Modulation via PIVOT chords uses chords that are common chords of the two chords considered for modulation. PIVOT chords serve as "portals" making possible a smooth transition from one chord to another. In the following, we consider the keys of the circle of fifths and indicate all the chords that are shared by two neighbor keys.

• Example: The common triads of C and G are CI = GIV, Ciii = Gvi, CV = GI, $C^b = B$, Cvi = Gii, $G^b = F^\#$, i.e., $C^\# = D^b$ CEG, EGB, GBD, ACE.

key	C	C_D	D	D_E	E	F	F_G	G	G_A	A	A_B	B	C
C	I		ii		iii	IV		V		vi		vii°	I
G	IV		V		vi		vii°	I		ii		iii	IV
D		vii°	I		ii		iii	IV		V		vi	
A		iii	IV		V		vi		vii°	I		ii	
E		vi		vii°	I		ii		iii	IV		V	
B		ii		iii	IV		V		vi		vii°	I	
$F^\#$		V		vi		vii°	I		ii		iii	IV	
D^b	vii°	I		ii		iii	IV		V		vi		vii°
A^b	iii	IV		V		vi		vii°	I		ii		iii
E^b	vi		vii°	I		ii		iii	IV		V		vi
B^b	ii		iii	IV		V		vi		vii°	I		ii
F	V		vi		vii°	I		ii		iii	IV		V
C	I		ii		iii	IV		V		vi		vii°	I

The left table shows the seven key-inherent triads for the 12 major keys. For example, the row for the key of D maj. lists vii°, I, ii, iii, IV, V, vi. This is the triad list $C^\#EG$, $DF^\#A$, EGB, $F^\#AC^\#$, GBD, $AC^\#E$, $BDF^\#$. Thus, these seven triads include the two accidentals of D maj., i.e., $F^\#$ and $C^\#$.

• IT IS POSSIBLE TO MODULATE FROM ANY KEY TO ANY OTHER KEY - VIA A SEQUENCE OF KEY PAIRS.

THE FOUR TRIADS THAT ARE SHARED BY TWO "NEIGHBOR KEYS" — LISTED IN TWO "NEIGHBOR ROWS" IN THE TABLE — APPEAR IN COLUMNS WHERE ONE SEES A PAIR OF TWO MAJOR OR OF TWO MINOR TRIADS. THE SYMBOL '|' INDICATES PAIRS.

• Modulation

The table of PIVOT triads also shows that triads having a "distance of two" in the circle of fifths only have two chords in common. For example, the keys of C maj. and D maj. only share C iii (= G vi) = D ii = EGB and C V (= G I) = D IV = GBD. Triads with a "distance larger than two" no longer share major or minor triads; thus, in order to smoothly modulate from one key to another key having a "large distance," one must perform a sequence of pivot triad modulations — of "distance two" or "distance one."

(It is also important to realize that the three triads C I – G I – D I = CEG – GBD – DF#A $\hat{=}$ $\underline{\underline{CEG}}$ $\underline{\underline{D}}$ F#A permit the transition between C maj. and G maj. and D maj. — via the pivot triad G I = GBD.)

BASIC MODULATION PRINCIPLES

→ Modulating a key to a "neighbor key" in the circle of fifths — or to a key that is "close" — is "simple" as only a small number of #s or bs must be considered: C ↦ G: F ⇝ F#; G ↦ C: F# ⇝ F; C ↦ F: B ⇝ Bb; F ↦ C: Bb ⇝ B.

→ Pivot-chord modulation supports "smooth" transitions. It is based on identifying chords — or possibly parts of chords — that are shared by the two keys considered for modulation. One ends with a cadence of the first key, moves to the shared pivot chord to add (or subtract) an accidental (# or b), and one starts with a cadence of the new, second key.

→ Modulation of minor keys should generally be done with the harmonic-minor scale, since this minor scale has a lead note — unlike the natural, relative minor scale. The harmonic-minor scale makes possible smooth HARMONIC transitions.

→ One can easily visualize the "linked" nature of triads/chords when using the circle-of-fifths order:

C E G D F♯ A E G♯ B F♯ A♯ C♯ G♯ B♯ D♯ A♯ C♯♯ E♯ B♯ D♯♯ F♯♯
G B D A C♯ E B D♯ F♯ C♯ E♯ G♯ D♯ F♯♯ A♯ E♯ G♯♯ B♯

→ Modulation in a key's "neighborhood" generally involves the triads/chords on steps I, IV, V, vi (major keys) to enable smooth pivot-based modulations:

$F_{I,IV,V,vi}$ = FAC, B♭DF, CEG, DFA; $C_{I,IV,V,vi}$ = CEG, FAC, GBD, ACE;

$G_{I,IV,V,vi}$ = GBD, CEG, DF♯A, EGB. ⇒ Possibilities for C↦G or C↦F.

→ Seventh chords and also 9th chords also support smooth modulations. For each of the seven notes of a key, one consider the (up to five) 7th and 9th chords that have a given note as root. For example, for the key of C major, one has these possibilities:

CEGB(D), DFAC(E), EGBD(F), FACE(G), GBDF(A), ACEG(B), BDFA(c);

CEGB♭(D), DFAC♭(E♭), EGBD♭(F), FACE♭(G), GBDF♭(A♭), ACEG♭(B♭), BDFA♭(c);

...CE♭G♭B♭♭ ≙ CE♭G♭A, ..., BDFA♭.

These and other 7th and 9th chords enable chord resolution that make possible smooth transition to a new key.

Cmaj. and Gmaj. have triads C, G, e and a in common

→ CEG-DFA-EGB-FAC-GBD-ACE-BDF-CEG ≙ C d-e-F-G-a-b°-C ≙
CEG-DF♯A-EGB-F♯AC-GBD-ACE-BDF♯-CEG C-D-e-f♯°-G-a-b-C

≙ 

is a compact "graph representation" of the 7 triads of Cmaj. and Gmaj.

→ One can use an equally compact representation to also show shared-chord behavior of three subsequent (major) keys in the circle of fifths. We consider Cmaj., Gmaj., Dmaj.:

Cmaj.: C E G | D F A | E G B | F A C | G B D | A C E | B D F | C E G

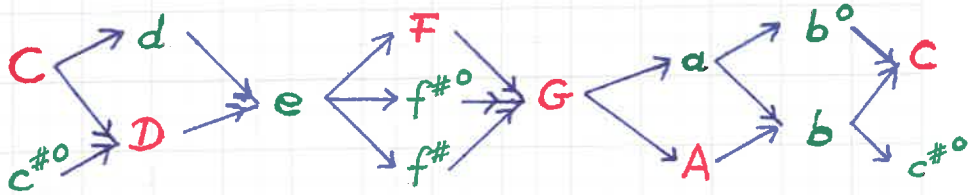
Gmaj.: C E G | D F♯ A | E G B | F♯ A C | G B D | A C E | B D F♯ | C E G

Dmaj.: C♯ E G | D F♯ A | E G B | F♯ A C♯ | G B D | A C♯ E | B D F♯ | C♯ E G

≙ chord/triad names

C	d	e	F	G	a	b°	C
C	D	e	f♯°	G	a	b	C
c♯°	D	e	f♯	G	A	b	c♯°

≙ graph representation with "merged" common triads



≙ Roman numerals associated with all triads

I	ii	iii	IV	V	vi	vii°	I
IV	V	vi	vii°	I	ii	iii	IV
vii°	I	ii	iii	IV	V	vi	vii°

The compact "directed graph representation" of the triads of the three keys visualizes possible transitions/modulations via triads that are are shared by two or three keys. The directed graph edges indicate how one might establish a modulation by using a "triad path" (from "left to right" in the visualized scenario) that passes through a shared "triad node in the graph," moving to another key. As before, the pairs or triplets of triads shared by two or three keys, respectively, are indicated via '|'.

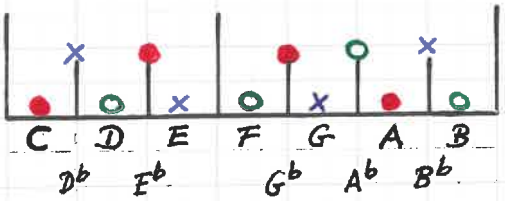
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➔ Modulation via the use of shared/partially shared 7th chords (or even 9th chords) opens up many possibilities, since these "tension chords" make possible the utilization of multiple possible resolutions to various new keys to be transitioned to. Further, one can consider (at least) five such 7th chords for any given root (root ≙ key). Of particular interest and importance in this context is the FULLY DIMINISHED 7th CHORD: This 7th chord (and its three inversions) can be "harmonically interpreted in several ways" and can be resolved to multiple resolution chords/keys. We consider the three fully diminished 7th chords: CE^bG^bA, FA^bBD and GB^bD^bE. (Note. The Cmaj7, Fmaj7 and Gmaj7 chords are CEGB, FACE and GBDF#. Turning these three chords into fully diminished chords requires one to "apply the operator 'b' once to the second and third notes and twice to the fourth note of the chords, leading to $CE^bG^bB^{bb} = CE^bG^bA$, $FA^bC^bE^{bb} = FA^bBD$ and $GB^bD^bF^b = GB^bD^bE$. Each one of these chords is a stack of three minor 3rds. Together, the notes of these three 7th chords cover all 12 notes of a "chromatic octave," and thus the chords do not have any note in common. This fact can be visualized by indicating the chords' notes (•, ○, x) in the left figure. Chord inversions are:

CE^bG^bA, E^bG^bAC, G^bACE^b, ACE^bG^b;

FA^bBD, A^bBDF, $BDFA^b$, DFA^bB;

GB^bD^bE, B^bD^bEG, D^bEGB^b, EGB^bD^b.



3 fully dim. 7th chords cover all 12 notes of an octave.

→ For simplicity, we will write these three 7th chords slightly differently, to avoid the need of using the '#' and 'b' symbols. We use $^{C}_{D}$, $^{D}_{E}$, $^{F}_{G}$, $^{G}_{A}$ and $^{A}_{B}$:

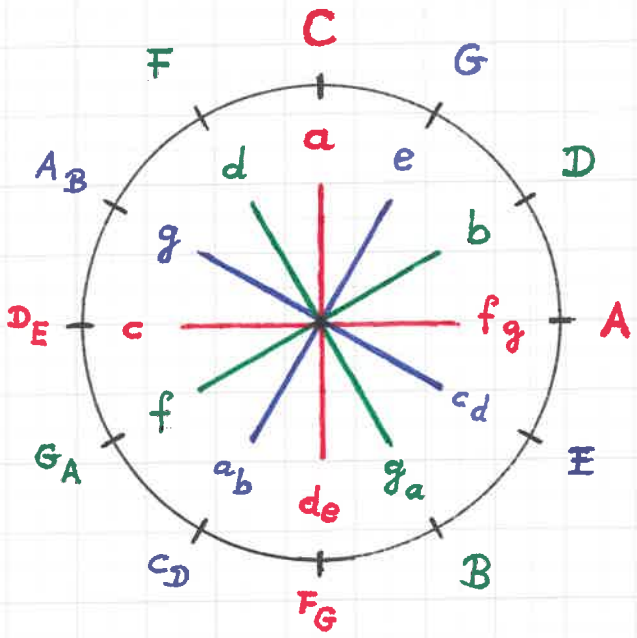
$CE^{b}G^{b}A \hat{=} C^{D}_{E}{}^{F}_{G}A$, $FA^{b}BD \hat{=} F^{G}_{A}BD$, $GB^{b}D^{b}E \hat{=} G^{A}_{B}{}^{C}_{D}E$.

One can assume that each of these three 7th chords can be resolved to possibly eight triads/keys. Resolutions are:

$C^{D}_{E}{}^{F}_{G}A$: C maj, $^{D}_{E}$ maj, $^{F}_{G}$ maj, A maj; c min, $^{d}_{e}$ min, $^{f}_{g}$ min, a min ;

$F^{G}_{A}BD$: F maj, $^{G}_{A}$ maj, B maj, D maj; f min, $^{g}_{a}$ min, b min, d min ;

$G^{A}_{B}{}^{C}_{D}E$: G maj, $^{A}_{B}$ maj, $^{C}_{D}$ maj, E maj; g min, $^{a}_{b}$ min, $^{c}_{d}$ min, e min .



The left figure shows the circle of fifths, where the simplified notation for notes/triads/keys is used again — '#' and 'b' are avoided. The outer letters denote the 12 major keys; the inner letters denote the 12 minor keys.

IT IS RELEVANT TO OBSERVE THAT THE 24 RESOLUTIONS OF THE THREE FULLY DIMINISHED 7th CHORDS PRODUCE A PERFECTLY SYMMETRIC PATTERN IN THE CIRCLE-OF-FIFTHS REPRESENTATION! Again, these 7th-chord resolutions serve modulation to a new key.

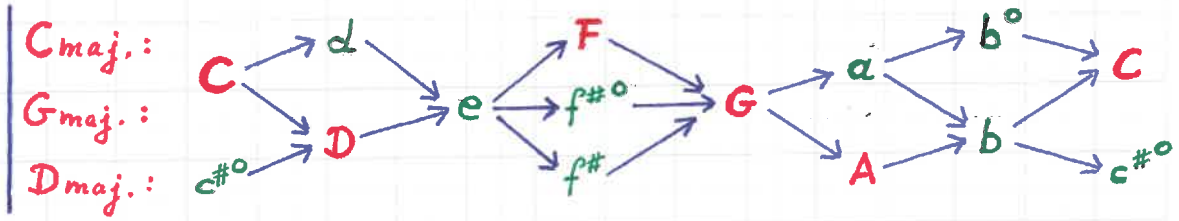
→ Since the fully diminished 7th chord and its three inversions are all defined by m 3rds, one can use/view any of its four notes as its root/its key. For example, the "note sequence" ... $C-{}^{D}_{E}-{}^{F}_{G}-A-C-{}^{D}_{E}-{}^{F}_{G}-A-C-{}^{D}_{E}-{}^{F}_{G}-A-C$... stacks m 3rd-generating the chords $C^{D}_{E}{}^{F}_{G}A$, $^{D}_{E}{}^{F}_{G}AC$, $^{F}_{G}AC^{D}_{E}$, $AC^{D}_{E}{}^{F}_{G}$,

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• Modulation

21/10/2025

➜ We re-consider the directed graph representation where common triads are shown as merged nodes. This flow graph can guide the definition of smooth modulations. Again, we use the three keys C maj., G maj. and D maj.:



In this flow graph, one sees the following behaviors:

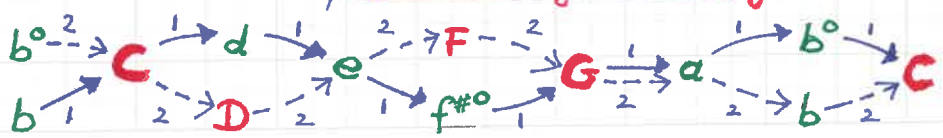
- 1 edge leaves a node: e.g., 1 edge leaves $c^{\#\circ}$, d, b°.
- 1 edge enters a node: e.g., 1 edge enters d, a.
- 2 edges leave a node: e.g., 2 edges leave C, G, b.
- 2 edges enter a node: e.g., 2 edges enter D, e, C.
- 3 edges leave a node: e.
- 3 edges enter a node: G.

One could also use the terms "splitting" and "merging":

- C splits into d and D. ⇒ C can transition to d or D.
- e splits into F, $f^{\#\circ}$ and $f^{\#}$. ⇒ e can transition to F, $f^{\#\circ}$ or $f^{\#}$.
- d and D merge in e. ⇒ Both d and D transition to e.
- F, $f^{\#\circ}$ and $f^{\#}$ merge in G. ⇒ F, $f^{\#\circ}$ and $f^{\#}$ transition to G.

It is now possible to identify and select various paths from a start key to a new key. For example, one can use:

- From C maj. to G maj.: C - d - e - $f^{\#}$ - G.
- From C maj. to D maj.: C - d - e - $f^{\#}$ - G - a - b - $c^{\#\circ}$ - D.
- Possibly from D maj. to C maj.: D - e - $f^{\#\circ}$ - G - a - b° - C.



➜ The left figure shows a potential "meandering": C-G-C-G-C...

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• Modulation 2/11/2025

→ Considering modulation C maj. ↔ G maj., the pivot chords serving as "modulation portals" are e, G, a and C. In the immediate key-inherent "triad neighborhoods," one sees:

- Pivot e: D F A (1) → E G B (2) → F A C; D F# A (2) → E G B (1) → F# A C
  1 Cmaj. ↦ Gmaj.
  2 Gmaj. ↦ Cmaj.

- Pivot(s) G-a: F A C (1) → G B D (1/2) → A C E (2) → B D F; F# A C (2) → G B D (1/2) → A C E (1) → B D F#
  1 Cmaj. ↦ Gmaj.
  2 Gmaj. ↦ Cmaj.

- Pivot C: B D F (1) → C E G (2) → D F A; B D F# (2) → C E G (1) → D F# A
  1 Cmaj. ↦ Gmaj.
  2 Gmaj. ↦ Cmaj.

These "portal paths" provide ideas for potential modulations.

→ Note. It is possible to define 9th chords that "contain as sub-chords" (triads) key-inherent chords (triads) of the two keys considered for modulation. For example, C E G B D = C E G ∪ G B D or F A C E G = F A C ∪ C E G; thus, it should be possible to consider these 9th chords for potential modulations C maj. ↔ G maj. and F maj. ↔ C maj., respectively.

→ $G^b\ D^b\ A^b\ E^b\ B^b\ F\ C\ G\ D\ A\ E\ B\ F^\#$
$e^b\ b^b\ f\ c\ g\ d\ a\ e\ b\ f^\#\ c^\#\ g^\#\ d^\#$
Of course, one can define a multitude of modulation sequences that involve all 12 major and 12 minor keys/scales. For example, one can consider the sequence C e G b D $f^\#$ A $c^\#$ E $g^\#$ B $d^\#$ (e^b) $F^\#$ (G^b) b^b D^b f A^b c E^b g B^b d F a C – shown in the figure. Each step in this sequence requires one to add/take away only one accidental (#, b) at most – since the sequence performs modulation steps according to the circle-of-fifths order.

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→ It is also interesting to consider the progression of all (major) triads, following the circle-of-fifths order:

CEG DF#A EG#B F#A#C# G#B#D# A#C##E# B#D##F##(CEG)
GBD AC#E BD#F# C#E#G# D#F##A# E#G##B#

This sequence of triads can inform smooth modulations.

→ We briefly consider the "circle-of-fifths neighborhood" F-d-C-a-G-e to show pivot chords/triads in the context of both major and minor keys. As before, keys share major and minor triads that can play a role when "pivoting."

key	C	C_D	D	D_E	E	F	F_G	G	G_A	A	A_B	B	C
F	V		vi		vii°	I		ii		iii	IV		V
d	VII	vii°	i		ii°	III		iv		v	VI		VII
C	I		ii		iii	IV		V		vi		vii°	I
a	III		iv		v	VI		VII	vii°	i		ii°	III
G	IV		V		vi		vii°	I		ii		iii	IV
e	VI		VII	vii°	i		ii°	III		iv		v	VI

This table shows that the six keys F, C, G and d, a, e share the triad CEG, and also have the triad ACE in common. "Unconventionally," the table uses the notation "vii°" to indicate that one can raise the seventh note of the natural, relative minor key/scale — to produce the harmonic minor key/scale and introduce a lead note. In the three minor keys included in the table, this change means: d min.: VII = CEG → C#EG = vii°; a min.: VII = GBD → G#BD = vii°; e min.: VII = DF#A → D#F#A = vii°. Considering these three minor keys/scales in more detail, these are all triad changes when moving from relative to harmonic minor: d min.: FAC, ACE, CEG → FAC#, AC#E, C#EG; a min.: CEG, EGB, GBD → CEG#, EG#B, G#BD; e min.: GBD, BDF#, DF#A → GBD#, BD#F#, D#F#A.

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➔ The modulation from the key of C maj. to the key of G maj. is described in more detail in the following – using a pivot.

F maj.: FAC (FI)

C maj.: FAC (CII)

G maj.:

D maj.:

PIVOTS	
CEG (CI)	GBD (CV)
CEG (GIV)	GBD (GI)

DF#A (GV)

DF#A (DI)

First, one can consider a full, authentic cadence for the C maj. key, e.g., CI – CIV – CV – CI = CEG – FAC – GBD – CEG.

Second, one can consider a full, authentic cadence for the G maj. key, e.g., GI – GIV – GV – GI = GBD – CEG – DF#A – GBD.

Thus, when one views these two cadences in a "combined" form, one moves effectively from CI to GI – where the triads CEG (CI, GIV) and GBD (CV, GI) serve as pivot triads to "blend" C maj. and G maj. smoothly. The triad GV = DF#A introduces the needed accidental note F#.

Further, one must also keep in mind that the VOICING of the triads used in the "modulation cadences" should move notes minimally when moving from triad to triad. Therefore, one must use root position and 1st and 2nd inversions of all triads. Superscripts are used to indicate these 3 positions, e.g., $CI^0 = CEG$, $CI^1 = EGC$, $CI^2 = GCE$.

For example, the considered cadences can be voiced more smoothly by using the following triad sequences:

C maj. cadence: $CI^1 - CIV^0 - CV^0 - CI^2 = EGC - FAC - GBD - GCE$;

G maj. cadence: $GI^0 - GIV^2 - GV^2 - GI^1 = GBD - GCE - ADF^{\#} - BDG$.

When viewing the combined sequence of these 8 triads, one sees that notes move up/down by 0, 1 or 2 semi-tones only.

Instead of this pair of full, authentic cadences, one can also consider the use of the shorter authentic cadences:

C maj. cadence: $CI^1 - CV^0 - CI^2 = EGC - GBD - GCE$;

G maj. cadence: $GI^0 - GV^2 - GI^1 = GBD - ADF^{\#} - BDG$.

This triad progression can be improved, as far as voicing is concerned, by using the following triads:

C maj. cadence: $CI^1 - CV^2 - CI^1 = EGC - DGB - EGC$;

G maj. cadence: $GI^2 - GV^0 - GI^2 = DGB - DF^{\#}A - DGB$.

This second progression of 6 triads ensures that triad notes move up/down by only 0, 1 or 2 semi-tones.

→ One can repeatedly use a chosen, smooth modulation, to move from a key to another key to another key etc. We use the "smooth-voicing" cadence for I-IV-V-I as an example:

C maj. cad.: $CI^1 - CIV^0 - CV^0 - CI^2 = EGC - FAC - GBD - GCE$;

G maj. cad.: $GI^0 - GIV^2 - GV^2 - GI^1 = GBD - GCE - ADF^{\#} - BDG$;

G maj. cad.: $GI^1 - GIV^0 - GV^0 - GI^2 = BDG - CEG - DF^{\#}A - DGB$;

D maj. cad.: $DI^0 - DIV^2 - DV^2 - DI^1 = DF^{\#}A - DGB - EAC^{\#} - F^{\#}AD$;

D maj. cad.: $DI^1 - DIV^0 - DV^0 - DI^2 = F^{\#}AD - GBD - AC^{\#}E - ADF^{\#}$;

A maj. cad.: $AI^0 - AIV^2 - AV^2 - AI^1 = AC^{\#}E - ADF^{\#} - BEG^{\#} - C^{\#}EA$.

Only three 6-triad pro-gression

This example represents a smooth overall modulation from C maj. to A maj. keys/scales. Effectively, one moves from the triad EGC to triad $C^{\#}EA$ – via the sequence $EGC - BDG - F^{\#}AD - C^{\#}EA$. The introduced accidentals $F^{\#}$, $C^{\#}$ and $G^{\#}$ are "double-underlined" in the triads that introduce them in the modulation. The entire sequence of triads (24!) included in the "modulation sequences" provided follows the principle of smoothest-possible voicing. Ultimately, one can modulate through the entire circle of fifths.

→ The full, authentic cadences one can use for a smooth modulation can be "reduced" to these essential triads:

C maj. cad.: $CI^1 - CIV^6 - CV^6 - CI^2$

G maj. cad.: $GI^6 - GIV^2 - GV^2 - GI^1$

≙ EGC - FAC - GBD - GCE - ADF# - BDG

This example for the modulation Cmaj. to Gmaj. can be repeated:

	Modulation	Triad 1	2	3	4	5	Triad 6
1	C→G	EGC	FAC	GBD	GCE	ADF#	BDG
2	G→D	BDG	CEG	D F# A	DGB	EAC#	F# A D
3	D→A	F# A D	GBD	AC# E	ADF#	BEG#	C# E A
4	A→E	C# E A	D F# A	EG# B	EAC#	F# B D#	G# B E
5	E→B	G# B E	AC# E	B D# F#	BEG#	C# F# A#	D# F# B
6	B→F#	D# F# B	EG# B	F# A# C#	F# B D#	G# C# E#	A# C# F#
7	F#→C#	A# C# F#	B D# F#	C# E# G#	C# F# A#	D# G# B#	E# G# C#
8	C#→G#	E# G# C#	F# A# C#	G# B# D#	G# C# E#	A# D# F##	B# D# G#
9	G#→D#	B# D# G#	C# E# G#	D# F## A#	D# G# B#	E# A# C##	F## A# D#
10	D#→A#	F## A# D#	G# B# D#	A# C## E#	A# D# F##	B# E# G##	C## E# A#
11	A#→E#	C## E# A#	D# F## A#	E# G## B#	E# A# C##	F## B# D##	G## B# E#
12	E#→B#	G## B# E#	A# C## E#	B# D## F##	B# E# G##	C## F## A##	D## F## B# = EGC
7	Gb→Db	Bb Db Gb	Cb Eb Gb	Db F Ab	Db Gb Bb	Eb Ab C	F Ab Db
8	Db→Ab	F Ab Db	Gb Bb Db	Ab C Eb	Ab Db F	Bb Eb G	C Eb Ab
9	Ab→Eb	C Eb Ab	Db F Ab	Eb G Bb	Eb Ab C	F Bb D	G Bb Eb
10	Eb→Bb	G Bb F	Ab C Eb	Bb D F	Bb Eb G	C F A	D F Bb
11	Bb→F	D F Bb	Eb G Bb	F A C	F Bb D	G C E	A C F
12	F→C	A C F	Bb D F	C E G	C F A	D G B	E G C

In each row, the "start" key is shown in the column "Triad 1"; the "goal" key is shown in the column "Triad 6." The last, 3rd note of "Triad 1" and "Triad 6" define the "start" and "goal" keys. Performing all modulations, from 1 to 12, represents a "modulation circle" that moves through all major keys in the sequence of the circle of fifths. ONE CAN MOVE FORWARD OR BACKWARD

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• Modulation

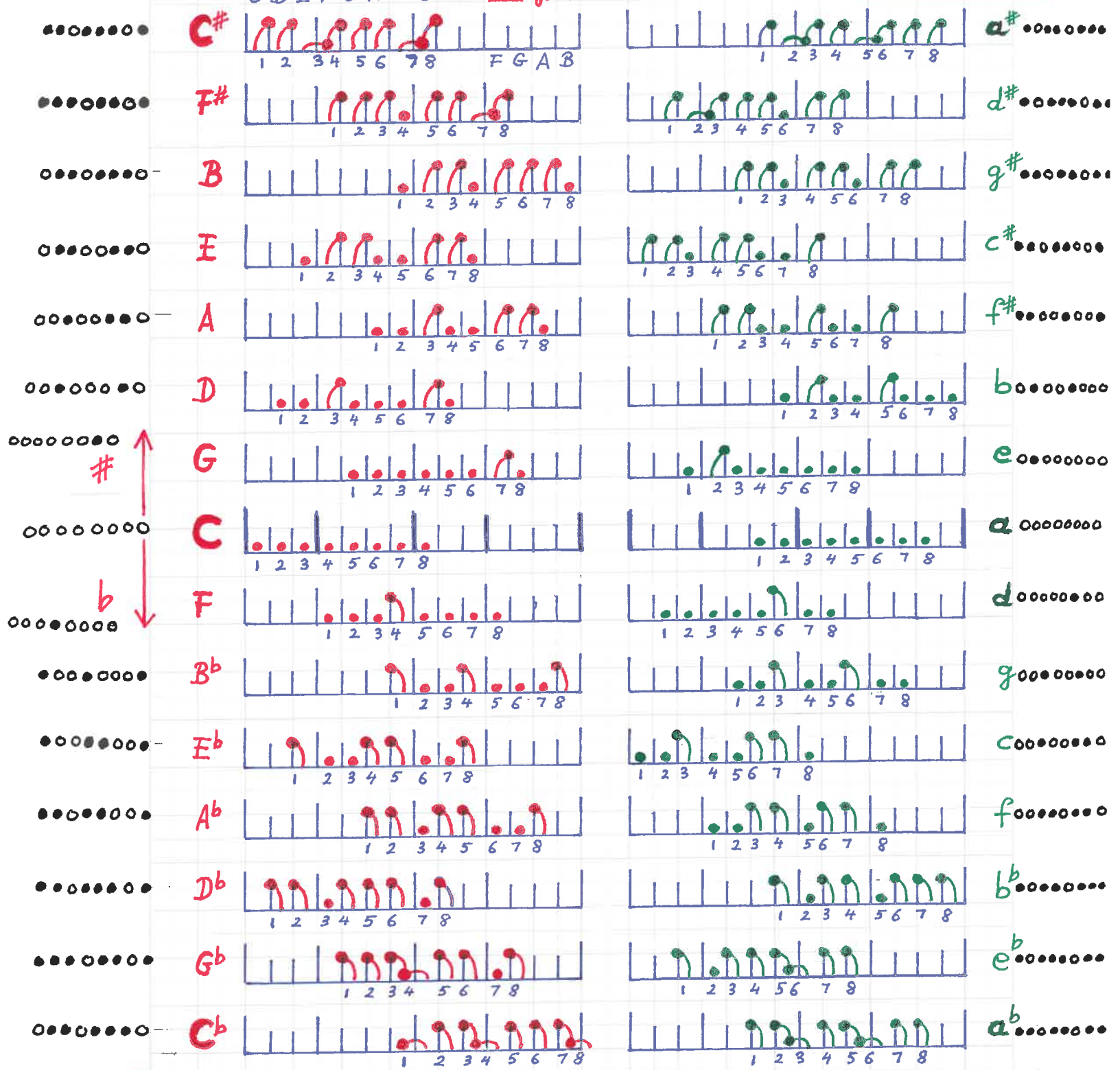
2/13/2025

→ In order to perform modulations on the keyboard with all 24 major and minor keys, "effective, efficient memorization" is needed.

Key color sequence:

C D E F G A B C D E MAJOR

MINOR (relative)



1) Memorize #'s and b's of all keys.

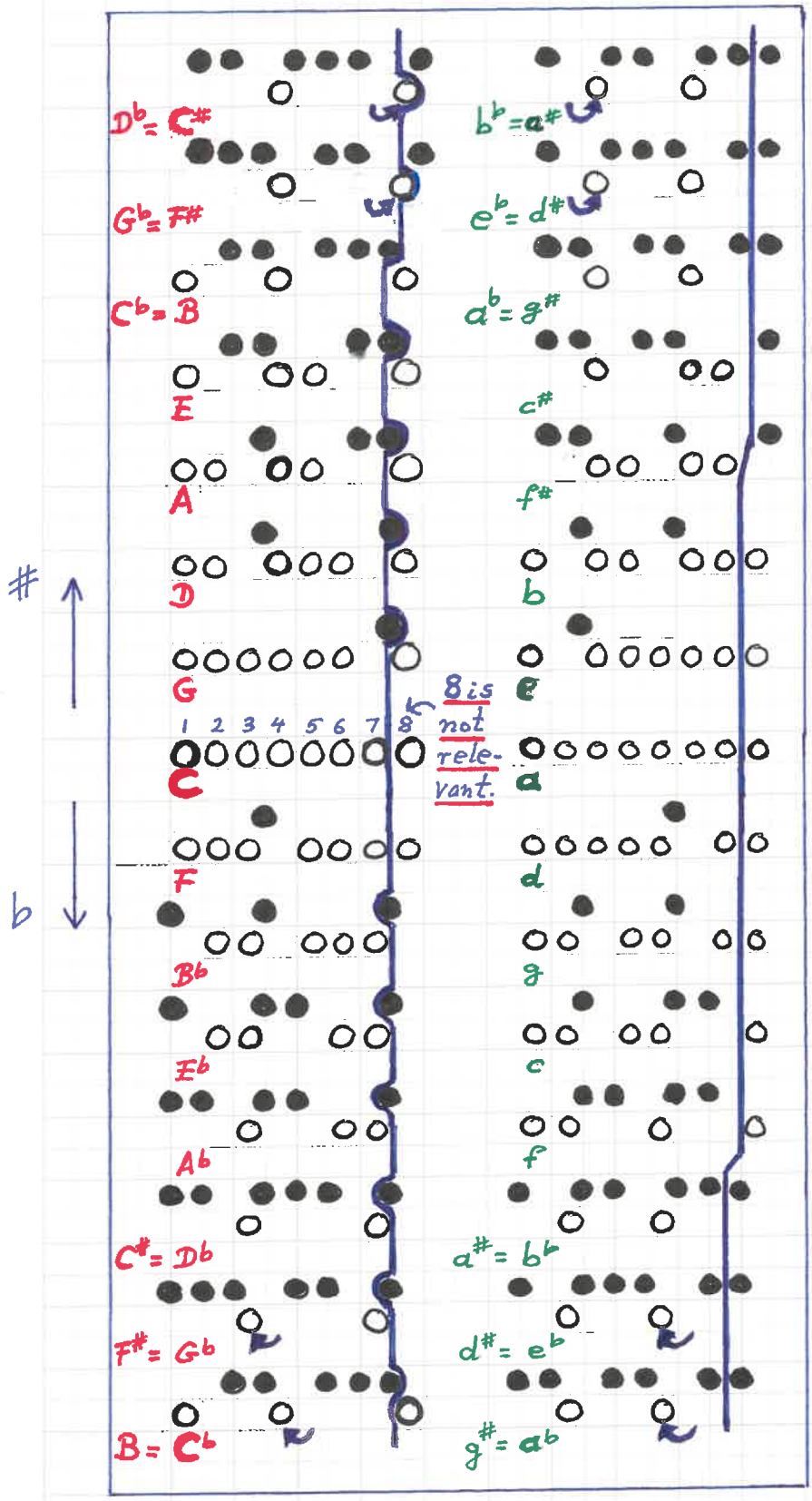
2) Memorize locations of all #'s, b's.

3) Number octave notes 1, 2, 3, ..., 8.

4) #'s: move "up-right"; b's: move "up-left."

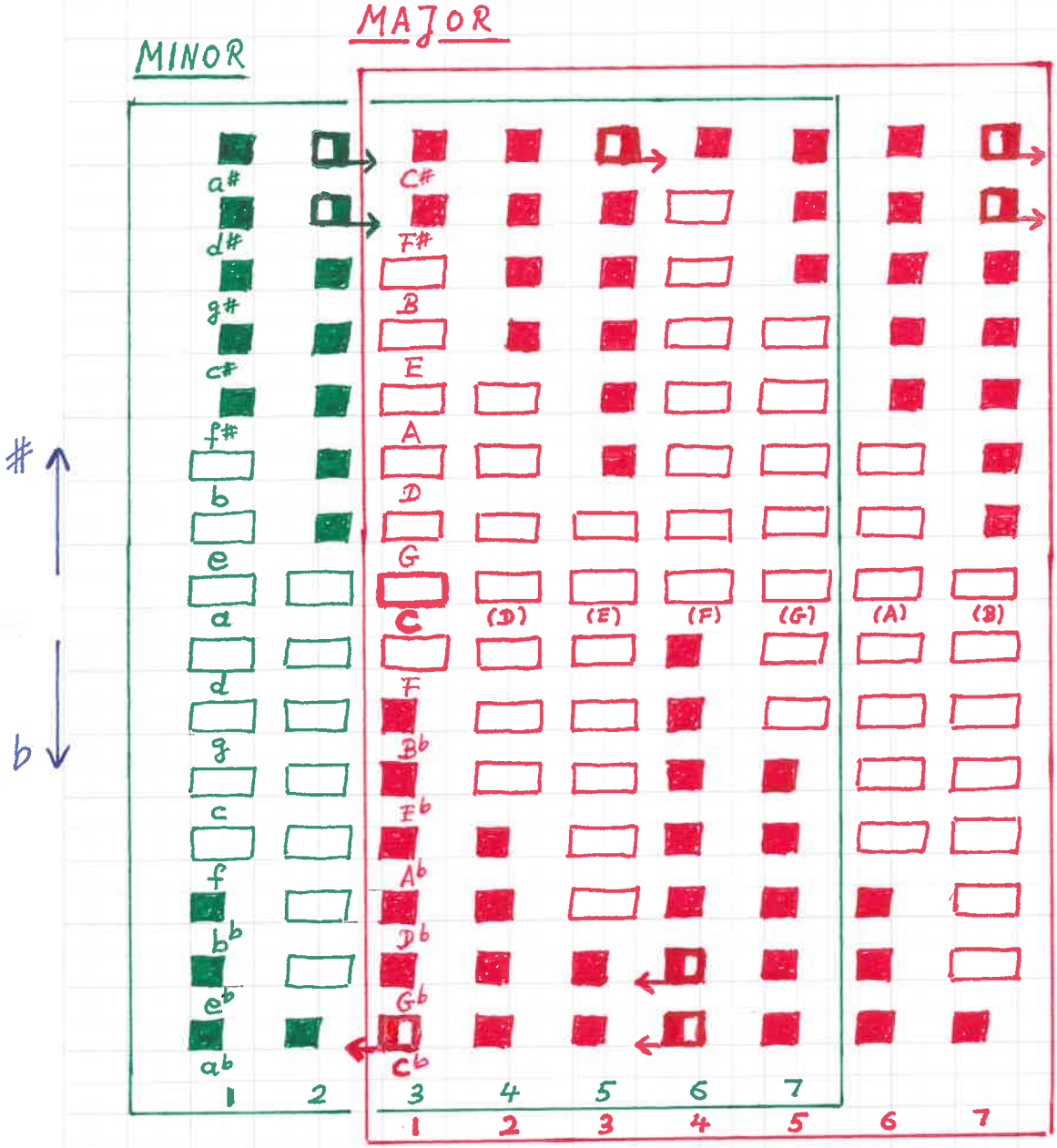
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→ One can also memorize the "shapes and patterns" of the keys (on the keyboard) of the 24 scales by "extracting" the 8 white and black keys that constitute one octave of a specific key/scale. The left table shows the "shapes and patterns" of all 12 major keys (left column) and all 12 relative minor keys. The order of the keys is using the circle-of-fifths order, to emphasize and "see" the gradual changes of key "patterns and shapes" when '#s' and 'bs' are added in a one-by-one fashion. Therefore, one can also focus on and memorize how a "pattern and shape" changes when "the next '#' ('b')" is introduced. The 8th key/note is not "pattern-relevant."



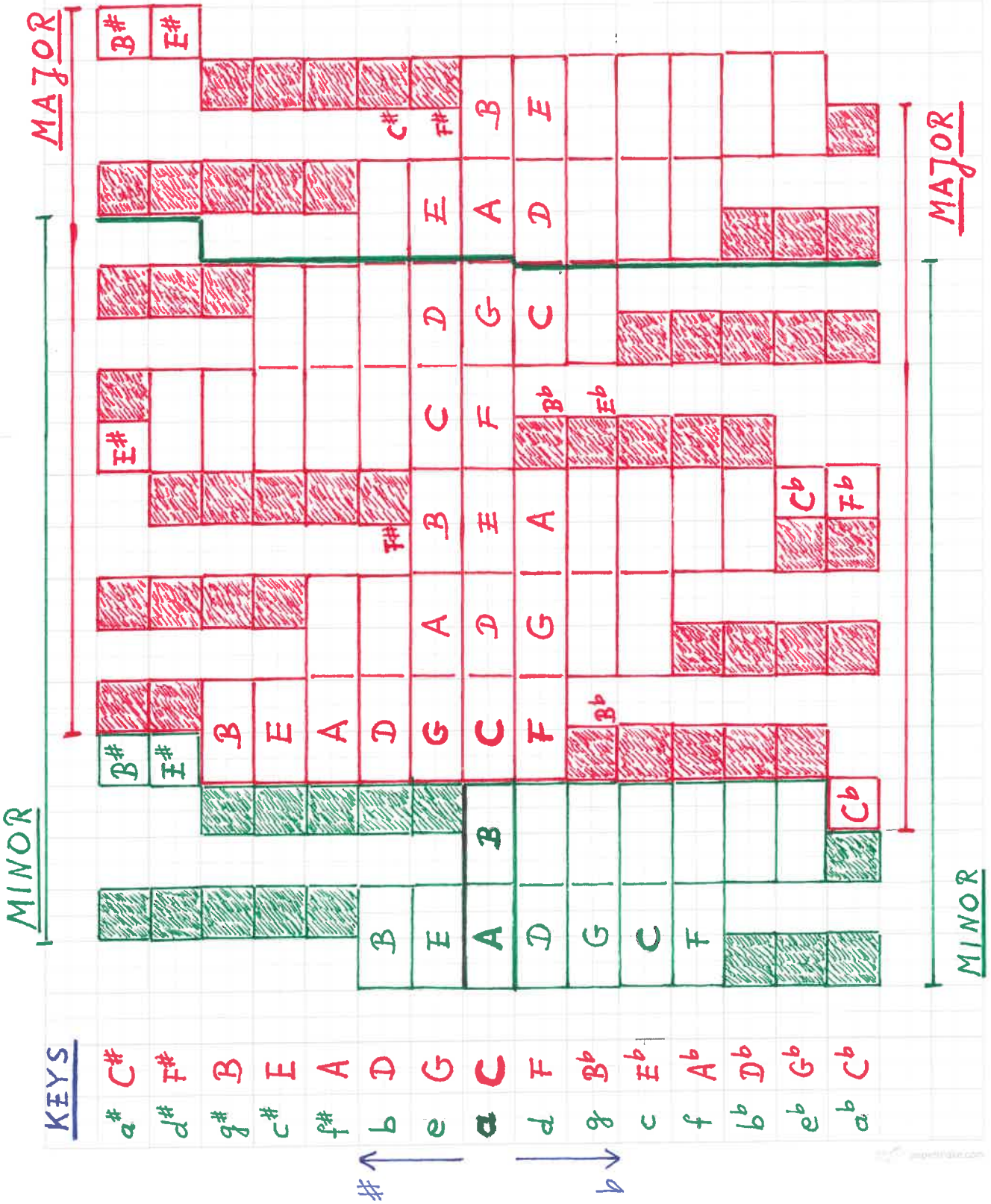
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→ The figure below is a compact representation of all 12 major and 12 relative minor keys/scales – i.e., a representation of the keys on the keyboard to be played.



The figure shows that B, F#, C# (g#, d#, a#) and Cb, Gb, Db (ab, eb, bb), respectively, are indeed "not equal" – concerning their harmonic functions.

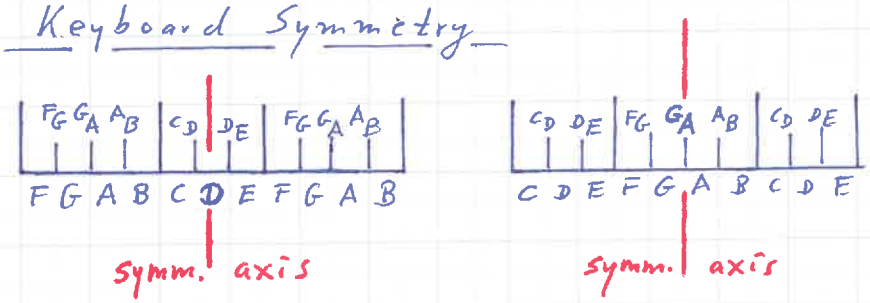
The visualization on this page shows another possibility to represent the notes and corresponding keys on the keyboard of all 12 major and associated 12 relative minor keys/scales. Again major and minor scales are overlaid.



• Mathematical Transformations & Effects

2/17/2025

1) Keyboard Symmetry



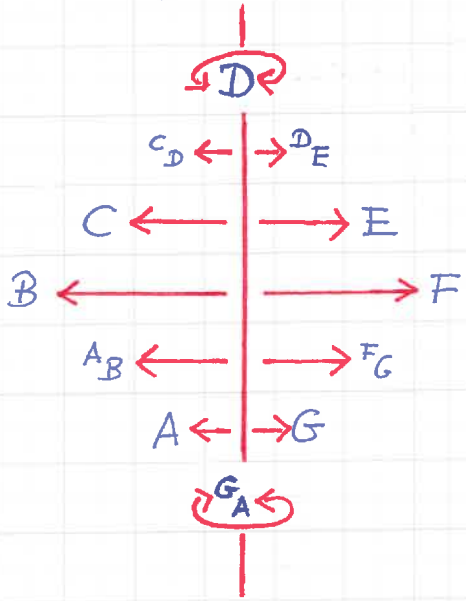
• Symmetry axis at notes D and G♯

• The two symmetry axes produce the same note mappings:

C	C♯	D	D♯	E	F	F♯	G	G♯	A	A♯	B
E	D♯	D	C♯	C	B	A♯	A	G♯	G	F♯	F

mirror images / reflections

• The "cyclic keyboard" makes it possible to use a circle:



• The left representation of "axis symmetry" shows the note pairs of notes that are "mirror images" of each other.

• It is interesting to understand how this mapping affect chords. For example, one can mirror the key-inherent triads of C major:

$C^1 E^2 G^3 \mapsto A'^3 C'^2 E'^1$ — I ↦ vi

$D^1 F^2 A^3 \mapsto G'^3 B'^2 D'^1$ — ii ↦ V

$E^1 G^2 B^3 \mapsto F'^3 A'^2 C'^1$ — iii ↦ IV

$F^1 A^2 C^3 \mapsto E'^3 G'^2 B'^1$ — IV ↦ iii

$G^1 B^2 D^3 \mapsto D'^3 F'^2 A'^1$ — V ↦ ii

$A^1 C^2 E^3 \mapsto C'^3 E'^2 G'^1$ — vi ↦ I

$B^1 D^2 F^3 \mapsto B'^3 D'^2 F'^1$ — $vii^o \mapsto vii^o$

C maj. triads mapped to other C maj. triads !

• A chord $N_1 N_2 \ldots N_n$ is "mirrored" to the chord $N_n' N_{n-1}' \ldots N_1'$ – where N_i' is the mirror image of note N_i. Example:

$CEG = N_1 N_2 N_3 \longmapsto$

$N_3' N_2' N_1' = G' E' C' = ACE$.

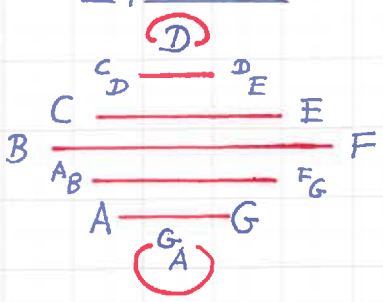
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1) TRANSLATION

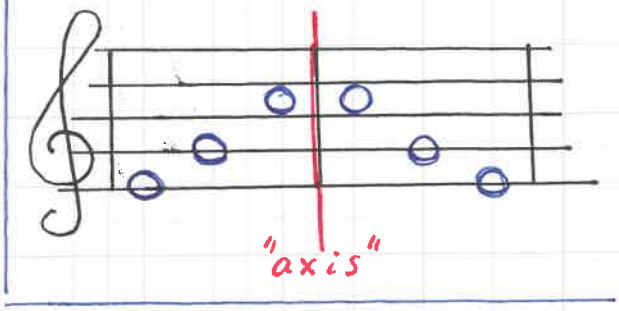
- "spatial": transposing a note/chord/key "up or down" – e.g., modulation to another key
- "temporal": voices (soprano, alto, tenor, bass) are "offset"/"delayed" relative to each other (canon, fugue)

2) REFLECTION

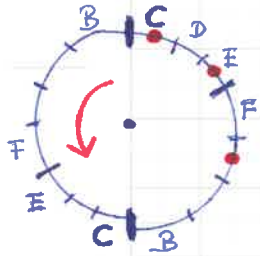
- "spatial"



- "temporal"

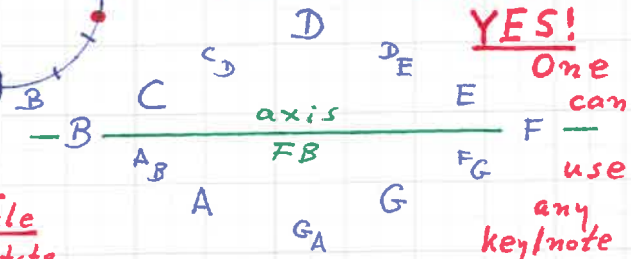


ROTATION:



View the keyboard as a circle ⇒ can "rotate notes/chords"

- Besides the axis DG_A, are other axes possible? ⟺ For example, if one could view the line FB (left figure) as an "allowable axis", one could concatenate two reflections:



YES! One can use any key/note as reflection axis.

i) $C^1 E^2 G^3 \xrightarrow[D G_A]{\text{axis}} A^{3'} C^{2'} E^{1'} \xrightarrow[FB]{\text{axis}} F_G\, A_B\, C_D$?

⇒ ii) $Ref_{FB}(Ref_{DG_A}(CEG)) = F_G A_B C_D$.

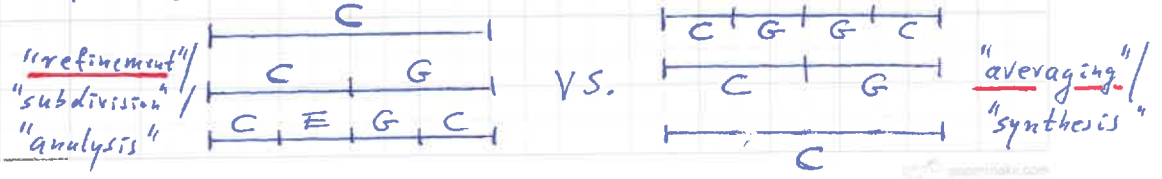
⇒ Possibility to define "ROTATION" VIA TWO REFLECTIONS?

3) SCALING – w.r.t. a scaling CENTER

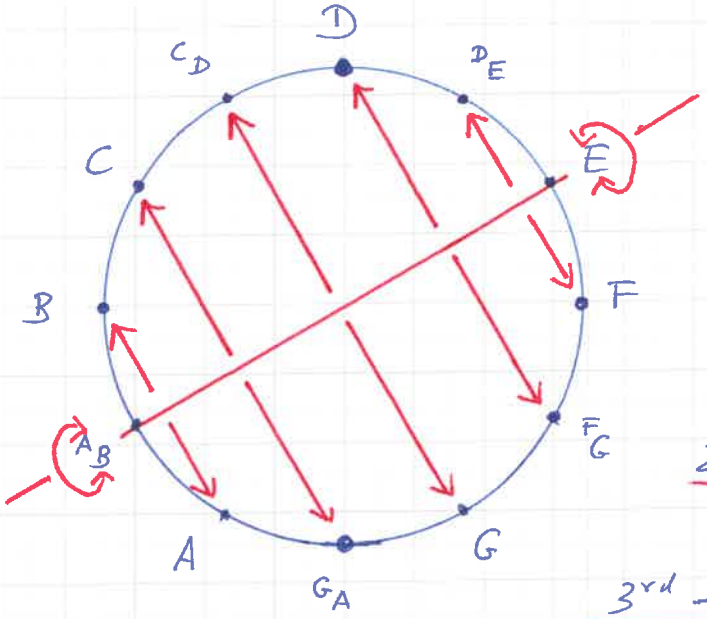
- "spatial": e.g., scale intervals relative to a "scaling center":

(center = C:) $C \overset{4}{\underset{4}{\rightarrow}} E \overset{7}{\underset{3}{\rightarrow}} G$ $\xrightarrow{\text{scale by 2}}$ $C \overset{8}{\underset{8}{\rightarrow}} G_A \overset{14}{\underset{6}{\rightarrow}} D$ ⇐ Sounds "good"?

- "temporal": similar to "multi-resolution" methods – longer & shorter notes:



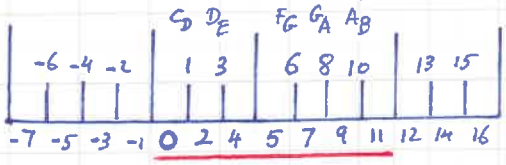
4) REFLECTION w.r.t. any reflection axis



- A general reflection can be viewed as a "geometrical concatenation":
  1st - "translate" to a default center of reflection / standard axis of reflection;
  2nd - "reflect" relative to this default standard axis;
  3rd - "translate" back.

⇒ Example: [see figure above]: reflect relative to axis E^{A_B}
⇒ note C is reflected to $^{G}_{A}$.

- Performing this general reflection with note "indices":



⇐ This figure shows a possible indexing / numbering of keys / notes. C ≙ 0 is used as default center / axis of reflection.

⇒ Example: "Reflect F (≙ 5) w.r.t. axis F_G (≙ 6)
1st - "translate by -6" ⇒ F ↦ B (≙ -1)
2nd - "reflect w.r.t. axis C (≙ 0)": B ↦ C_D (≙ 1)
3rd - "translate by +6" ⇒ C_D ↦ G (≙ 7)
⇒ F ⟼ G ✓

- Considering the above figure again, we can reflect triads that are key-inherent to C major relative to the E^{A_B} axis:

C_{maj} { I $C^1 E^2 G^3 \mapsto {}^{3'}C_D\ E\ {}^{2'}G_A = E$vi
ii $D F A \mapsto B\ D_E\ F_G = E$V
iii $E G B \mapsto A\ C_D\ E = E$IV
IV $F A C \mapsto G_A\ B\ D_E = E$iii

C_{maj} { V $G B D \mapsto F_G\ A\ C_D = E$ii
vi $A C E \mapsto E\ G_A\ B = E$I
vii° $B D F \mapsto D_E\ F_G\ A = E$vii°

⇒ C maj. triads to E maj. triads!

- Modulation

3/1/2025

Similarity of Seventh Chords and Keys/Scales Defined by the Numbers of Chord Notes Belonging to Specific Keys/Scales

7th Chord	C-E-G-B	C-E-G-Bb	C-Eb-G-Bb	C-Eb-Gb-Bb	C-Eb-Gb-A
F# [d#]	*	*	* *	***	* *
B [g#]	* *	* *	* *	***	**
E [c#]	* *	*	*	**	**
A [f#]	* *	*		*	**
D [b]	***	**	*	*	**
G [e]	****	***	* *	* *	* **
C [a]	****	***	* *	*	* *
F [d]	***	****	* **	* *	* *
Bb [g]	* *	* **	****	** *	** *
Eb [c]	* *	* **	****	** *	**
Ab [f]	* *	* **	****	** *	**
Db [bb]	*	* *	** *	****	***

Note. The numbers of chord notes belonging to a specific key/scale are equal to the numbers of '*' (0, 1, 2, 4). For example, all 4 notes of 7th chord C-E-G-Bb belong only to F maj. (d min.). Thus, this C7 chord can be used as a "pivot chord" to modulate from C maj. to F maj.: C-E-G → C-E-G-Bb → C-F-A.