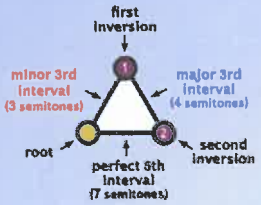
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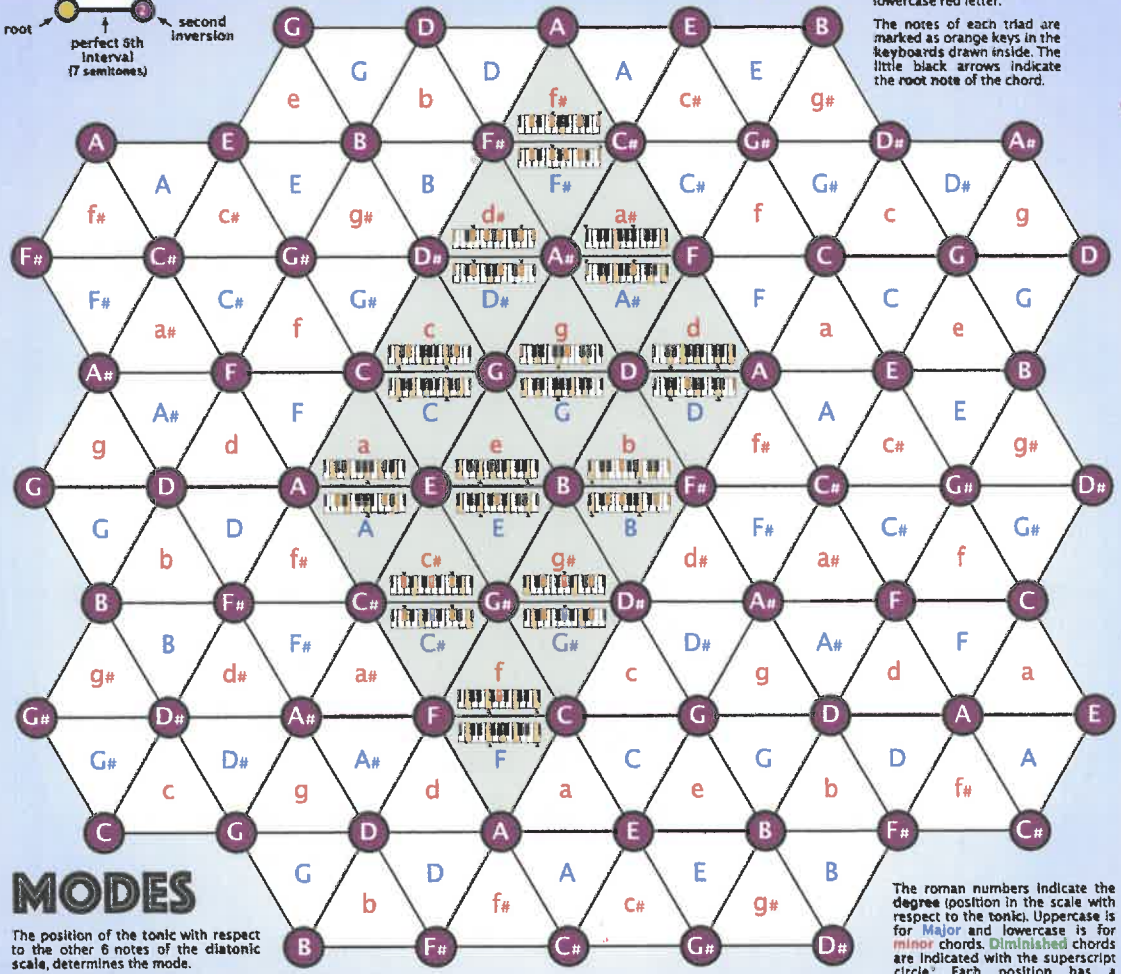
TONNETZ

In 1739 the mathematician Leonhard Euler discovered that the 12 notes of the western music system can be arranged in an hexagonal grid, the **Tonnetz**. The notes are at the nodes of the grid (purple disks) and the interval between adjacent notes in each direction is the same across the entire diagram. For example, there is a perfect 5th interval between C and G, as well as between G and D. The shadowed parallelogram represents the unit cell, the block of notes that is repeated infinitely in both directions of the plane.



Each triangle in the Tonnetz is a triad (group of 3 notes), corresponding to either a Major chord, labelled with an uppercase blue letter, or a minor chord, labelled with a lowercase red letter.

The notes of each triad are marked as orange keys in the keyboards drawn inside. The little black arrows indicate the root note of the chord.

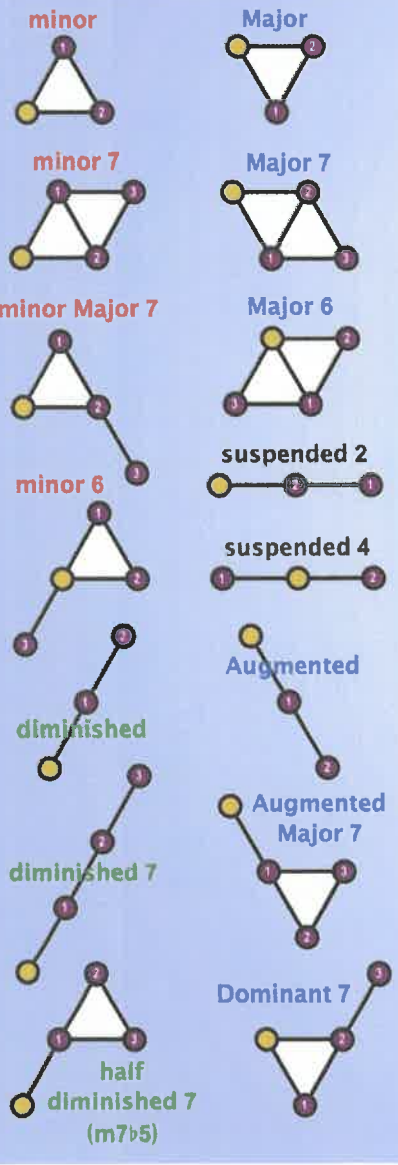


CHORDS

A **chord** is a set of 3 or more **notes** that sound harmonically when played together. Since chords are defined by the **interval** between its notes, in the **Tonnetz** the same type of chord (for example a Major chord) has always the same shape, regardless of where you put the **root note** (Cmaj, Dmaj, Emaj, ...).

The **root note** gives name to the chord and has the lowest pitch, unless the chord is **inverted**. The first inversion of a chord has the same notes, but the note at position 1 (see diagrams below) has the lowest pitch instead. Similarly, the second inversion gets note 2 as its lowest pitch note, etc.

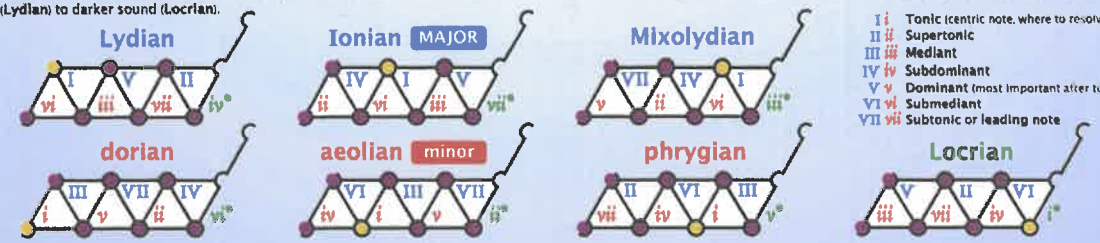
Inverted chords are indicated with the lowest pitch note as part of the name. For example: 2nd inversion of C minor = Cm/G.



MODES

The position of the tonic with respect to the other 6 notes of the diatonic scale, determines the mode.

Each of the 7 main modes of the diatonic scale has a different flavor in western music since the middle ages, from brighter sound (Lydian) to darker sound (Locrian).

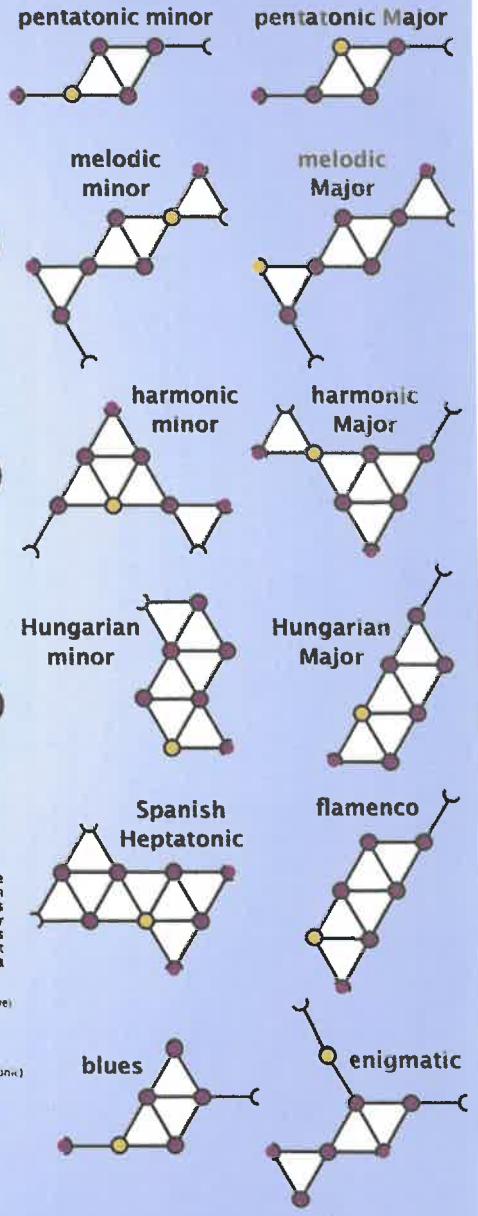


The roman numbers indicate the **degree** (position in the scale with respect to the tonic). Uppercase is for Major and lowercase is for minor chords. Diminished chords are indicated with the superscript circle°. Each position has a function in western tonal system:

- I i Tonic (centric note, where to resolve)
- II ii Supertonic
- III iii Mediant
- IV iv Subdominant
- V v Dominant (most important after tonic)
- VI vi Submediant
- VII vii Subtonic or leading note

SCALES

Any set of notes forms a **scale**. Scales are characterized by how many notes they have and by their relative positions. In the Tonnetz, each scale has a shape, and all the chords that fit into those shapes, are chords of the scale. The most common scale is the diatonic scale (see its 7 modes at the bottom). Here are some other common scales. Yellow disks mark the tonic note of the scale.



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Circle of Fifths (maj and min)

G^b	D^b	A^b	E^b	B^b	F	C	G	D	A	E	B	F#
e^b	b^b	f	c	g	d	a	e	b	f#	d^b	a^b	e^b

C# D# F# G# A#

C$_D$ D$_E$ F$_G$ G$_A$ A$_B$

D^b E^b G^b A^b B^b

12maj
12min

circle of m3rds

circle of 5ths

circle of M3rds

5th m3rd M3rd

circle of fifths

CE^{b}G minor

CEG major

notes of chromatic scale

"harmonic neighborhood" of triad at center

Five 7th chords:
CEGB
CEGA$_B$
D$_E$GA$_B$
D$_E$F$_G$A$_B$
D$_E$F$_G$A

Chords in the Tonnetz

-12-

0 1 2

012 root pos.
120 1st inv.
201 2nd inv.

3 M3rds = 1 octave
4 m3rds = 1 octave
12 5ths = 7 octaves

12/31/2024

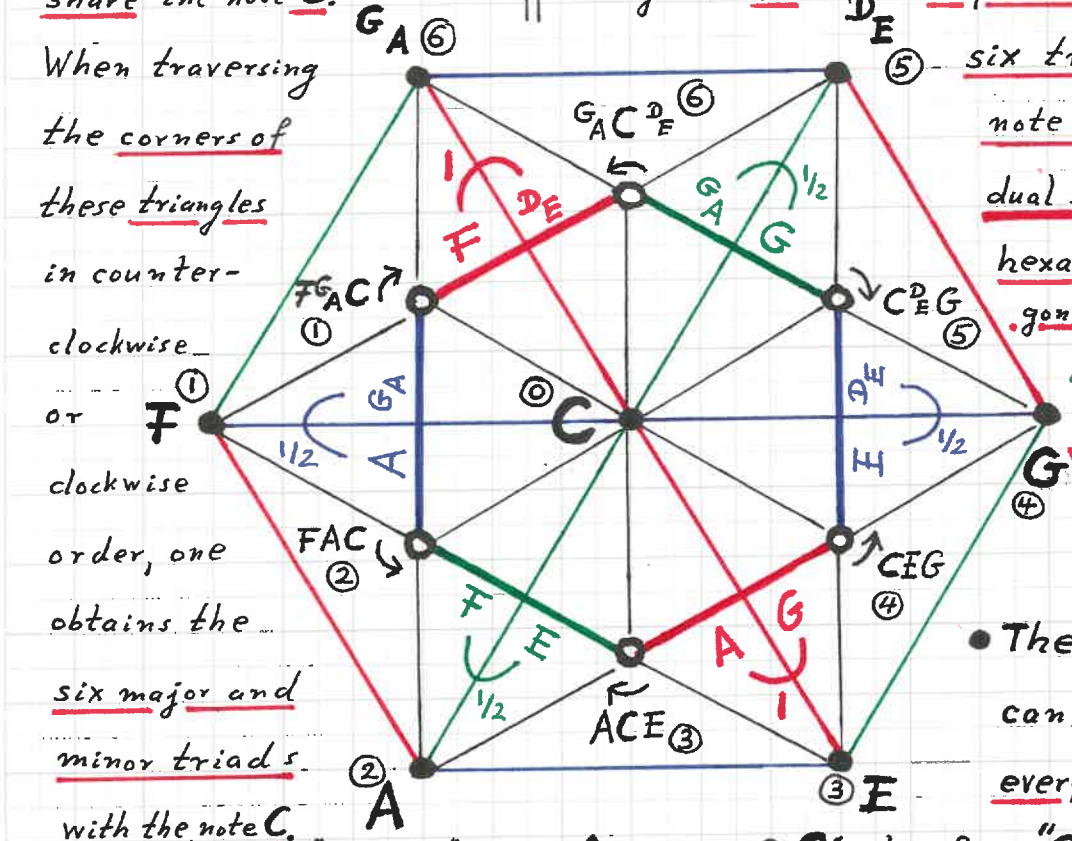
The 4×3 and larger 12×12 "structures" are periodic/cyclic in the horizontal/vertical directions.

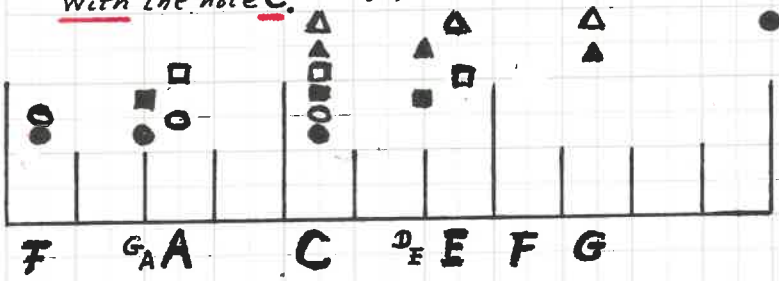
"Neighbors" of triads:
C maj: a, c, e min
a min: F, A, C maj

notes of chromatic scale

• Six Tonnetz-triangles share the note C. When traversing the corners of these triangles in counter-clockwise or clockwise order, one obtains the six major and minor triads with the note C.

• The Tonnetz structure (graph) is "designed" to represent/visualize the six triads that each note belongs to. The dual structure consists of hexagons.





• Circle of six "C-triads":

①	F	G_A	C	=	C	F	G_A
②	F	A	C	=	C	F A	1/2
③	A	C	E	=	C	A E	1/2
④	C	E	G	=	C	E G	1
⑤	C	D_E	G	=	C	D_E G	1/2
⑥	G_A	C	D_E	=	C	D_E G_A	1/2
①	F	G_A	C	=	C	F	G_A

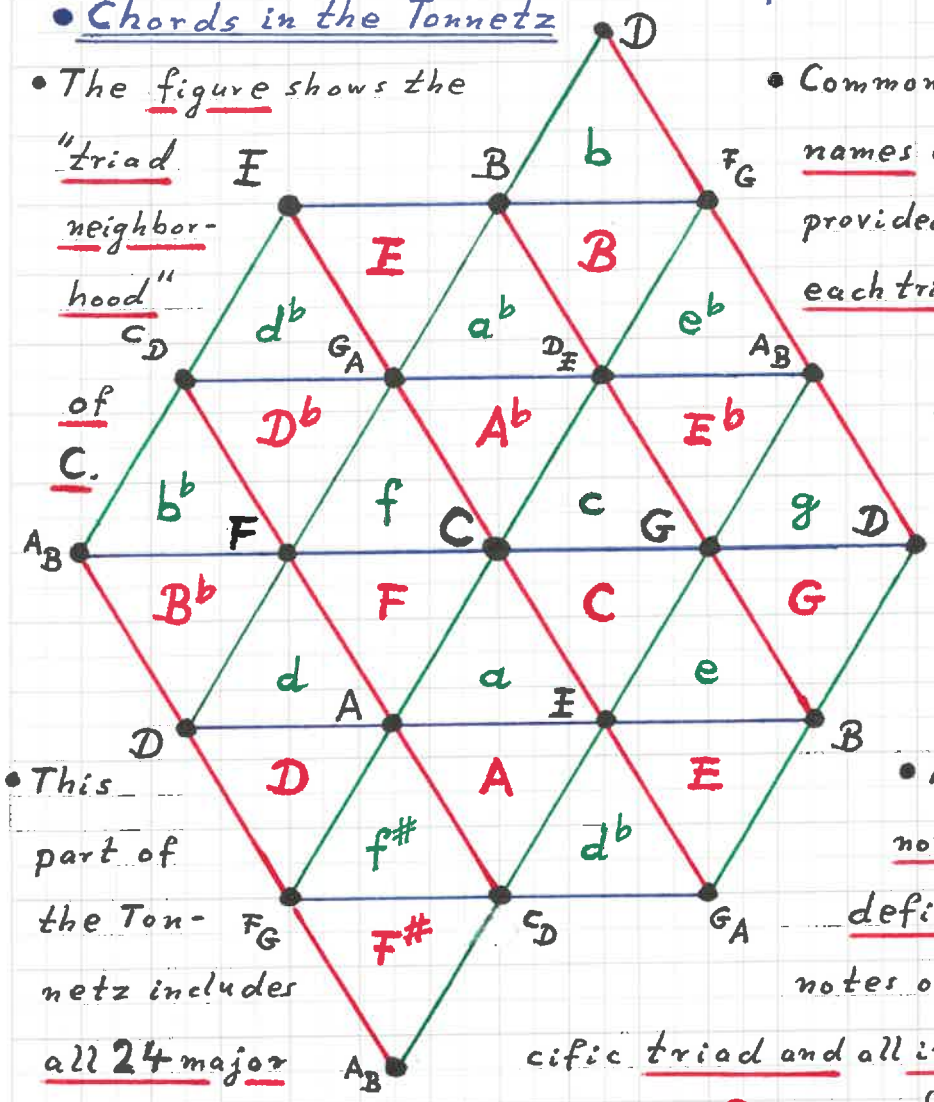
• The same structure can be defined for every note being at the center.

• The note C is part of six triads, shown in the keyboard sketch and the list of "C-triads" (right). The order of these triads is defined by the oriented "C-triad hexagon" in the figure above. By performing triad inversions of the "C-triads" such that C is the first note, one sees that a pair of subsequent/neighboring "C-triads" have have "total note distance" of either one (1/2) or two ($2 \cdot 1/2 = 1$) halftones. Thus, when "moving" forward or backward on the ordered "C-triad hexagon," the overall "smoothest C-triad movement" is guaranteed – ideal for modulation.

11/4/2025

• Chords in the Tonnetz

• The figure shows the "triad neighborhood" of C.



• This part of the Tonnetz includes all 24 major and minor triads.

• Common triad names are provided in each triangle.

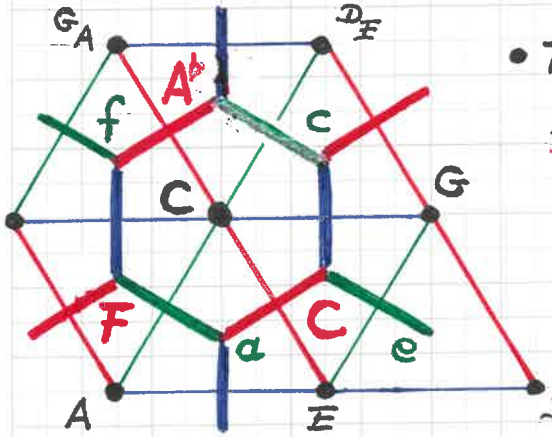
• A triad note set defines the notes of a specific triad and all inversions.

• Triad note sets:

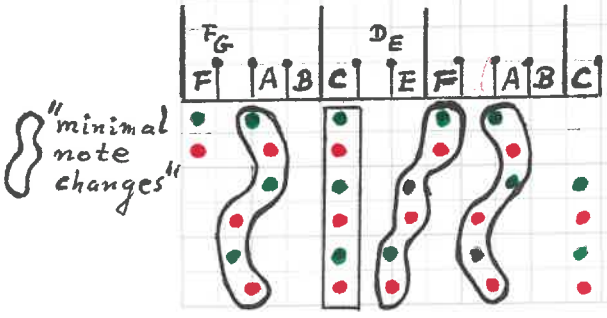
C = C, E, G
G = G, B, D
D = D, F♯, A
A = A, C♯, E
E = E, G♯, B
B = B, D♯, F♯
F♯ = F♯, A♯, C♯
F = F, A, C
B♭ = A♯, D, F
E♭ = D♯, G, A♯
A♭ = G♯, C, D♯
D♭ = C♯, F, G♯

c = C, D♯, G
g = G, A♯, D
d = D, F, A
a = A, C, E
e = E, G♯, B
b = B, D♯, F♯
f♯ = F♯, A♯, C♯
f = F, G♯, C
b♭ = A♯, C♯, F
e♭ = D♯, F♯, A♯
a♭ = G♯, B, D♯
d♭ = C♯, E, G♯

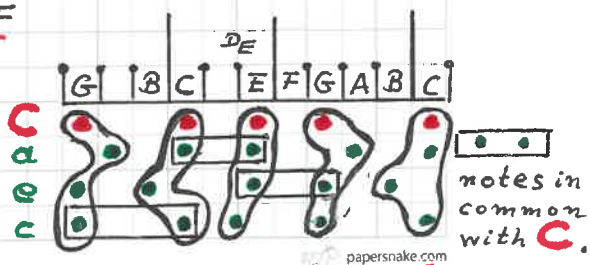
For example, C = C, E, G = {C, E, G} ⇒ CEG, EGC, GCE.



• The left figure shows the TRIAD NEIGHBORHOOD OF THE NOTE C (C, F, A♭, c, f, a) and the TRIAD NEIGBORHOOD OF THE TRIAD C (a, e, c) via the "dual of the Tonnetz." The illustrations below show that these neighborhoods make possible "SMOOTH CHANGES/PROGRESSIONS" OF TRIADS (and keys) - ideal for modulation.



Neighborhood of note C.



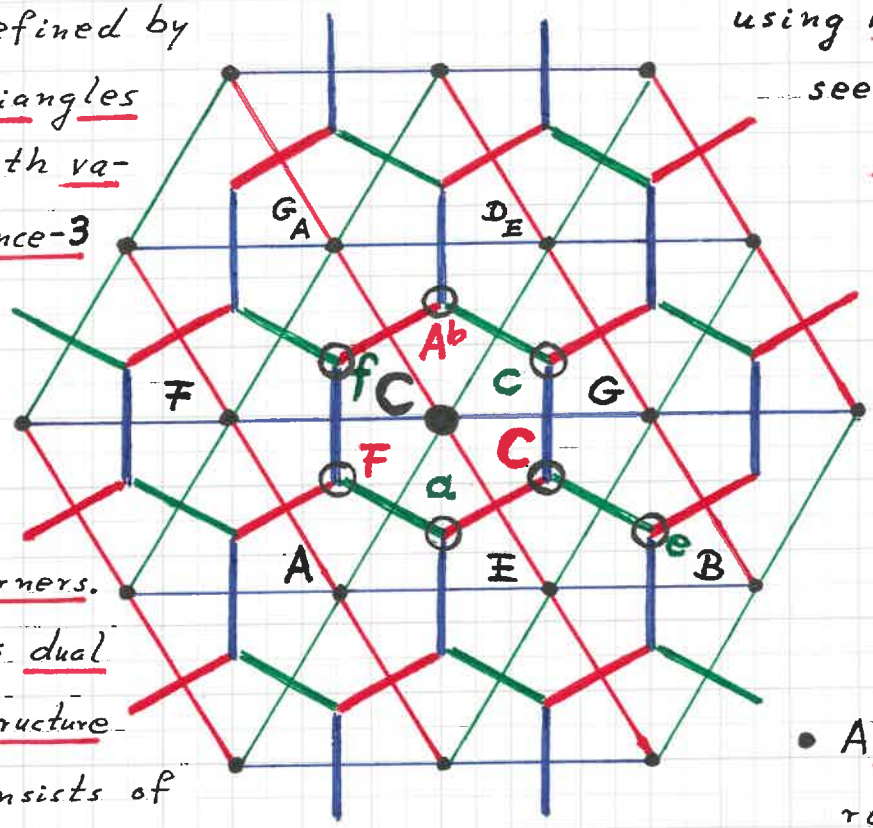
Neighborhood of triad C.

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• Chords in the Tonnetz

11/5/2025

• The Tonnetz is defined by triangles with valence-3 corners. Its dual structure consists of hexagons with valence-6 corners.



• Considering the chromatic scale using halftones, one sees that these structures capture a multitude of relevant intervals:



C→G:	5th
G→C:	4th
C→E:	M3rd
E→C:	m6th
C→D_E:	m3rd
D_E→C:	6th

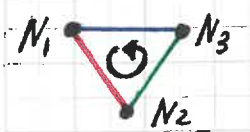
• A Tonnetz point '•' represents a note, having 6 "neighbor notes" — defining 3 major and 3 minor triads sharing the note '•'.

• A point 'O' of the Tonnetz' dual structure represents a triad, having 3 "neighbor triads" — where a major triad has 3 minor "neighbor triads" and a minor triad has 3 major "neighbor triads." A PAIR OF "NEIGHBOR TRIADS" HAS A "TOTAL NOTE DISTANCE" OF EITHER 1 OR 2 HALFTONES.

• Various interesting facts concerning the dual structures are:



The clockwise-oriented triangle with notes N_1, N_2 and N_3 defines the minor triad $N_1 N_2 N_3$ and its two inversions $N_2 N_3 N_1$ and $N_3 N_1 N_2$.



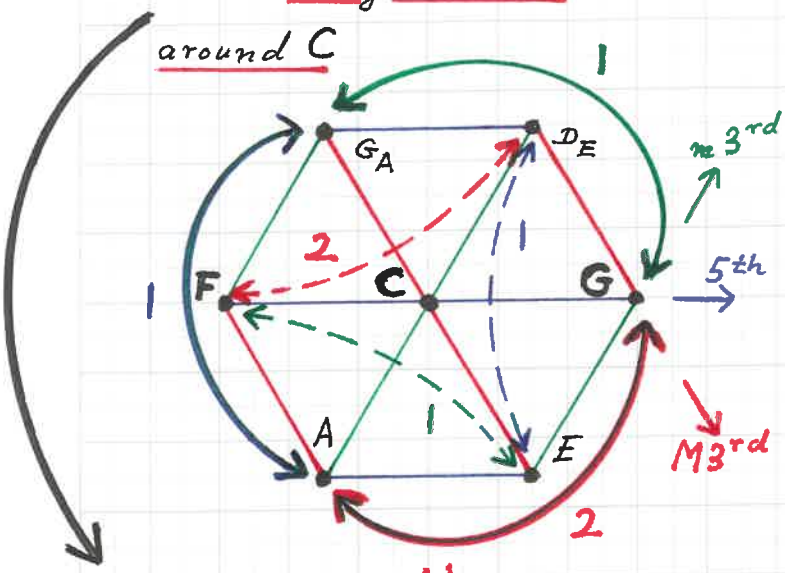
The counter-clockwise-oriented triangle with notes N_1, N_2 and N_3 defines the major triad $N_1 N_2 N_3$ and its two inversions $N_2 N_3 N_1$ and $N_3 N_1 N_2$.

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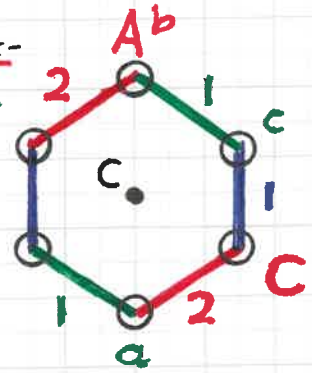
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• Chords in the Tonnetz

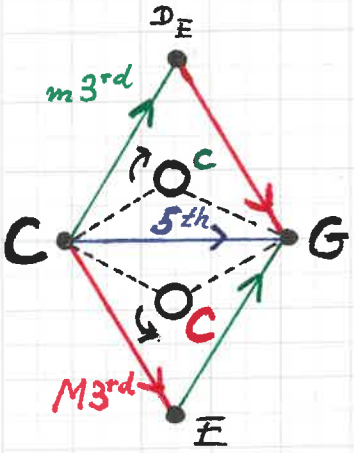
Local neighborhood around C



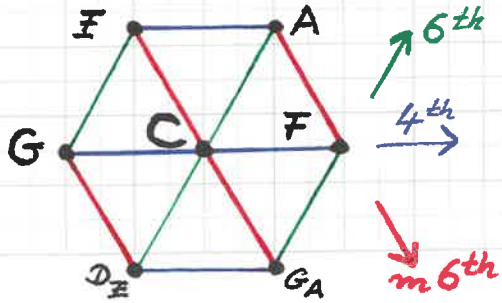
Dual structure of local neighborhood: a hexagon



The left figure shows that the corner points/notes of the hexagon around C have "note distances" of either 1 or 2 halftones — when considering "pairs of every other point/note." Since the set of six triangles/triads around C can be ordered, one can effectively construct/define a "loop of triads around C" (left figure). As already discussed above, the "total note distances" between consecutive triads are 1 or 2 halftones.



The left figure shows the neighborhood of the C major and C minor triads (C and c). The "3-direction network" is defined by the circles of m 3rds, M 3rds and 5ths. The "parallelogram" CCGc (edges shown by broken lines) has the 5th CG as its diagonal — and the minor triad c on its "left side" and the major triad C on its "right side."



The left visualization is obtained by "inverting/reversing directions." One obtains the circles of 4ths (distance = 5 halftones), 6ths (distance = 9 halftones) and m 6ths (distance = 8 halftones).

• Chords in Keyboard Neighborhoods

1/16/2025

The table lists triads $2 \times 4 \times 3 = 24$ chromatic-lexicographically.



Triad			Name	
C	D	F♯ G♭		
		G	C sus 2	
		G♯ A♭		
	D♭ E	F♯ G♭	C dim	
		G	C min	
		G♯ A♭	A♭ dim 1st	
	E	F♯ G♭		
		G	C maj	
		G♯ A♭	C aug	
	F	F♯ G♭		
		G	C sus 4	
		G♯ A♭	F min 1st	
C♯ D♭	D	F♯ G♭		
		G		
		G♯ A♭		
	D♭ E	F♯ G♭		
		G		
		G♯ A♭	D♭ sus 2	
	E	F♯ G♭		
		G	D♭ dim	
		G♯ A♭	D♭ min	
	F	F♯ G♭		
		G		
		G♯ A♭	D♭ maj	

Triads in 9-key/-note neighborhood.

In order to play a "multitude" of triads (in root, 1st inversion or 2nd inversion position) one must also know how to play them "densely and compactly" in just one octave, for example. The left table is based on three "note sets," $\{C, C\sharp/D\flat\}$, $\{D, D\sharp/E\flat, E, F\}$ and $\{F\sharp/G\flat, G, G\sharp/A\flat\}$, and it lists all possible triads — when using one note from each set. When playing the resulting 24 triads on the keyboard, using only keys between the C-key and $G\sharp/A\flat$-key, one can obtain triads played in root or first/second inversion (1st, 2nd). Not all triads are harmonically meaningful or have "standard names."

• Chords in Keyboard Neighborhoods

1/6/2025

Key	Notes	Inversion	C C_D D D_E E F F_G G G_A A A_B B
C	C E G		o o o
a	A C E	1	o o o
G	G B D	2	o o o
e	E G B		o o o
D	D F_G A		o • o
b	B D F_G	1	o • o
A	A C_D E	1	• o o
f#	F_G A C_D	2	• • o
E	E G_A B		o • o
d^b	C_D E G_A		• o •
B	B D_E F_G	1	• • o
a^b	G_A B D_E	2	• • o
F#	F_G A_B C_D	2	• • •
e^b	D_E F_G A_B		• • •
D^b	C_D F G_A		• o •
b^b	A_B C_D F	1	• o •
A^b	G_A C D_E	1	o • •
f	F G_A C	2	o o •
E^b	D_E G A_B		• o •
c	C D_E G		o • o
B^b	A_B D F	1	o o •
g	G A_B D	2	o o •
F	F A C	2	o o o
d	D F A		o o o

24 triads in circle-of-5ths order.
'1' = first, '2' = second inversion.

→ Every note appears in 6 triads.

→ Every note-pair appears in 2 triads.

→ Every triad shares 2 of its notes with 3 other triads.

→ 10 triads are in root position, 7 are in 1st and 7 in 2nd inversion.

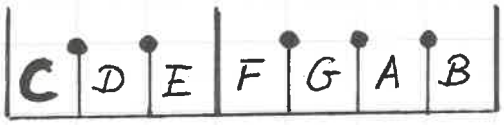
→ The 7 notes defining a key can be "covered" by 3 triads with only

The left table lists the 12 major and 12 minor triads, using the order of the circle-of-fifths. The right column provides a visualization of all 24 triads when played in one octave, using root position, first inversion or second inversion of a triad — to "make it fit" into one octave (on the keyboard). The 24 key patterns shown in the right column make it possible to use an extremely "dense and compact playing style" for these triads — staying within one octave and thus minimizing hand movement across the keyboard. Of course, depending on the **desired** or necessary harmonization, such a compact way to play triads might not always support the intended effects.

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• Chords in Keyboard Neighborhoods 1/7/2025

"Harmonization and Chord/Key Progressions Using Key-inherent Triads or 'Triad Parts'"



Harmonization can be defined and described in many ways, using more or less formal approaches. In the following, basic principles, ideas and "rules" are informally summarized – for the key of C major:

1) The 7 key-inherent triads are I = CEG, ii = DFA, iii = EGB, IV = FAC, V = GBD, vi = ACE, vii° = BDF.
2) Each of these triads must be understood as a root position that also represents its first and second inversions, i.e., I = {CEG, EGC, GCE} etc.
3) Several triad subsets of 3 (or more) triads of the set {I, ii, iii, IV, V, vi, vii°} include (together) all 7 notes of the C major scale.
4) One can establish various triad subsets of 3 (or more) triads for harmonizing the notes of C major – as long as these 3 (or more) triads jointly include all 7 scale notes.
5) Different selections of such subsets of 3 (or more) triads lead to different "colors," "emotions," "effects" when harmonizing a melody (the same melody) with them.
6) The selected triads in such a subset might lead to a PROGRESSION that uses them in some arbitrary sequence or in a repetitive/periodic way.
7) Each of the 7 key-inherent triads can/must also be understood as chords defining "sub-chords," e.g., I = {CEG, EGC, GCE; CE, EC, CG, GC; (EG); C, (E), (G)}.

• Chords in Keyboard Neighborhoods 1/7/2025

Harmonization Principles ...

8) One can view a triad progression as the result of harmonizing a given melody; alternatively, one can understand a melody as the result produced by a given triad progression.

Based on these principles, we consider examples of possible triad progressions for the C major scale:

→ I, IV, V — CEG, FAC, GBD
CG, FAC, GBD
CE, FA, GBD

→ I, vi, V, IV — CEG, ACE, GBD, FAC
CG, AE, GBD, FA
GCE, ACE, GBD, ACF

→ I, ii, V — CEG, DFA, GBD
GCE, FAD, GBD
GC, ADF, BDG

→ vii°, I, IV — BDF, CEG, FAC
BDF, CEG, CFA

→ I, IV, vi, I, V — CEG, CFA, CEA, CEG, BDG

One uses such progressions most commonly in a periodic fashion, e.g., I, IV, V, I, IV, V, I, IV, V, ...

From a combinatorics-driven viewpoint, one is interested in the numbers of possible C major-scale harmonizing triad progressions — consisting of 3, 4, 5, 6 or all maximally possible 7 key-inherent triads. One must keep in mind that one must determine both the set of 3 or more triads and the order defining the specific sequence/cycle of the triads.

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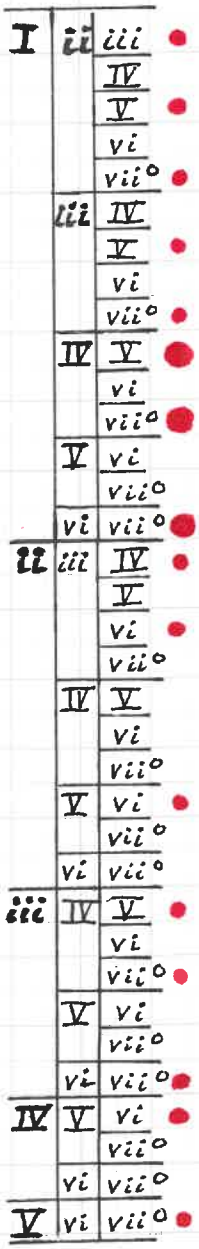
• Chords in Keyboard Neighborhoods 1/8/2025

Harmonization Principles...

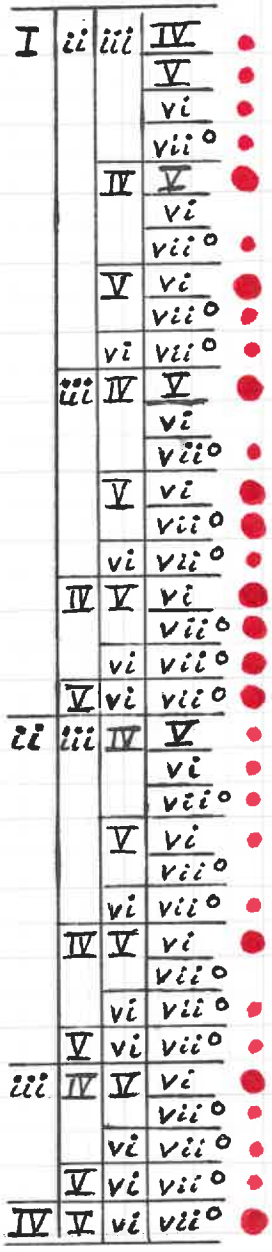
Note. It is often desirable to play the triads (or note subsets) of a progression "densely and compactly" on the keyboard – minimizing finger movements / key distances and also leading to a more "pleasing sound."

We now determine possible triad selections and possible sequences / cycles to play them. The number of possibilities to select 3, 4, 5, 6 or 7 triads from

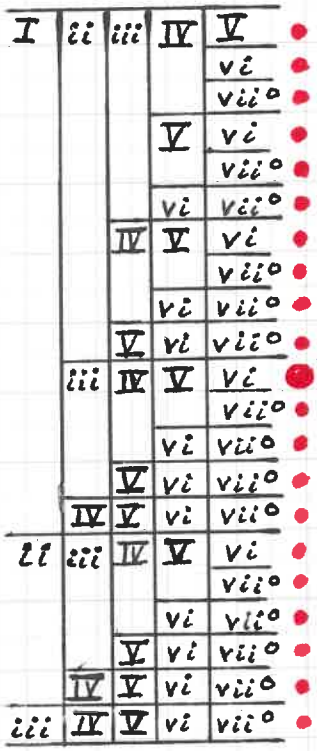
• 3 triads



• 4 triads

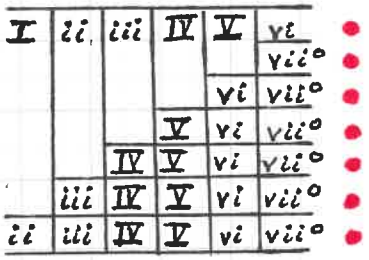


• 5 triads



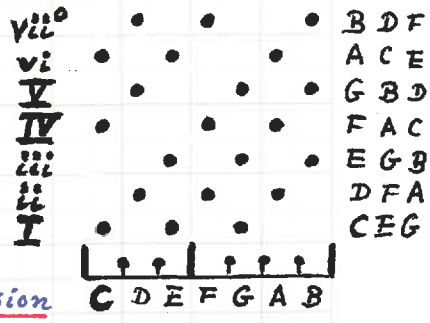
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• 6 triads



• 7 triads





P Pachelbel's progression

the 7 key-inherent triads is $\binom{7}{3}=35$, $\binom{7}{4}=35$, $\binom{7}{5}=21$, $\binom{7}{6}=7$ and $\binom{7}{7}=1$, respectively. These numbers are the lengths of the five tables on this page. SELECTED TRIADS THAT TOGETHER "COVER" ALL 7 NOTES OF C MAJ. ARE MARKED!

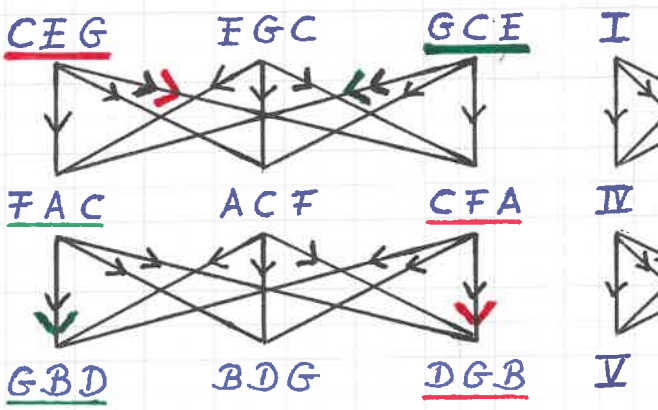
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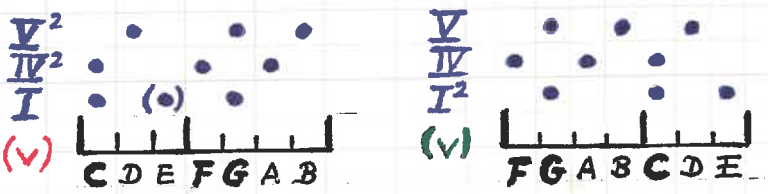
Harmonization Principles...

Notes. 1) The total number of selections of 3, 4, 5, 6 or 7 triads from the 7 key-inherent triads is $35+35+21+7+1=$ **99**. It is important to note that only **16** (of 35) triad selections cover all notes of C major when selecting 3 triads and only **31** (of 35) triad selections cover all notes of C major when selecting 4 triads.

2) A set of k triads can be ordered — and played — in $k!=1\cdot 2\cdot\ldots\cdot k$ ways, e.g., the triad set {I, IV, V} can be ordered in $3!=6$ ways: I-IV-V, I-V-IV, IV-I-V, IV-V-I, V-I-IV, V-IV-I. This fact implies, for example, that the 16 triad selections (when selecting 3 triads) can all be played in 6 ways — defining a total of 96 possibilities to play these 16 triads.

3) One should use a triad's root position and inversions to determine ways to play a triad-progression "compactly and densely" on the keyboard to minimize hand and finger movements. The $3\times 3\times 3=$ **27** possibilities to play the I-IV-V progression with second inversions I^2, IV^2 and V^2 are shown in the figure.





Two possibilities to play I-IV-V densely. I=CEG could be played partially as CG.

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Harmonization Principles...

4) The possibility to "play a triad partially" is powerful. For example, I = CEG can be played as CEG, CG or simply C. Depending on the "harmonic context" of a specific composition, the "human ear/brain can complete the intended triad when played only in part."

5) Many commonly used progressions use a specific triad/chord multiple times. This possibility further increases the number of available progressions to harmonize in a given key. For example, I-IV-V could be "extended" via repeated use of I: I-IV-I-V, I-IV-V-I or I-IV-I-V-I are all viable. The triad progression of "Pachelbel's Canon" also uses triads multiple times. It is defined as **I-V-vi-iii-IV-I-IV-V**.

6) Additional comments concerning progressions/harmonization:

- A very common progression is I-V-vi-IV.
- 12-bar rock and blues progressions usually consist of three "groups," e.g., I-I-I-I; IV-IV-I-I; V-IV-I-I.
- Chord progressions can also include 7th chords — or more complex chords — e.g., V could also be V^7.
- One can harmonize a major scale with 3 major triads, i.e., I (tonic), IV (subdominant) and V (dominant).
- One can analogously harmonize a minor scale with 3 minor triads, i.e., i, iv and v. (Note: 3rd of v often raised by a halftone, to include the "Leitton".* Example: i=ACE, iv=DFA, v=EGB ⇒ V= $EG^{\#}B$, including "Leitton" $G^{\#}$.)

*Leitton = leading tone

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Harmonization Principles...

- A progression chord can **include** a note(s) not belonging to the specific diatonic scale. Often, including such a note(s) "prepares" modulation to another key.
- Typically, progressions have a length of only a few bars.
- Two-triad/-chord progressions are "most simple," and alternating between I and V(7) is commonly done.
- Three-triad progressions can capture all 7 notes of a melody in a specific key. One commonly uses them as four-triad "groups" — to match a "binary rhythm." Examples: I-IV-V-V, I-IV-I-V, I-IV-V-IV.
- ONE CAN USE A MELODY TO DETERMINE A PROGRESSION; ONE CAN USE A PROGRESSION TO DETERMINE A MELODY.
- One can use "groups" of four triads as "building blocks" and append/concatenate them, e.g., (I-IV-V-V)-(I-IV-V-I). This can be done repeatedly, to satisfy the need of a 4-/8-/16-/32-bar melody.
- Blues often uses a "12-bar form" with 3 "groups." A typical progression is (I-I-I-I)-(IV-IV-I-I)-(V-IV-I-I). (Chuck Berry and Little Richard have used it.)
- So-called "1950s progressions" include minor triads/chords. Examples are: I-vi-IV-V and I-vi-ii-V.
- CIRCLE-OF-FIFTHS PROGRESSIONS use "circle adjacency" to define the sequence of triads/chords of a progression. Thus, the Circle makes possible ascending or descending using 5^{th}s/4^{th}s. Examples: vi-ii-V-I = $_{C_{maj}}$ ACE-DFA-GBD-CEG; I-IV-vii°-iii-vi-ii-V-I (full-circle progression).

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Harmonization Principles...

- Sometimes, major triads/chords are used in a progression, instead of the "correct" minor triads/chords, e.g., using I-VI-II-V instead of I-vi-ii-V, when considering the key/scale of C major, for example. Again, this is done to "prepare" modulations.
- Perceptually, harmonization via progressions that "move up/move down a scale step-by-step" is pleasing. ("Pachelbel's Canon" is an example that employs such a principle.)
- All these principles apply equally to major and minor keys. (An example of a descending minor progression is the "Andalusian Cadence," defined as $i-VII-VI-V^{(7)}$ = $\underset{A\,min.}{=}$ ACE - GBD - FAC - EG#B (D). Here, V = EG#B is used instead of v = EGB, since G# is the leading tone in A minor.)

---

7)

	C	D	E	F	G	A	B	
I	•	o		•	o		o		i
ii		•	o		•	o		•	o	ii°
iii		o		•	o		•	o		III
IV	•		o		•	o		•		iv
V		•		o		•	o		•	v
vi	•	o		•		o		•	o	VI
vii°		•	o		•		o		•	VII

"Harmonization table."

The table shown here is a "7×7 matrix" that summarizes compactly the essence of harmonization with triads of the C major scale ('•', '•') and the A minor scale ('o', 'o'). The first row of the table shows that I can be used to harmonize the notes C, E and G ('•') and that i can be used to harmonize the notes A, C and E ('o'). The first column shows that C can be harmonized with the triads I, IV and vi of the C major scale and that C can be harmonized with the triads i, III and VI of the A minor scale—for example.

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Harmonization Principles...

The following examples demonstrate the use of the "harmonization table" on the previous page:

→ Given the progression I-IV-V-I, one can select notes from the "note set sequence" {C, E, G} - {F, A, C} - {G, B, D} - {C, E, G} to define a melody. Of course, when making note selections from one of these "note sets, one can select one, two or all notes in the set - possibly also selecting a note(s) multiple times (defining a simple "one-note-sequence" of a melody or a more complex "triad-/chord-sequence").

→ Given a sequence of single notes defining a melody, e.g., C-E-F-B-D-A, one can define implied triad progressions — using either the C major scale or the A minor scale for harmonization. Considering the six-note sequence example, the two sequences of "triad sets" are:

C major harmony: {I, IV, vi} - {I, iii, vi} - {ii, IV, vii°} - {iii, V, vii°} - {ii, V, vii°} - {ii IV vi}.

A minor harmony: {i, III, VI} - {i, III, v} - {ii°, iv, VI} - {ii°, v, VII} - {ii, iv, VII} - {i, iv, VI}.

THE PROBLEM OF DETERMINING A "GOOD" — OR "THE BEST" — TRIAD PROGRESSION TO HARMONIZE A MELODY (WHEN ONLY A NOTE SEQUENCE IS GIVEN) IS DIFFICULT! FOR EXAMPLE, GIVEN THE NOTE SEQUENCE / MELODY $N_1-N_2-N_3-N_4-N_5-N_6-N_7-N_8$, HOW DOES ONE DECIDE THAT $N_1-N_2 \Rightarrow$ I; $N_3-N_4-N_5-N_6 \Rightarrow$ V; $N_7-N_8 \Rightarrow$ I IS "GOOD"? IS $N_1-N_2 \Rightarrow$ III; $N_3-N_4-N_5-N_6 \Rightarrow$ VII; $N_7-N_8 \Rightarrow$ III ALSO "GOOD"???

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8) A "simple harmonization algorithm." The problem of determining a (formally) viable harmonization / progression for a given melody is discussed in more detail in the following.

- It is assumed that a melody is given — a sequence of notes (some of them possibly played in las/with chords).
- The key of the melody is known. (C major / A minor is assumed and used in the example below.)
- We "move a window" of fixed width over the melody-notes from "left" to "right," e.g., we move a window that contains 2 or 3 or 4 or... consecutive notes and "slide it" over the melody notes.
- For each window position "along the line of melody notes," we determine for how many of the k considered notes lying inside the window, one could use triad I, ii, iii, IV, V, vi, vii° or i, ii°, III, iv, v, VI or VII to harmonize the k notes. ⇒ "Triad counters" for each position!
- Since the key of the melody is known, one might eventually be interested in a harmonization that only uses a simple set of triads / chords to define a / the possibly underlying progression — e.g., I-IV-V-I or i-iv-v-i.

(• Using such a "sliding window" method resembles a type of "discrete convolution.")

- For each window position along the melody, determine the triad / triads with maximal "triad-counter" values.
- We can now determine sequences ("lines" / "poly-lines") of triads "along the melody" connecting those with maximal "triad-counter" values. (Consider all 7 or only certain triads.)

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- The triad sequence(s) resulting from this "purely formal" algorithmic method for determining harmonizing triads (and a progression(s)) might not be "perceptually pleasing" (to the ear). Thus, one also must consider "requirements of rhythm and human perception" to determine, modify and finalize a harmonization — that often must support a desired effect as well.
- We use the first 14 notes (of the melody) of "Alle Vögel sind schon da" von Heinrich Hoffmann von Fallersleben to demonstrate the described technique. (Originally, the song's key is D major; we have transposed it to C major to establish the 14-note opening sequence.)

- Used window width for counter value computation: 4 notes

C maj	A min	C	E	G	C	A	C	A	G	F	G	E	C	D	C
I	i	1x 1x	2x 2x	3x 2	4x 3	3x 3x	3x 3x	2 4x	2 3x	2 2	2x 1	3x 1	3x 2	3x 2	3x 3x
ii	ii°					1	1	2	2	2 1	2 1	1 1	1 1	1 1x	1 1
iii	III	1x	1 2x	2 3x	2 4x	2 3x	1 3x	2	1 2	1 1	2x 2x	3x 3x	2 3x	2 3x	1 3x
IV	iv	1x	1	1	2	2 0	3x 1	2x 2	3x 2	3x 2	2 2	1 1	1 1	1 1	2 1
V	v		1	1 2	1 2	1 2	1 1		1 1	1 1	2x 2x	2 3x	1 2	1 2	1 1
vi	VI	1x 1x	2x 1	2 1	3 2	3x 2	3x 3	3x 2x	3x 3x	2 3x	1 2	1 1	1 2	2 1	3x 2
vii°	VII			1	1	1	1		1	1 1	1 2x	1 2	1 1	2 1	1 1

- "Correct harmonization":

C E G C → I
A C A → IV
G → I
F G → V
E C → I
D → V
C → I

The counter value **0** is not included in the table; all "blank spaces" in the table represent this value. Under each note, maximal counter values of triads of the C maj (A min) key are marked with an (x); The associated triads can be viewed as "harmonization candidates."

⇒ ONE MUST USE THIS TABLE TO DERIVE VIABLE CHOICES FOR HARMONIZATION — POSSIBLY MODIFYING WINDOW WIDTH.

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• We use the same 14-note example to demonstrate that it is HELPFUL / IMPORTANT TO APPLY THIS METHOD TO THE GIVEN NOTES BY TRAVERSING THEM FORWARD – FROM LEFT TO RIGHT – AND BACKWARD – FROM RIGHT TO LEFT to make the resulting "combined" "triad-counter" values DIRECTION-INDEPENDENT. In the following example we use a note-window width of 2 when moving forward or backward.

C maj	A min	C	E	G	C	A	C	A	G	F	G	E	C	D	C
I	i	1 1 2 2	2 2 2 1	2 1 2 1	2 1 1 2	1 2 1 2	1 2 1 2	1 2 1 1	1 1 1	1 1	1 2 1	2 1 2 2	2 2 1 1	1 1 1 1	1 1 1 1
ii	ii°				1	1 1	1 1	1 1	1 1 1	1 1 1 1	1 1		1 1	1 1 1 1	1 1
iii	III	1 1 2	1 2 2 2	2 2 1 2	1 2 1	1 1	1 1	1 1 1	1 1 1 1	1 1 1 1	1 1 2 2	2 2 1 2	1 2 1	1 1	1 1
IV	iv	1 1	1	1	1 2 1	2 1 2 1	2 1 2 1	2 1 1 1	1 1 1 1	1 1 1 1	1 1	1	1 1 1	1 1 1 1	1 1 1
V	v	1	1 1 2	1 2 1 1	1 1			1 1	1 1 1 1	1 1 1 1	1 1 1 2	1 2 1	1 1	1 1	1
vi	VI	1 1 2 1	2 1 1	1 1 1	1 1 2 2	2 2 2 2	2 2 2 2	2 2 1 1	1 1 1	1 1 1	1 1	1 2 1	2 1 1 1	1 1 1 1	1 1 1 1
vii°	VII		1	1 1	1				1 1 1	1 1 1 1	1 1 1 1	1		1 1 1 1	1 1

black number-pairs (top): window from left to right

blue number-pairs (bottom): window from right to left

• adding top and bottom rows of numbers

• maximal values of sums underlined in each column (blue)

C maj	A min	C	E	G	C	A	C	A	G	F	G	E	C	D	C
I	i	3 3	4 3	4 2	3 3	2 4	2 4	2 3	2 1	2	3 1	4 3	3 3	2 2	2 2
ii	ii°				1	2	2	2	2 1	2 2	1 1		1 1	2 2	1 1
iii	III	1 3	3 4	3 4	1 3	2	2	1 2	2 2	2 2	3 3	3 4	1 3	2	2
IV	iv	2	1	1	3 1	4 2	4 2	3 2	2 2	2 2	1 1	1	2 1	2 2	2 1
V	v	1	1 3	2 3	1 1			1 1	2 2	2 2	2 3	1 3	1 1	2	1
vi	VI	3 2	3 1	2 1	3 3	4 4	4 4	3 3	1 2	2	1 1	3 1	3 2	2 2	2 2
vii°	VII		1	2	1			1	1 2	2 2	1 2	1	1 1	2 2	1 1

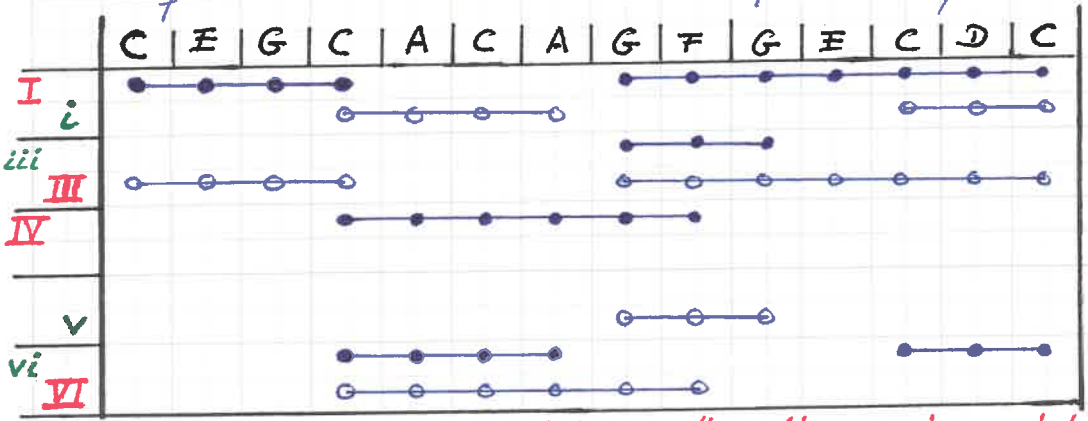
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C maj	A min	C	E	G	C	A	C	A	G	F	G	E	C	D	C
I	i	●○	●	●	●○	○	○	○	●	●	●	●	●○	●○	●○
ii	ii°								●	●○				●○	
iii	III	○	○	○	○				●○	●○	●○	○	○	○	○
IV	iv				●	●	●	●	●○	●○				●○	●
V	v								●○	●○	○			●	○
vi	VI	●			●○	●○	●○	●○	○	○			●	●○	●
vii°	VII								○	●○				●○	

This table marks the "winning and formally viable" triads that are the key-inherent triads of C major ('●') and A minor ('○') that are the "candidate triads" for harmonizing the given 14-note sequence. Considering the alternative to harmonize with C major-inherent triads, the number of possible triad sequences is therefore $2\times1\times1\times3\times2\times2\times2\times5\times6\times2\times1\times2\times6\times3 = 2^8\cdot3^4\cdot5 = 256\cdot81\cdot5$.

(The number of possibilities is the same when considering A minor-inherent triads.) Thus, an effective, efficient and practical method for determining a "truly acceptable and pleasing harmonizing triad sequence is necessary. Obviously, the $256\cdot81\cdot5 = 103680$ "triad candidate" sequences cannot be "tested practically." For example one can identify "spans of consecutive notes" that can be harmonized by one of the "candidate triads" marked '●'/'○'



This table shows such "spans" – those where at least three consecutive notes share a common "candidate triad."

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The "spans" shown in the figure at the bottom of the previous page suggest the following harmonization possibilities:

C maj. triads: CEGC → I, CACAGF → IV (or CACA → vi), GFGECDC → I

A min. triads: CEGC → III, CACAGF → VI (or CACA → i), GFGECDC → III

The method generates the "corresponding harmonization sequences" for C major and A minor triads, i.e., I-IV(or vi)-I and III-VI (or i)-III.

We also provide the results produced by the described method for a "window of width 1." In this extreme setting, one only needs to determine whether a note is or is not part of a particular triad. Again, in order to determine a formally viable harmonization of the 14 notes with a small (or minimal) number of "spans," one must first identify those triads that can be used to harmonize largest-possible subsets of consecutive notes, and then combine/connect/concatenate (some or all of) these triads such that all 14 notes are harmonized.

C A	C	E	G	C	A	C	A	G	F	G	E	C	D	C
I i	• o	• o	•	• o	o	• o	o	•		•	• o	• o		• o
ii ii°					•		•		• o				• o	
iii III	o	• o	• o	o	o		o	• o		• o	• o	o		o
IV iv	•			•	• o	•	• o		• o			•	• o	•
V v		o	• o					• o		• o	• o		•	
vi VI	• o	•		• o	• o	• o	• o		• o		•	• o		• o
vii° VII			o					o	•	o			• o	

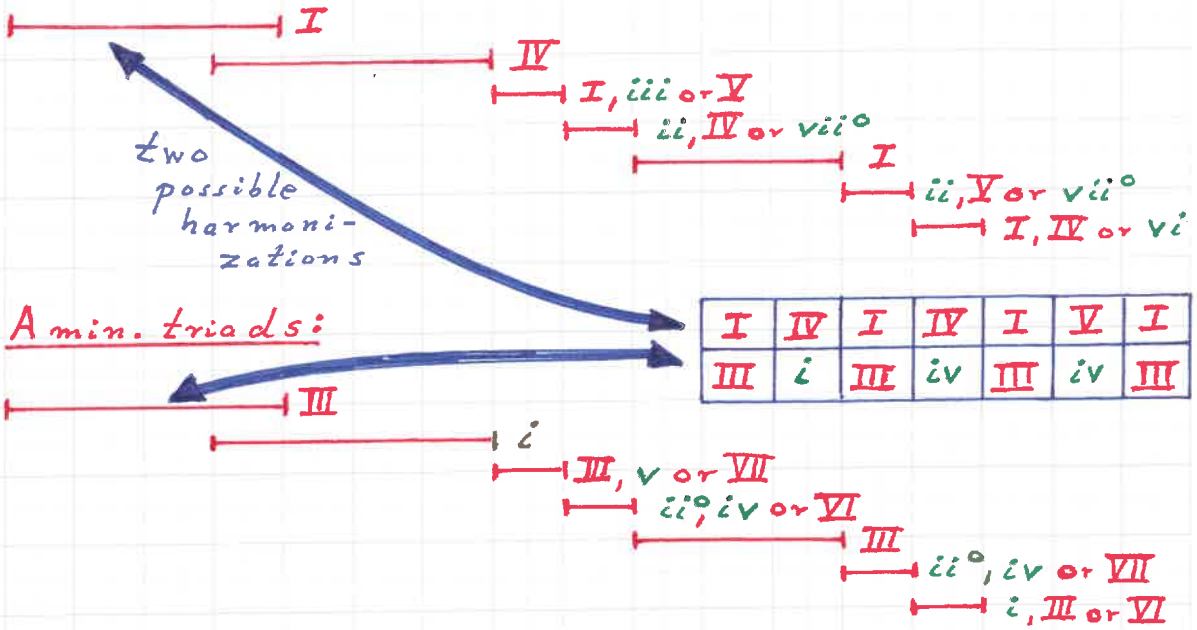
In the left table, a mark '•' or 'o' is made when a triad can be used to harmonize the note in the corresponding column.

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"Spans" can be identified by carefully looking at the table at the bottom of the previous page. These "spans" lead to the following formally viable harmonizations:

C maj. triads:

C E G C A C A G F G E C D C



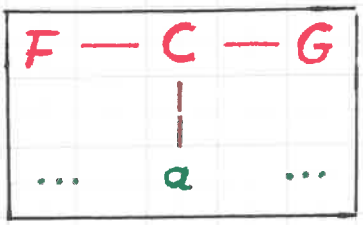
Note. First, the "1-note spans" can all be harmonized with three possible triads. Second, some "spans" overlap, i.e., the 4th note, C, can be harmonized with **I** or **IV** (or III or i). Thus, "span overlaps" provide various harmonization possibilities. Third, the "correct harmonization" (as published) is CEGC→I, ACA→IV, G→I, FG→V, EC→I, D→V, C→I; the harmonization that is closest to this sequence that can be constructed from the C major triad "spans" shown above is CEGC→I, ACA→IV, G→I, F→IV, GEC→I, D→V, C→I. Thus, the only difference in harmonization is FG→V and EC→I versus F→IV and GEC→I. Overall, one can see that the described method can generate "relatively small harmonization sequences" [illegible] select a practically desirable one(s).

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Additional comments concerning this harmonization method:

→ Considering harmonization with key-inherent triads of C major (or any other major scale), and knowing that the key of a composition/melody is C major, one could (initially) use a harmonization maximizing the use of C major triads I = CEG, IV = FAC and V = GBD — since I is the tonic, IV is the subdominant and V is the dominant (triad). (Analogously, one should use this approach also when harmonizing a melody in a known minor key — prioritizing triads i, iv and v.)

→ Considering harmonization with C major triads again, the circle of fifts and its "neighborhoods" also suggest harmonization triads that are "close":



C = I = CEG (tonic) has F = IV = FAC (subdominant) and G = V = GBD (dominant) as its "closest major neighbors"; but Amin = a = vi = ACE defines the "closest minor neighbor" of C and should thus also be used. Thus, {I, IV, V, vi} also defines a good 4-triad harmonization with triads inherent to the C major scale.

→ In order to incorporate desirable "tension" in a harmonization, one can also consider the use of 7th chords. For example, one has the possibility to "extend" I, IV and V, e.g., using $I^7 = CEGB^b$, $IV^7 = FACE^b$ and $V^7 = GBDF$, and $vi^7 = ACEG$.

→ Ultimately, rhythm and desired "harmonic effects" will impact the final harmonization greatly!

→ One must further decide whether it is "better" to harmonize with a small or large number of triad changes for a melody.