using the symmetry axis as reflection axis. Alternatively, one can understand a pattern on the right-hand side as the result of "traversing" the pattern on the left-hand side from right to left. Again, ONE MUST MEMORIZE ONLY SIX PATTERNS — G, D, A, E, B, F♯. Other key patterns are implied by reflection, and major and parallel minor keys have the same set patterns.

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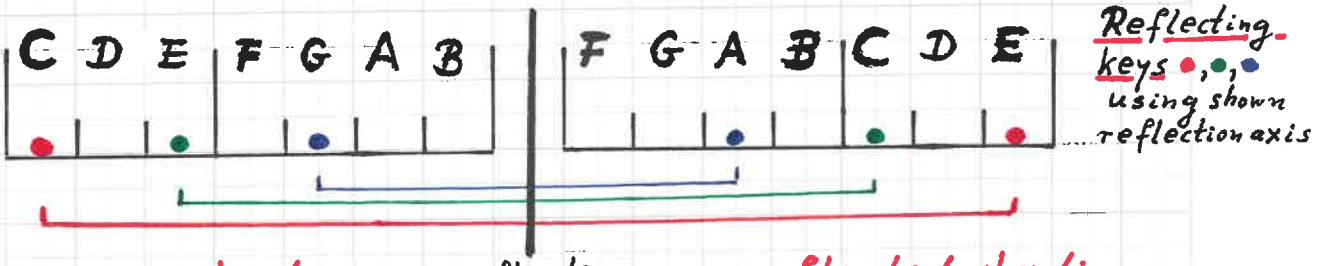


One can use the circle of fifts to represent/visualize the table on the previous page. The keys with symmetric patterns on the keyboard are shown in the circle of fifths by the (blue) edges that connect 4 keys. For example, remembering/knowing the scale for A major, one can "immediately play" the scales for E^b major, $f^\#$ minor and c minor: The scale for E^b major is obtained from the scale for A major by reflection; the scale for $f^\#$ minor is the scale of the parallel minor of A major, thus using the same keys as those used for A major — but using $f^\#$ as root key; the scale for c minor is the scale of the parallel minor of E^b major, thus using the same keys as those used for E^b major — but using c as root key. (This example is high-lighted in bold in the above figure.) The two special cases are shown at the top of the circle (C-a) and at the bottom ($(F^\#-d^\#)=(G^b-e^b)$). KNOWING THE CIRCLE OF FIFTHS AND THE SCALES FOR G, D, A, E, B, $F^\#$ SUFFICES.

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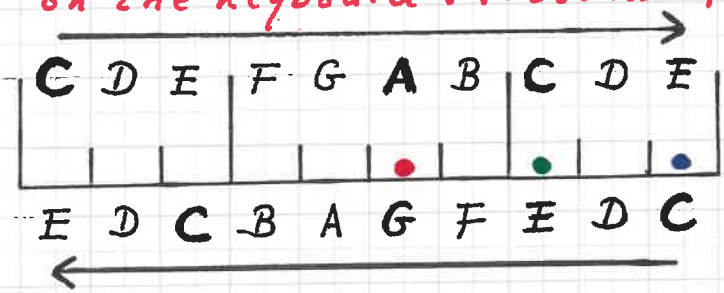
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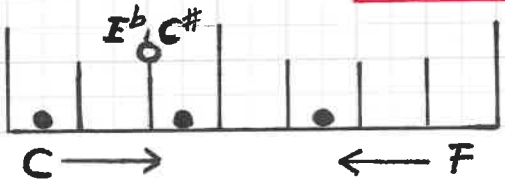
• ANOTHER INTERESTING POSSIBILITY FOR MEMORIZING "SHAPES AND PATTERNS" ON THE KEYBOARD IS THE USE OF REFLECTION: ONE CAN "INVERT" A KEY SEQUENCE BY "READING THE KEYBOARD FROM RIGHT TO LEFT." WHAT IS MEANT BY THIS STATEMENT? THE GENERAL PRINCIPLE IS DESCRIBED.



⇒ The chord CEG = Cmaj = C becomes the reflected chord ACE = Amin = a. ⇒ Major becomes minor.

• One can also think of the reflection operation as a reversal of the naming / labeling of the keys on the keyboard: Instead of using the "standard" left-to-right naming sequence CDEFGAB, one can consider the right-to-left naming sequence EDCBAGF for the same seven keys. The figure shows how these two naming / labeling options are applied to the keyboard. The figure shows how the "given chord" ACE = Amin = a becomes the "new chord" CEG = Cmaj = C. ⇒ Minor becomes major.





→ CEG, CE♭G The left figure

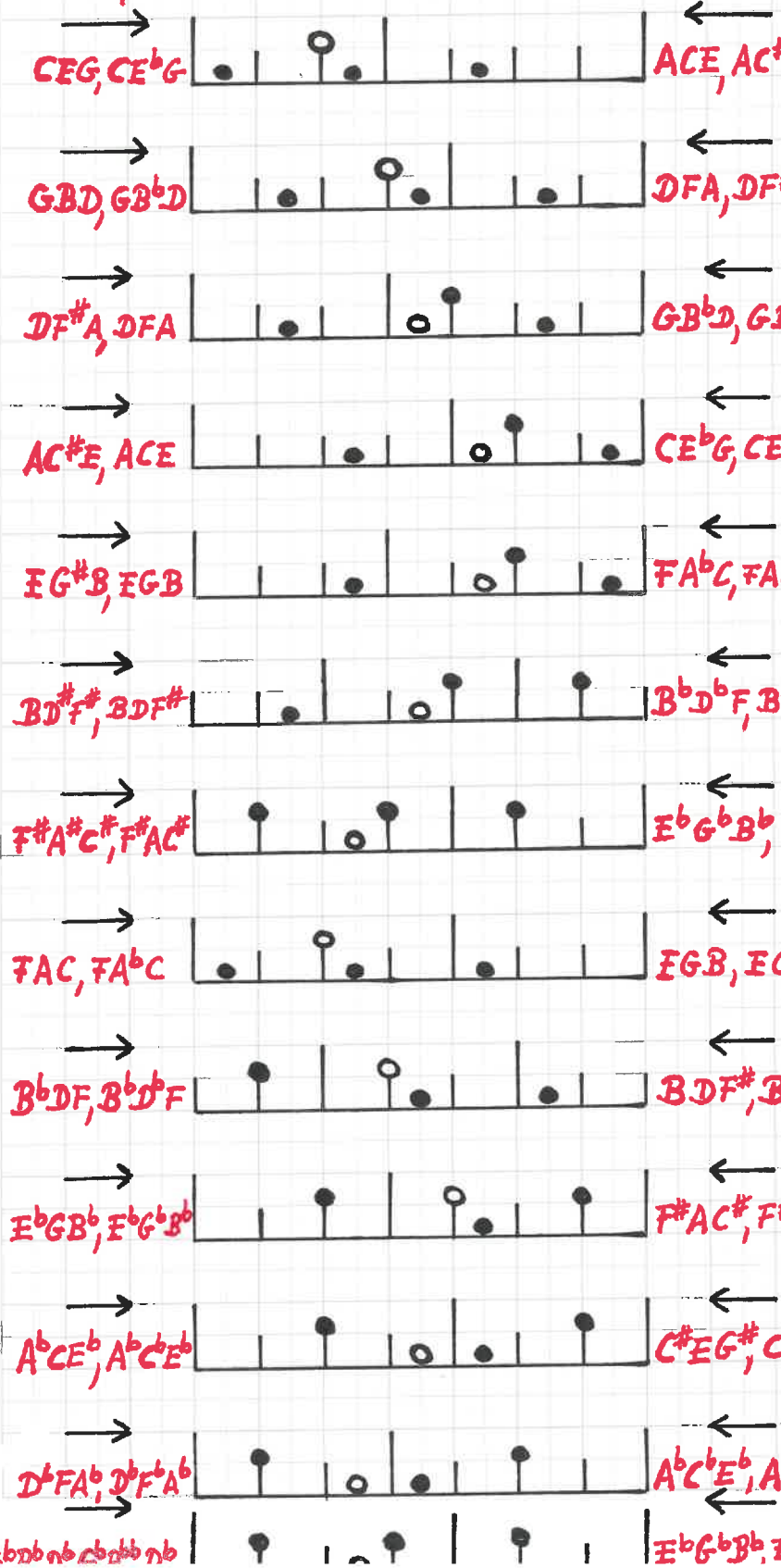
← ACE, AC♯E shows the chords CEG and CE♭G – mapped to ACE and AC♯E

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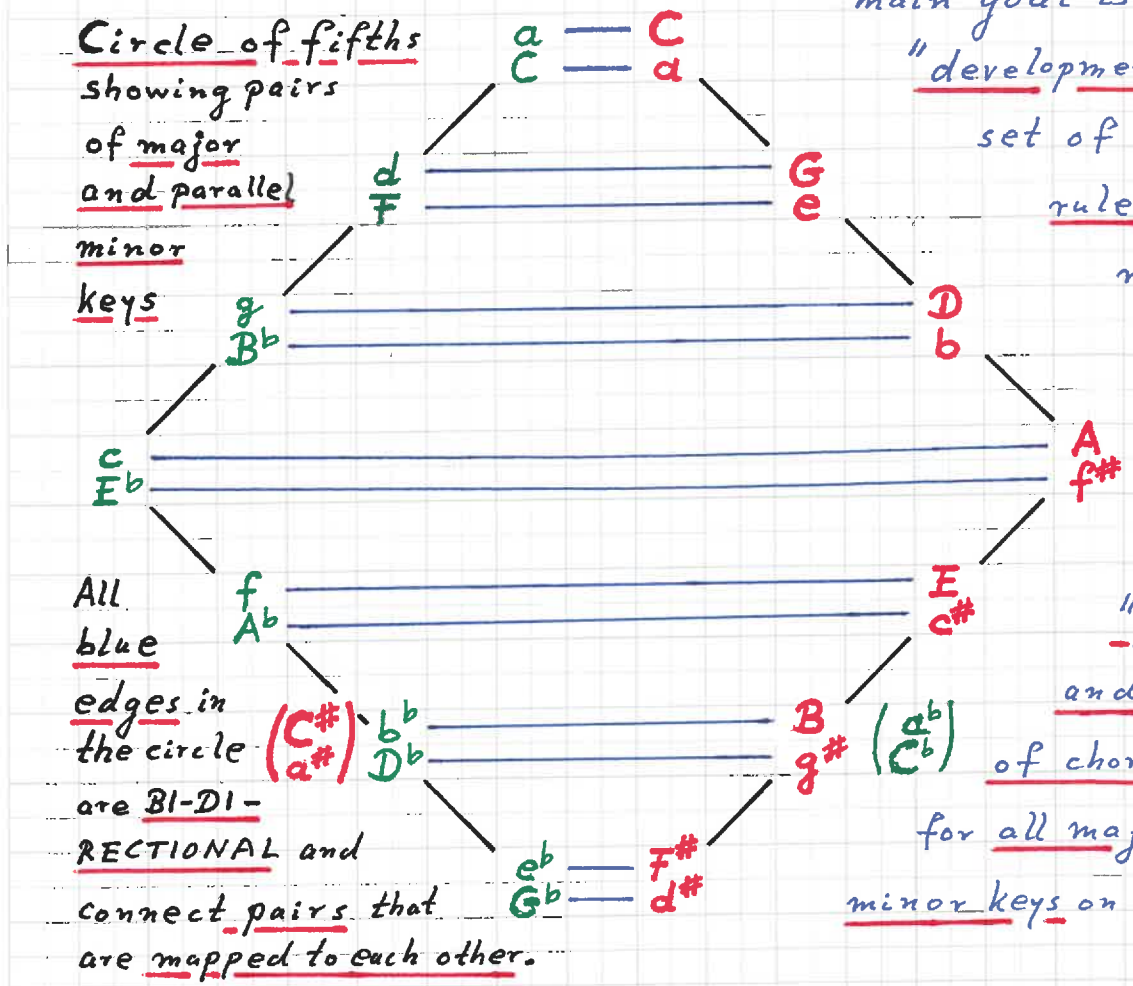
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In the following, the major and minor triads of all keys are mapped based on the principle described on the previous page. The circle-of-fifths order of keys is used to perform / list all mappings.



• Keys, Scales, Circles 12/22/2024

It is possible – and helpful – to represent the "harmonic relationships" resulting from this mapping in the context of the circle of fifths. One also must keep in mind that the following (maj, min) pairs are equivalent: $(F^{\#}, d^{\#}) \triangleq (G^{b}, e^{b})$; $(C^{\#}, a^{\#}) \triangleq (D^{b}, b^{b})$; $(B, g^{\#}) \triangleq (C^{b}, a^{b})$. Once again, the main goal is the "development of a set of SIMPLE rules" for making it easier to memorize the "patterns and shapes" of chords/triads for all major and minor keys on the keyboard.



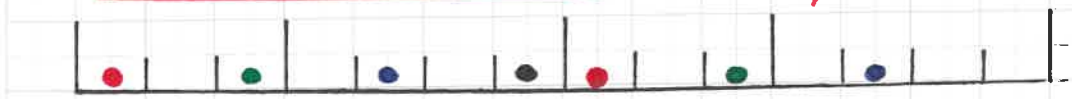
⇒ BY KNOWING AND MEMORIZING THE "MAPPING RELATIONSHIPS" INCLUDED IN THIS REPRESENTATION OF THE CIRCLE OF FIFTHS, THE TRIAD "SHAPES" OF CHORDS MAPPED TO EACH OTHER CAN BE PLAYED/DERIVED MORE EASILY.

→ C	C_D	D	D_E	E	F	F_G	G	G_A	A	A_B	B
← E	D_E	D	C_D	C	B	A_B	A	G_A	G	F_G	F

The left figure shows the principle. D–G_A remains as "axis."

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• In addition, one can use the described "reversal of the keyboard direction" for triad and 7th chord inversions. For example, we consider a C chord:



→ C D E F G A B C D E F G A B

← E D C B A G F E D C B A G F

7th chords:

C E G B	root pos.
E G B C	1st inv.
G B C E	2nd inv.
B C E G	3rd inv.

→ 7th's

C E G B
E G B C
G B C E
B C E G

← 7th's

F A C E	root pos.
A C E F	1st inv.
C E F A	2nd inv.
E F A C	3rd inv.

reverse order 7th chords:

A C E F	1st inv.
C E F A	2nd inv.
E F A C	3rd inv.
F A C E	root pos.

"Included triads":

→	←	
C E G	A C E	root pos.
E G C	C E A	1st inv.
G C E	E A C	2nd inv.
(C maj)	(A min)	

⇒ A given 7th chord implies 14 "patterns":

- 4 patterns for its root and 3 inversion forms
- 4 patterns for root and 3 inversion forms when using reverse keyboard order
- 3 patterns for its "included triad" in root and 2 inversion forms
- 3 patterns for root and 2 inversion forms for its "included triad" when using reverse keyboard order

Since there are 5 commonly used 7th chords for a specific root note, the number of "patterns" for a root note is 5×14 = 70. Since there are 12 possible root notes, the total number of "patterns" is 12×70 = 840.

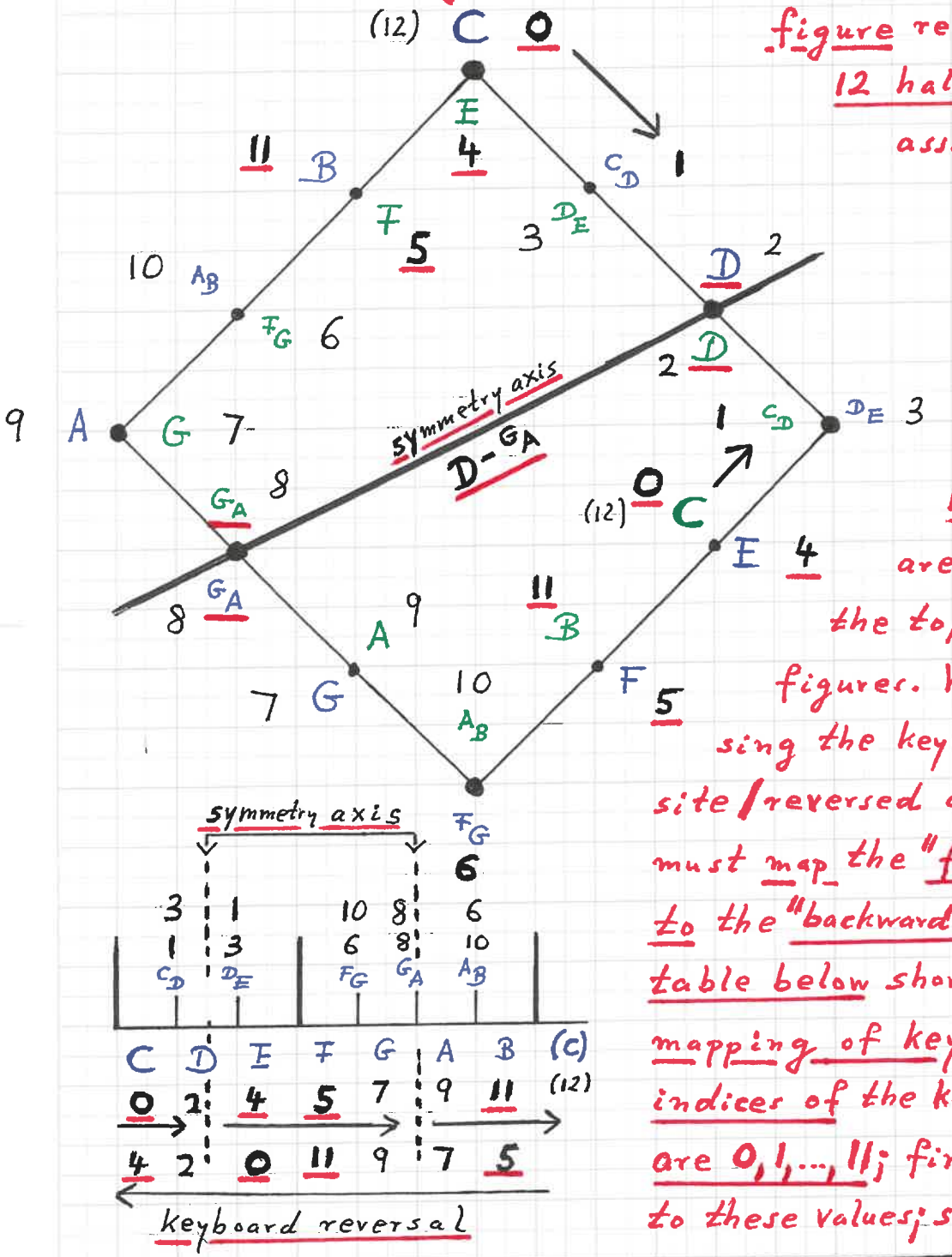
⇒ It is highly desirable to understand relationships between these "patterns" in order to simplify the process of memorizing and playing the "patterns." Thus, one must determine these relationships and derive "simple pattern rules" for playing.

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• One can also view the "reversal of the keyboard direction" from a purely geometrical / linear transformation / algebraic perspective. The left figure represents the 12 halftones and associated keys on the keyboard in an abstract way. Note names, keys, numbering / indexing are included in the top and bottom figurer. When "traversing the keyboard in opposite / reversed direction" one must map the "forward index" to the "backward index." The table below shows the needed mapping of key indices. The indicer of the keys for C, C_D, ..., B are 0, 1, ..., 11; first, one adds 16 to these values; second, one uses the operation "modulo 12" to obtain the "backward index."

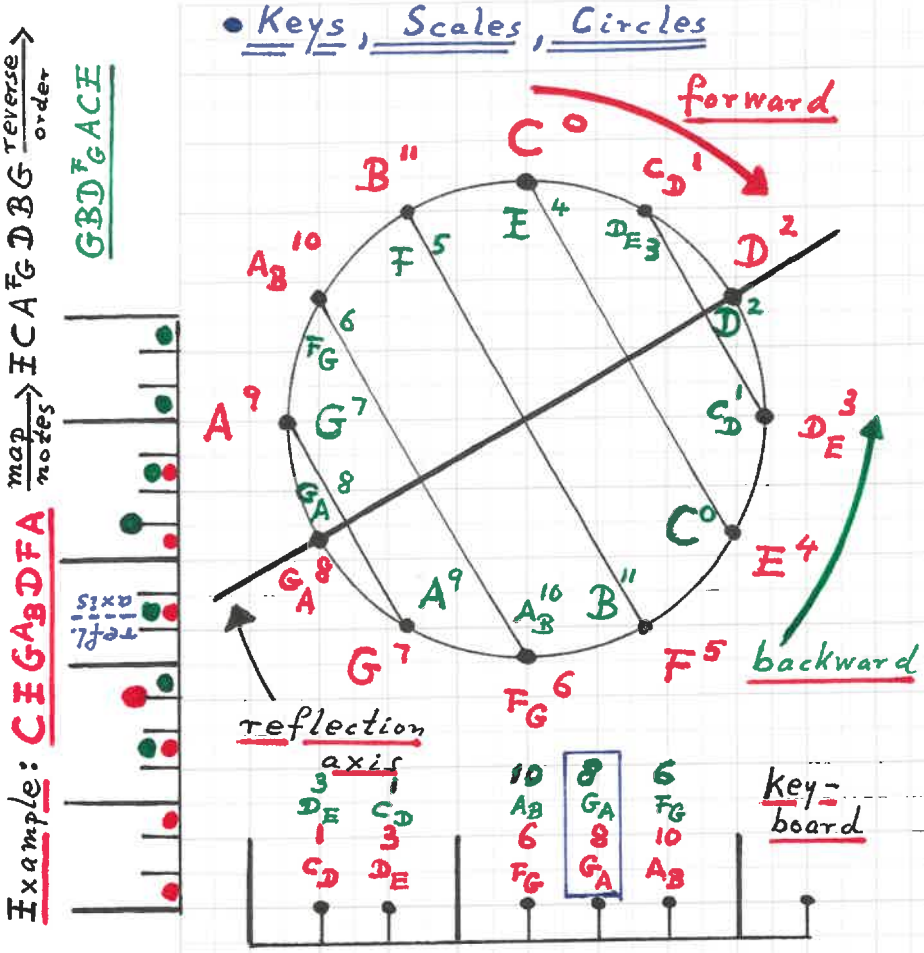


forward index i	0	1	2	3	4	5	6	7	8	9	10	11	12
$I = 16 - i^*$	16	15	14	13	12	11	10	9	8	7	6	5	4
backward ind. (I mod 12)	4	3	2	1	0	11	10	9	8	7	6	5	4

*One could also define I as $I = 4 - i$.

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mapping note names	C	D	E	F	G	A	B	C	← index i
	0	2	4	5	7	9	11	12	mapping key indices
	E	D	C	B	A	G	F	E	
index j →	4	2	0	11	9	7	5	4	

Index mapping: $j = (16 - i) \bmod 12$

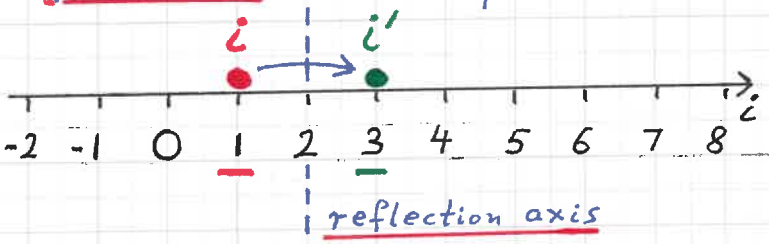
The left figure shows how the symmetry axis D-G_A can geometrically be viewed as a "reflection axis" that maps "key indices and note names from one side to the other side" of the axis. The symmetry of the arrangement of the physical keys of the keyboard leads to the index-mapping formula provided in the left figure. Since we have established a correspondence between notes/keys and index values for the FORWARD and BACKWARD directions, one can "invert/reverse/reflect" any given sequence of notes/chord by the defined mapping of key indices. For example, a given index tuple $(i_1, \ldots, i_n)$ is mapped to the tuple $(j_1, \ldots, j_n) = ((16-i_1) \bmod 12, \ldots, (16-i_n) \bmod 12)$. Considering the triad CEG, the triad's index tuple is (0,4,7). The mapping defines the new tuple $(16 \bmod 12, 12 \bmod 12, 9 \bmod 12) = (4, 0, 9)$. THE ORDER OF THIS TUPLE'S COMPONENTS MUST BE REVERSED, LEADING TO (9,0,4) ≙ ACE.

$(i_1, \ldots, i_n) \to (j_n, \ldots, j_1) = ((16-i_n) \bmod 12, \ldots, (16-i_1) \bmod 12)$

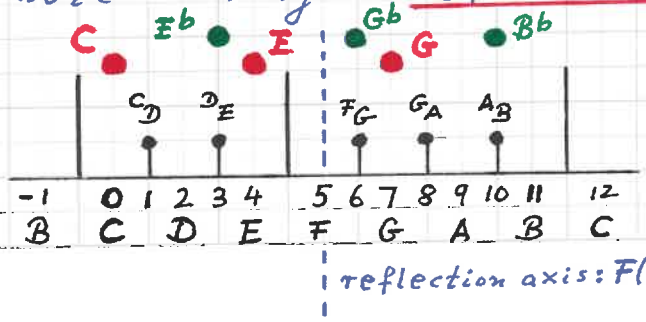
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• It is possible to understand and perform reflections in a general one-dimensional "line setting": Since one can assign a specific integer value to the keys (and notes), one can define a reflection of a key/note, chord or sequence of keys/notes relative to a chosen "reflection axis" (= a point / integer / key). The linear transformation is briefly described first. In the example shown in the left figure, the i-value used to define the reflection axis is i=2.



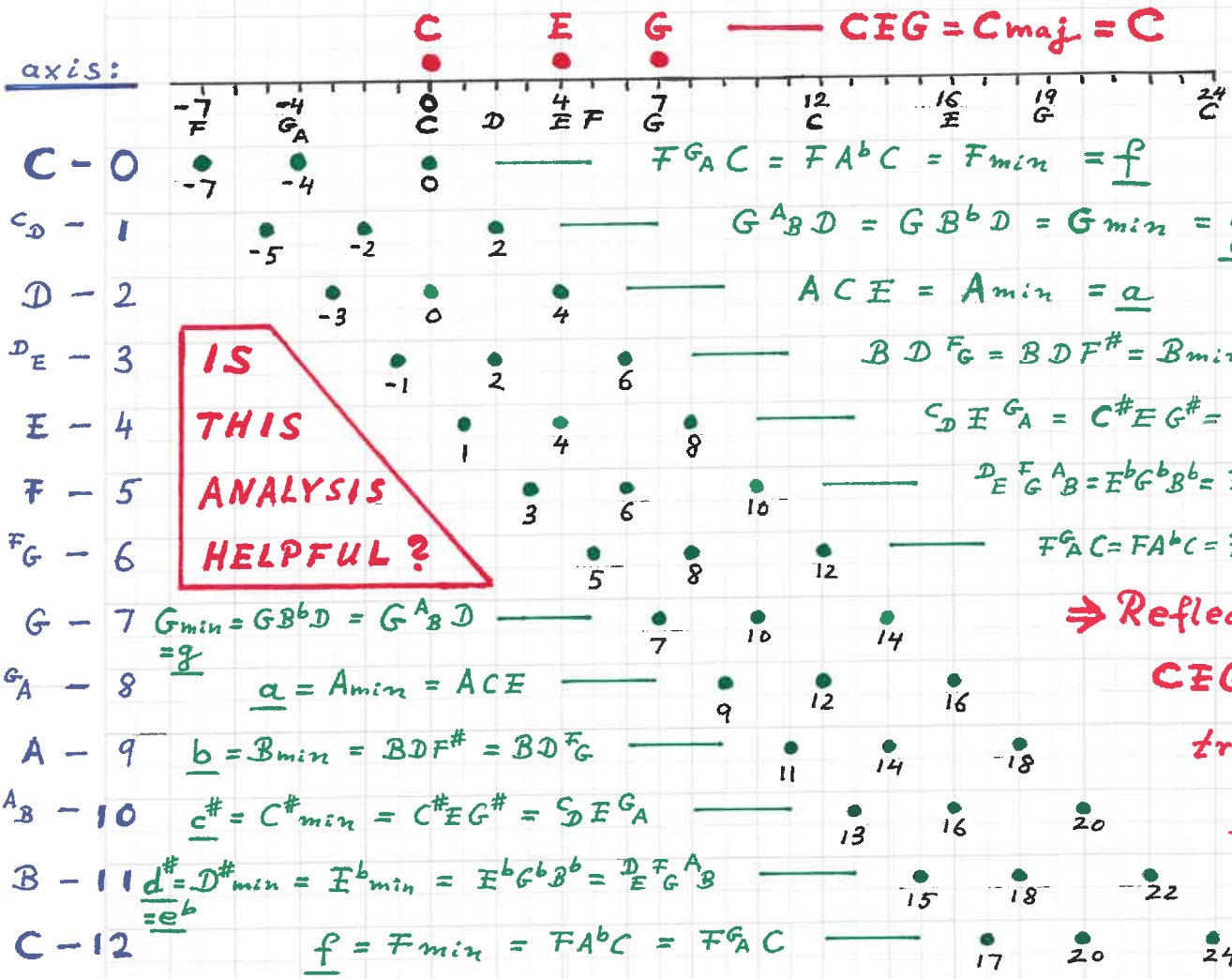
When reflecting the specific value i=1 relative to this axis, it is mapped to i'=3. One can understand the mapping as the concatenation of three "simple" steps: "Subtract 2 from the given i-value, then invert the sign, then add 2": $i' = (-(i-2)) + 2 = 4 - i$. Of course, on the keyboard such a reflection (of a specific key/note) relative to a reflection axis (a specified key/note) must be "seen and played immediately" — without performing "calculations." Obviously, one can use this transformation to reflect any group of keys/notes (chords!) relative to any key/note serving as reflection axis. Reflecting the Cmaj triad CEG using F(5) as reflection axis yields the triad $E^bG^bB^b$, i.e., the E^bmin triad, see left figure.



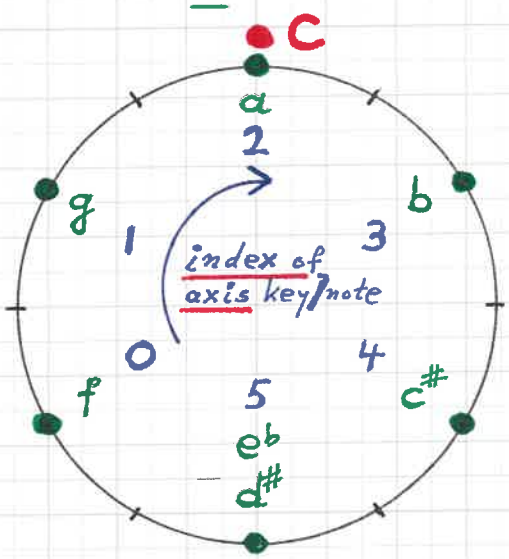
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To obtain a better understanding of the reflection of chords, we reflect the Cmaj triad using several keys/notes to serve as reflection axis.



⇒ Reflecting the CEG major triad can only yield 6 minor triads!



Using circle-of-fifths illustration to show location of resulting triads.

When using the 12 halftones of the chromatic scale one obtains only 6 "reflection triads" of Cmaj (repeated in different octaves). The 6 distinct resulting triads are Fmin, Gmin, Amin, Bmin, C#min, D#min.

Thus, all triads are minor triads.

The circle-of-fifths layout nicely shows the "harmonic location" of the 6 minor triads, see left figure.

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