

Visualizing local vector field topology

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Abstract. *The visualization of vector fields has attracted much attention over the last decade due to the vast variety of applications in science and engineering. Topological methods have been used intensively for global structure extraction and analysis. Recently, there has been a growing interest in local structure analysis due to its connection to automatic feature extraction and speed. We present an algorithm that extracts local topological structure of arbitrary regions in a two-dimensional vector field. It is based on a mathematical analysis of the topological vector field structure in these regions. The algorithm deals with piecewise linear vector fields and arbitrary polygonal regions. We have tested the algorithm for well known analytic vector fields and data sets resulting from computational fluid dynamics. © 2000 SPIE and IS&T. [S1017-9909(00)01104-1]*

1 Introduction

Fluid mechanics is a major application for vector field visualization. A velocity field contains the answers to many of the important questions of physicists and engineers and, due to rotation, the velocity can usually not be described by a gradient field, so an analysis of a single scalar field does not capture the whole structure. Since fluid mechanics is an essential part of the aerospace and automotive industries, there is a strong need for better analysis and visualization methods. Topology has been used in fluid mechanics for several years to interpret experiments and deduct theoretical results.^{1–3} These ideas provided the foundation for the use of vector field topology for the analysis and visualization by Helman and Hesselink⁴ as well as Globus *et al.*⁵

Mathematically, vector fields are geometric representations of differential equations, and the number of experimental and numerical data sets defined by the discretized vector field is growing rapidly. The analysis and visualiza-

tion of the resulting data sets still pose challenges to the visualization community. One standard method is based on topological analysis of vector field data.^{4–6} These methods require an analysis of the whole vector field to provide answers on the structure, and certain methods may also miss certain features.⁷ For this reason, several local analysis algorithms have been developed that are based on topology or related concepts like derivative and eigenvector analysis.^{8,9}

In this paper, we localize the concept of topology analysis by concentrating on an arbitrary region inside a two-dimensional (2D) vector field that we analyze without using information outside the region. It turns out that a substantial extension of the standard algorithm for topology analysis is necessary to accomplish correct local analysis. Besides the critical points, one has to analyze the boundary of a local region based on *inflow* or *outflow* conditions. This analysis allows us to determine additional separatrices that make, in a topological sense, a separation of the local region into areas of topologically uniform flow possible.

The mathematical background is developed in Secs. 2 and 3. Some special cases concerning piecewise linear vector fields are discussed in Sec. 4. In Sec. 5, we prove the correctness of the algorithm for several analytical examples. We especially highlight the effect of including the boundary of a polygonal region into the analysis of a field. Section 6 shows results for two computational fluid dynamics data sets. In Sec. 7, we provide conclusions and allude to further research.

2 Vector Field Topology

The study of topology of vector fields is based on several basic theorems from the theory of ordinary differential equations.^{10–15} We survey the important terms and results for planar, steady vector fields.

Definition 2.1: A planar vector field is a map

$$v: \mathbb{R}^2 \rightarrow \mathbb{R}^2,$$

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$$x \mapsto v(x). \tag{1}$$

Usually, one is not so much interested in the vector field per se but in its integral curves.

Definition 2.2: An integral curve through a point $x \in \mathbb{R}^2$ of a vector field $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a map

$$\alpha_x: \mathbb{R} \supset I \rightarrow \mathbb{R}^2, \tag{2}$$

where

$$\alpha_x(0) = x_0, \tag{3}$$

$$\dot{\alpha}_x(t) = v(\alpha(t)), \quad \forall t \in I. \tag{4}$$

Concerning the theorem on existence and uniqueness of integral curves, the Lipschitz condition has to be satisfied:

Definition 2.3: Let $U \subset \mathbb{R}^2$ be an open subset. A continuous vector field $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ satisfies a Lipschitz condition on U provided there exists a real number $K > 0$ such that

$$\|Dv(x) - Dv(y)\| \leq K \|x - y\| \tag{5}$$

holds for all $x, y \in U$ and all $t \in J$, where Dv denotes the differential of the vector field v . The constant K is called the Lipschitz constant. One can now formulate the existence and uniqueness theorem.

Theorem 2.4: (Existence and uniqueness of integral curves). Let $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a vector field satisfying the Lipschitz condition on any open neighborhood around point $x \in \mathbb{R}^2$. Then there exists one and only one integral curve through any $x_0 \in \mathbb{R}^2$.

Proof: See Ref. 13 pp. 66–68.

If a vector field satisfies the Lipschitz condition for an open neighborhood of every point, then the integral curves are defined over the whole time line.

Lemma 2.5: If $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is Lipschitz-continuous around each point $a \in \mathbb{R}^2$, then every integral curve is defined over the whole time line \mathbb{R} .

Proof: See Ref. 13 p. 90. This leads to an analysis of the asymptotic behavior of integral curves. The following terms are used to study asymptotic behavior:

Definition 2.6: Let $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field and $\alpha: \mathbb{R} \rightarrow \mathbb{R}^2$ an integral curve. The set

$$\{a \in \mathbb{R}^2 \mid \exists (t_n)_{n=0}^\infty \subset \mathbb{R}, t_n \rightarrow \infty, \lim_{n \rightarrow \infty} \alpha(t_n) \rightarrow a\} \tag{6}$$

is called the ω -limit set of α . The set

$$\{a \in \mathbb{R}^2 \mid \exists (t_n)_{n=0}^\infty \subset \mathbb{R}, t_n \rightarrow -\infty, \lim_{n \rightarrow \infty} \alpha(t_n) \rightarrow a\} \tag{7}$$

is called the α -limit set of α . Standard algorithms limit topological analysis to α - and ω -limit sets consisting of critical points.

Definition 2.7: A critical point of $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a point $p \in \mathbb{R}^2$ with $v(p) = 0$. Two cases are of interest: (1) sinks and (2) sources.

Definition 2.8: Let $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field and $z \in \mathbb{R}^2$ a critical point. If there is a neigh-

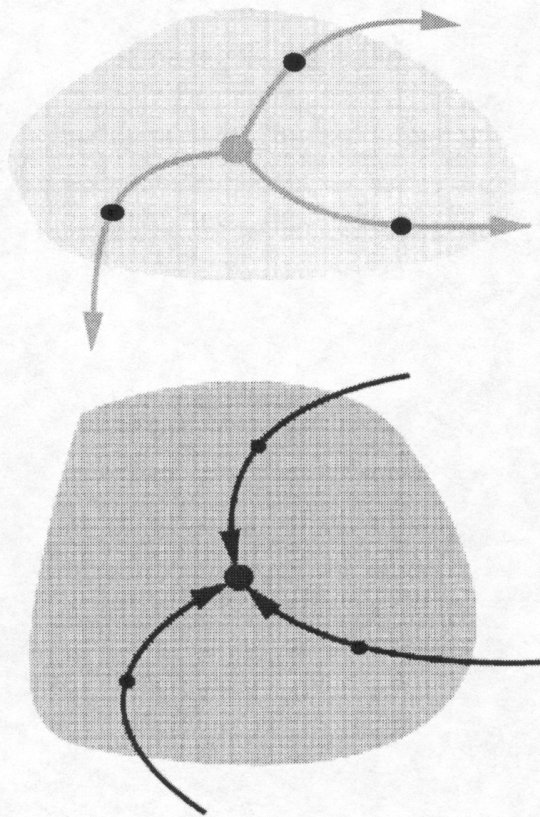


Fig. 1 Around a source, every integral curve starts at the source, around a sink, every integral curve ends at the sink.

borhood $U \subset \mathbb{R}^2$ of z such that all integral curves in U have an α -limit set consisting only of z , then z is called a source. If there is a neighborhood $U \in \mathbb{R}^2$ of z such that all integral curves in U have an ω -limit set consisting only of z , then z is called a sink.

Figure 1 shows a source in the upper part. All integral curves around the critical point start at this critical point. In the lower part, a sink is shown. Every integral curve in the neighborhood ends at the sink. We restrict our consideration further to simple critical points. For the analysis of a vector field, one has to determine the asymptotic behavior of all its integral curves. This is done by analyzing the union of all integral curves starting or ending at the same critical point, called the basin.

Definition 2.9: Let $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field and $a \in \mathbb{R}^2$ a critical point. The union of all integral curves of v that converge to a for $t \rightarrow -\infty$ is called the α -basin of a , denoted by $B_\alpha(a)$. The union of all integral curves of v that converge to a for $t \rightarrow \infty$ is called the ω -basin of a , denoted by $B_\omega(a)$. For sinks and sources, one can prove the following lemma:

Lemma 2.10: Let v and a be as defined in Definition 2.9. If a is a source then $B_\alpha(a)$ is an open subset of \mathbb{R}^2 . If a is a sink then $B_\omega(a)$ is an open subset of \mathbb{R}^2 .

Proof: See Ref. 12 pp. 181–182.

Figure 2 shows a typical α -basin of a source.

Definition 2.11: If a subset $S \subset \mathbb{R}^2$ allows a description as a pure n -dimensional manifold, we define the dimension of S as n .

Remark 2.12: We need the term dimension only for ba-

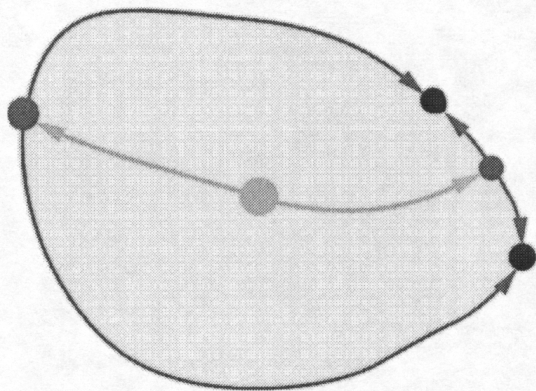


Fig. 2 A typical α -basin of a source. The light gray basin consists of all integral curves starting at the source.

sins. The previous definition means that an open basin in \mathbb{R}^2 is a 2-manifold, a basin consisting of a finite number of stream lines is a 1-manifold, and a basin consisting of a finite number of points is a 0-manifold.

If all limit sets are critical points and one regards infinity as an additional critical point, then the whole plane is separated into basins of sources, sinks and infinity. The boundaries of open basins cannot belong to the basins themselves. Since there are no other possibilities, the boundaries start at the remaining critical points or at infinity. With the assumption that all critical points are simple, these remaining critical points are saddle points.

Theorem 2.13: Let $v: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field. Let all α - and ω -limit sets be simple critical points and ∞ . Let a_1, \dots, a_j be the sources, s_1, \dots, s_k be the saddles, and z_1, \dots, z_m be the sinks. Then \mathbb{R}^2 can be divided into disjoint α -basins

$$\mathbb{R}^2 = \bigcup_{i=1}^j B_{\alpha}(a_i) \cup \bigcup_{i=1}^k B_{\alpha}(s_i) \cup \bigcup_{i=1}^l \{z_i\} \cup B_{\alpha}(\infty). \quad (8)$$

It can also be divided into disjoint ω basins

$$\mathbb{R}^2 = \bigcup_{i=1}^l B_{\omega}(z_i) \cup \bigcup_{i=1}^k B_{\omega}(s_i) \cup \bigcup_{i=1}^j \{a_i\} \cup B_{\omega}(\infty). \quad (9)$$

Proof: Every point belongs to exactly one stream line. By this assumption, the stream line has to start at one of the critical points or at the boundary, so it is in one of the basins. This gives the first partition. The second partition is given by a look at the end of a stream line. \diamond

The topological structure is given by all intersections between α - and ω -basins. The description of these intersections uses the dimension of the different basins. The α -basins of the sources and the ω -basins of the sinks alone can contain open subsets besides the basins of infinity. It follows from Lemma 2.10 that their intersections form 2D (open) "faces" of the structure. The boundaries do not belong to these basins and consist of curves and points. A curve can only be part of a basin of a saddle. The method by Helman and Hesselink⁴ uses this fact to determine the structure by finding all critical points and then drawing all basins of the saddles. Apart from the critical points, the resulting plane consists of faces of the resulting graph,

sometimes called "topological skeleton." These faces are the intersections between the α -basins of the sources and the ω -basins of the sinks. It can be shown that they can be deformed into a topologically uniform flow. This provides the mathematical explanation for the description of the whole process as separating the vector field into *domains of topologically uniform flow*.

3 Local Vector Field Topology

In most practical applications, the domain of a vector field is finite. It is usually either the union of all the cells of a finite-element or finite-difference grid or the convex hull of scattered data points without connectivity. Topologically, this means that a vector field is defined over a closed and bounded, i.e., compact, subset of the real plane. The analysis of a vector field should reflect this property. However, current visualization algorithms used to date do not do this. The main reason for this is the fact that standard algorithms operate on the whole domain of a vector field, and the flow is typically very simple on the boundaries. The boundary is either a solid boundary, where the flow is set to tangential flow (often zero), or it is an open boundary, where the flow has a simple structure. In a wind tunnel experiment, for example, there is one inflow region and one outflow region. We, by contrast, want to analyze an arbitrary compact region inside the domain of a vector field. We will show that one has to analyze the boundary completely to obtain a correct analysis. For this purpose, we generalize some of the terms of the last section. This explains also some possible structures at infinity, since infinity plays the role of the boundary.

The first change concerns Lemma 2.5. Since we must consider a boundary now, an integral curve in a Lipschitz-continuous, steady vector field might start or end on the boundary. One might imagine a vector field over the whole plane with a compact region cut out of it resulting in the given vector field. The integral curves are also cut; they still can be continued inside the local region, but they may end or start on the boundary.

Lemma 3.1. Let $D \subset \mathbb{R}^2$ be a compact subset and $v: D \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field. Then, for an arbitrary integral curve $\alpha: (a, b) \rightarrow D$ of maximal length, a will be finite only if $\alpha(a) \in \partial D$, where ∂D denotes the boundary of D . Analogously, b will be finite only if $\alpha(b) \in \partial D$.

Since we want to establish a theorem similar to Theorem 2.13, we distinguish three subsets of the boundary with respect to the vector field. These three subsets are the sets of inflow, outflow, and boundary flow points. The connected components of the inflow and outflow regions play the role of additional sources and sinks in the topological analysis. The end points lead to the additional separatrices that would be missed when using the standard approach.

Definition 3.2: Let $D \subset \mathbb{R}^2$ be a compact domain of a Lipschitz-continuous vector field $v: D \rightarrow \mathbb{R}^2$. Let $d \in \partial D$ be a point on the boundary. We define the following three entities:

1. Point d is called an **outflow point** if every integral curve through d ends at d and there exists an inte-

gral curve through d that does not contain another point of ∂D . The set of all outflow points is called the **outflow set**.

2. Point d is called an **inflow point** if every integral curve through d starts at d and there exists an integral curve through d that does not contain another point of ∂D . The set of all inflow points is called the **inflow set**.
3. Point d is called a **boundary flow point** if there exists an integral curve $\alpha_d: (-\epsilon, \epsilon) \rightarrow D$, $\epsilon > 0$, through d lying completely inside ∂D . The set of all boundary flow points is called the **boundary flow set**.

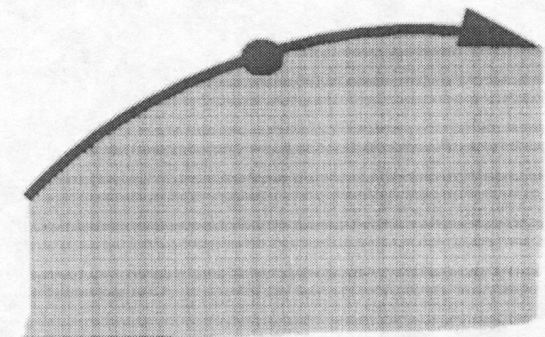
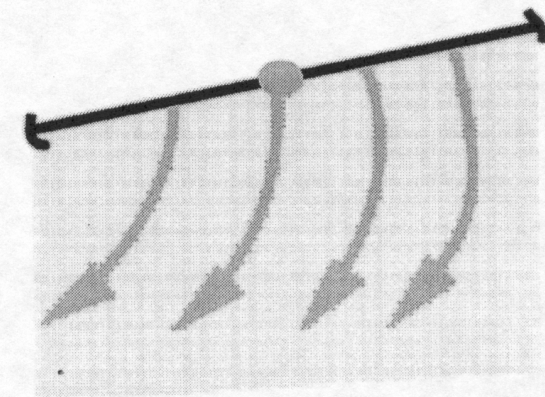
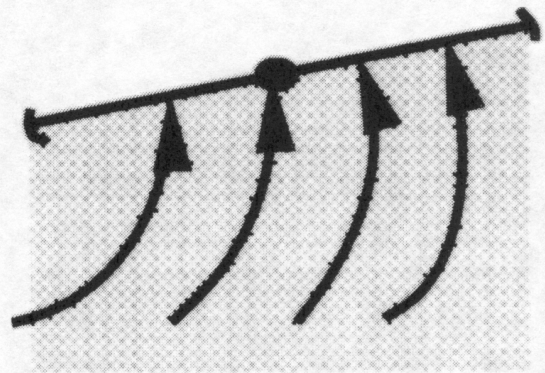


Fig. 3 Around an outflow point (dark gray), every integral curve enters the boundary from the inside of the domain. Around an inflow point (light gray), every integral curve enters the inside of the domain from the boundary. At a boundary flow point (medium gray), there exists an integral curve that stays on the boundary.

The topological situations around an outflow point, an inflow point and a boundary flow point are sketched in Fig. 3. In most practical applications, the vector field has a piecewise linear boundary. This case will be handled explicitly in Sec. 4. The inflow and outflow regions play the role of sinks and sources.

Lemma 3.3: Let I_i be a connected component of the inflow set I . The α -basin $B_\alpha(I_i)$ is the union of all integral curves starting inside I_i . This basin is open. Let O_i be a connected component of the inflow set $O \subset \partial D$. The ω -basin $B_\omega(O_i)$ is the union of all integral curves ending inside O_i . This basin is also open.

Proof: The sets are open due to the continuity of the solution of the initial value problem with respect to parameters like the starting point. If a point is in a small neighborhood of a curve in the basin, then its corresponding integral curve is close to this curve at another point. Since our basins are created by open subsets of the boundary, they are themselves open in D . \diamond

A boundary of an open basin cannot belong to the basin; thus, it must start at the boundary points between the inflow and outflow sets. We call these starting points **boundary saddles** to reflect the similarity to the saddle points inside the domain.

Definition 3.4: Let $D \subset \mathbb{R}^2$ be a compact subset with a smooth boundary. Let $v: D \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field. A point $d \in \partial D$ that is no inflow, outflow, or boundary flow point is called the **boundary saddle**.

It is now possible to describe D as a union of disjoint topologically uniform flow regions:

Theorem 3.5: Let $D \subset \mathbb{R}^2$ be a compact subset with a smooth boundary. Let $v: D \rightarrow \mathbb{R}^2$ be a Lipschitz-continuous vector field. Let a_1, \dots, a_j be the sources, s_1, \dots, s_k be the saddles, b_1, \dots, b_p be the boundary saddles, and z_1, \dots, z_l be the sinks. Furthermore, let I_1, \dots, I_m be the inflow components, and O_1, \dots, O_n be the outflow components. If we assume that there are no other α - and ω -limit sets, then D can be divided into α -basins

$$D = \bigcup_{i=1}^j B_\alpha(a_i) \cup \bigcup_{i=1}^m B_\alpha(I_i) \cup \bigcup_{i=1}^l B_\alpha(s_i) \cup \bigcup_{i=1}^p B_\alpha(b_i) \cup \bigcup_{i=1}^n O_i \cup \bigcup_{i=1}^k \{z_i\}. \quad (10)$$

Region D can also be divided into ω -basins

$$D = \bigcup_{i=1}^k B_\omega(z_i) \cup \bigcup_{i=1}^n B_\omega(O_i) \cup \bigcup_{i=1}^l B_\omega(s_i) \cup \bigcup_{i=1}^p B_\omega(b_i) \cup \bigcup_{i=1}^m I_i \cup \bigcup_{i=1}^j \{a_i\}. \quad (11)$$

Proof: Every point belongs to exactly one stream line. This stream line has to start at a critical point, i.e., a source, sink or saddle, or at the boundary, i.e., an inflow region, an outflow region or a boundary saddle, since we excluded other α -limit sets. This shows the first partition. The same argument for the ending of stream lines shows the second partition. \diamond

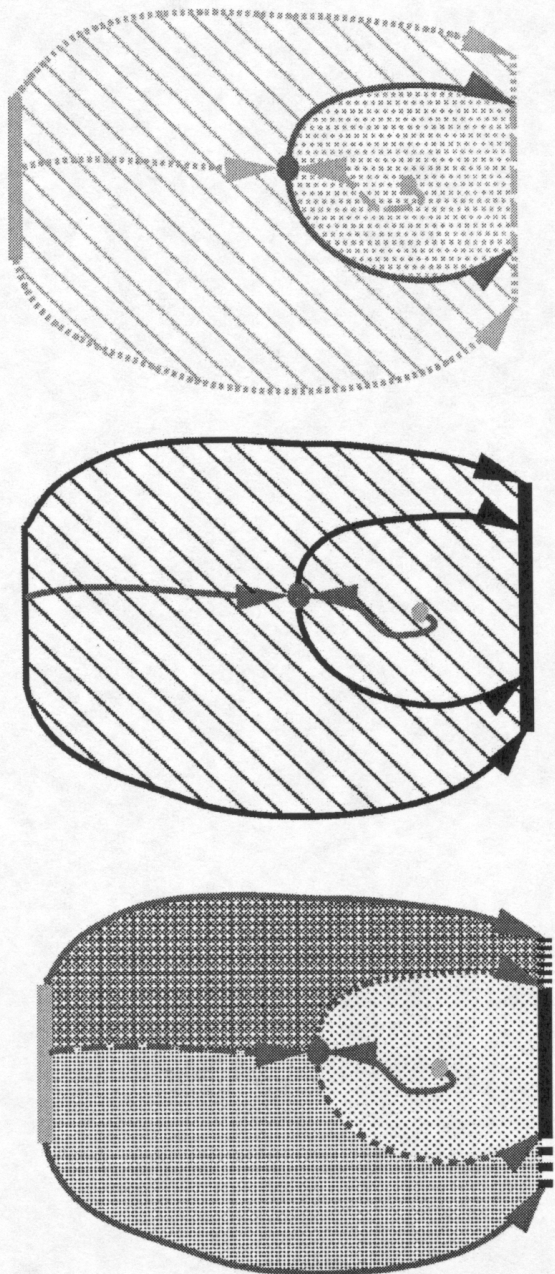


Fig. 4 A simple vector field is first partitioned into α -basins and then into ω -basins. The resulting different intersections of α - and ω -basins are shown in the bottom image. All α -limit sets and α -basins are colored light gray, all ω -limit sets and ω -basins are colored dark gray and the separatrices and saddles are colored medium gray. The intersections are shown in different black patterns.

Both parts of the theorem are illustrated in Fig. 4. In the upper part, the figure shows a decomposition of a vector field into α -basins. In the center, it shows the decomposition of the same vector field into ω -basins. We are interested in the connected components of the intersections between α - and ω -basins. Since these components consist of integral curves starting at a connected part of the boundary (or a critical point inside the domain) and ending at a specific connected part of the boundary (or a critical point inside the domain), one can transform the whole region into topologically uniform flow. Thus, one can separate the vector field into regions of topologically uniform flow. The 2D intersections are built by intersections of α -basins of

sources or α -basins of inflow regions with ω -basins of sinks or ω -basins of outflow regions. The boundaries of these regions consist of the critical points, the boundary saddles, the basins of the saddles, and connected subsets of boundary flow points. The lower part of Fig. 4 shows the decomposition into these intersections for the simple vector field used above. One can describe the topological structure of the vector field by drawing the critical points, boundary saddles and the integral curves starting or ending at the saddles and boundary saddles. In Fig. 4, the information about the vector field structure can be given by the red, blue and green elements.

4 Linear Local Vector Field Topology

This section deals with the specific cases to be considered for a piecewise linear vector field defined over a triangulation in the plane. This case will occur in all the examples shown in Secs. 5 and 6. The changes and additions to the previous section deal with the situation on the boundary. One has to deal with the vertices of a polygon, i.e., the boundary is not a smooth curve. We use scalar products of the vector field at certain positions and outward normals of the boundary edges to analyze the vector field. Together with the procedure from Ref. 4, one can construct an algorithm extracting the local topology.

We assume that the domain D is triangulated and that the vector field is defined by piecewise linear interpolation of vectors at the vertices of the triangulation. The analysis of the local vector field topology is simplified on the boundary by performing linear interpolation along the edges. For interior points of the edges, we use Lemma 4.1.

Lemma 4.1: Let $D \subset \mathbb{R}^2$ be a triangulated 2D domain and ∂D be its polygonal boundary consisting of one or more polygons. Let $v: D \rightarrow \mathbb{R}^2$ be a piecewise linear vector field defined over the triangulation of D . Let $d \in \partial D$ be an interior point on a boundary edge e and $n \in \mathbb{R}^2$ be the outward normal of the edge. There are four possibilities:

1. if $v(d) \cdot n > 0$ holds, d is an outflow point;
2. if $v(d) \cdot n < 0$ holds, d is an inflow point;
3. if $v(d) \cdot n = 0$ holds and d is the only point for which this condition holds on the edge, d is a boundary saddle. (The integral curve through d is a separatrix, and one has to integrate it in negative and positive temporal directions.)
4. If $v(d) \cdot n = 0$ holds for all points on the edge, then d is a boundary flow point.

Proof: Let $x \in D$ be any point. The above four statements are true because $v(x) \cdot n = 0$ is a linear condition along the edge, since

$$v(x) = (1-t)v_0 + tv_1, \quad (12)$$

where

$$x = (1-t)p_0 + tp_1, \quad (13)$$

where p_0 and p_1 denote the end points of the edge and v_0 and v_1 the vector values at these positions. It follows that

$$v(x) \cdot n = (1-t)v_0 \cdot n + tv_1 \cdot n, \quad (14)$$

which is a simple linear equation that can be either zero at one point or at all points. If $v(x) \cdot n$ is zero at all points, then the flow stays on the edge, proving possibility 4. If $v(x) \cdot n$ is zero at only one point, then $v(x) \cdot n$ is positive on one side and negative on the other side of d , which implies that d is a boundary saddle since the integral curve "touches" the edge at that point (or contains just that point). The first two cases imply that the integral curve ends or starts at d and, due to continuity, the integral curve leads into the interior of D or originates from there. \diamond

For the analysis of an endpoint p on a boundary edge, one has to check the vector field value with the normals m and n of the two adjacent boundary edges. One has to distinguish the convex, collinear, and concave cases. The analysis is always based on the behavior of the vector field on two consecutive boundary edges. If the vector at the vertex under consideration is not tangential to one of the edges, one can determine the type of the vertex by calculating the scalar products $m \cdot v(p)$ and $n \cdot v(p)$ of the vector at the vertex with the two outward normals of the edges. This defines the topological behavior of the vector field in a neighborhood of the vertex on the boundary. If the field is tangential to one edge (or both, in the collinear case), one has to analyze the vector at the other end of the edge(s) to determine the topological type of the vertex with respect to the vector field. We formulate three Lemmata to describe all possible cases and illustrate the situations in Figs. 5–7. The algorithm uses these properties to find the boundary saddles and the separatrices starting from there. The first Lemma deals with the convex case:

Lemma 4.2: *Let p be a convex boundary vertex of D , and let m and n be the normals of the two adjacent boundary edges. Furthermore, let o and q be the endpoints of the two boundary edges. We distinguish five cases.*

1. If $(v(p) \cdot m)(v(p) \cdot n) < 0$ holds, then the integral curve through p in D contains only p .
2. If $(v(p) \cdot m)(v(p) \cdot n) > 0$ holds, then p is an outflow or inflow point, depending on the common sign of the products.
3. If $v(p) \cdot m = 0$ and $v(o) \cdot m > 0$ holds, then p is an outflow point.
4. If $v(p) \cdot m = 0$ and $v(o) \cdot m = 0$ holds, then p is a boundary saddle. (The integral curve through p stays on the first edge and leaves D at p , so one does not have a separatrix entering the interior of D at p .)
5. If $v(p) \cdot m = 0$ and $(v(o) \cdot m) < 0$ hold, then p is a boundary saddle. (The integral curve through p consists only of p , so one has no separatrix entering the interior of D at p .)

An analysis of the cases with $v(p) \cdot n = 0$ yields the same results.

Proof: The typical situation $(v(p) \cdot m)(v(p) \cdot n) < 0$ is shown in the upper-left corner of Fig. 5. The vector and its inverse lead out of D such that the integral curve consists only of p . Concerning case A2, the vector field points ei-

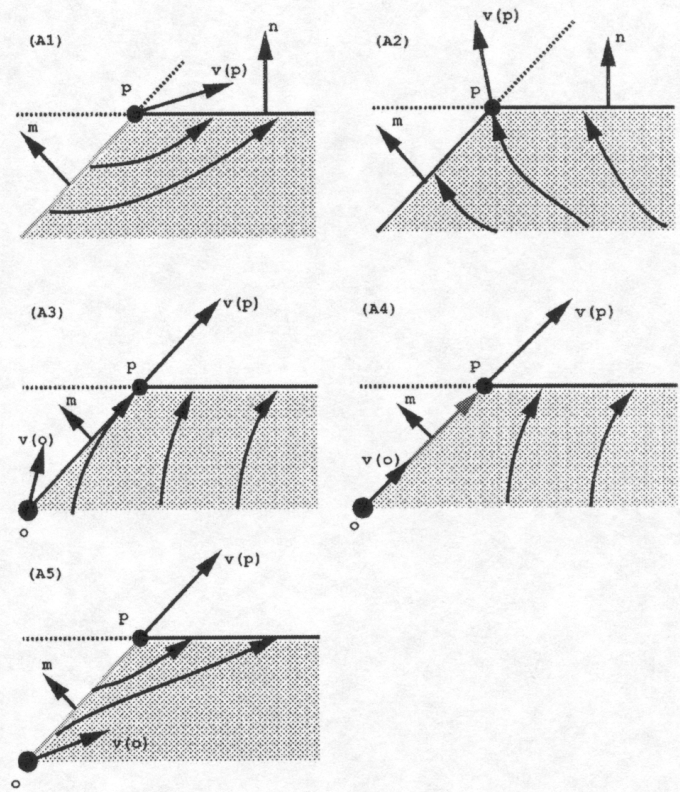


Fig. 5 Convex vertex. Case A1: inflow and outflow edges; case A2: outflow on both edges; case A3: flow at vertex being parallel to one edge—outflow inside both edges; case A4: boundary flow on one edge—outflow on the other edge; case A5: flow at vertex being parallel to one edge, inflow inside one edge, and outflow on the other edge. Inflow edges are marked light gray, outflow edges are marked dark gray, and boundary flow is marked medium gray.

ther outward at p (and in a neighborhood on both edges) or inward on a complete boundary neighborhood, as shown in the upper-right corner of Fig. 5. Then, p is an ordinary outflow or inflow point. Case A3 is shown in the left-middle image of Fig. 5. The point p is in the middle of an outflow region; therefore, it is an outflow point. In case A4, the integral curve through p stays on the edge from o to p and leaves D there, as can be seen in the right-middle image of Fig. 5. Finally, case A5 is shown in the lower-left corner of Fig. 5. All integral curves in a neighborhood of p enter D on one edge and leave it on the other. Thus, the integral curve through p consists only of p itself. \diamond

This proves that one does not need to calculate any separatrices in this case. The collinear case is treated in the next lemma.

Lemma 4.3: *Let p be a shared boundary vertex of two collinear edges, and let $v(p) \neq 0$. Furthermore, let $n \in \mathbb{R}^2$ be the outward normal of both edges and o, q the other two endpoints of the edges. One can distinguish eight cases.*

1. If $v(p) \cdot n > 0$ holds, then p is an outflow point.
2. If $v(p) \cdot n < 0$ holds, then p is an inflow point.
3. If $v(p) \cdot n = 0$, $v(o) \cdot n = 0$, and $v(q) \cdot n < 0$ holds, then p is a boundary saddle. (The integral curve through p is a separatrix, and one needs to calculate the part entering the interior of D .)

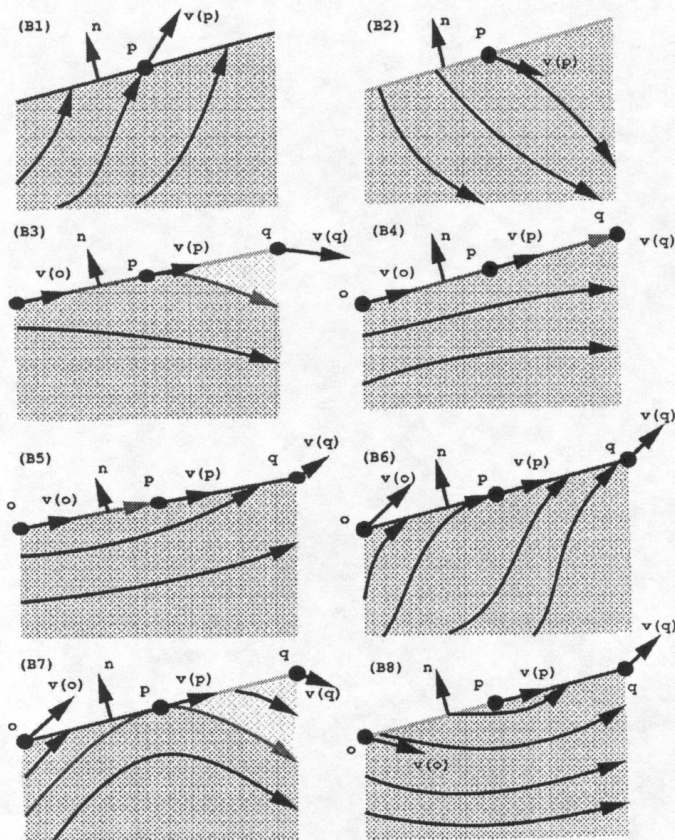


Fig. 6 Collinear vertex. Case B1: outflow at p ; case B2: inflow at p ; case B3: boundary flow on one edge and inflow on the other edge; case B4: boundary flow on both edges; case B5: boundary flow on one edge and outflow on the other edge; case B6: flow parallel to the edges at p and outflow inside both edges; case B7: flow parallel to the edges at p , inflow in the direction of the vector, and outflow on the other edge; case B8: flow parallel to the edges at p , outflow in the direction of the vector, and inflow on the other edge. All inflow edges are colored light gray, outflow edges are colored dark gray, and boundary flow and separatrices are colored medium gray.

4. If $v(p) \cdot n = 0$, $v(o) \cdot n = 0$, and $v(q) \cdot n = 0$ holds, then the integral curve through p stays on the boundary around p .
5. If $v(p) \cdot n = 0$ holds, then $v(o) \cdot n = 0$ and $v(q) \cdot n > 0$ holds, p is a boundary saddle. (The integral curve through p stays on one edge and ends at p . One does not have to calculate a separatrix.)
6. If $v(p) \cdot n = 0$ and $((v(o) \cdot n)(v(q) \cdot n)) > 0$ holds, p is an outflow or inflow point, depending on the common sign of the scalar products.
7. If $v(p) \cdot n = 0$ and $((v(o) \cdot n)(v(q) \cdot n)) < 0$ holds and $v(x) \cdot n$ changes from outflow to inflow in the direction of the tangential component for points x on the two edges, p is a boundary saddle. (The integral curve through p is a separatrix touching the boundary. One has to integrate it backward and forward in time.)
8. If $v(p) \cdot n = 0$ and $((v(o) \cdot n)(v(q) \cdot n)) < 0$ holds and $v(x) \cdot n$ changes from inflow to outflow in the

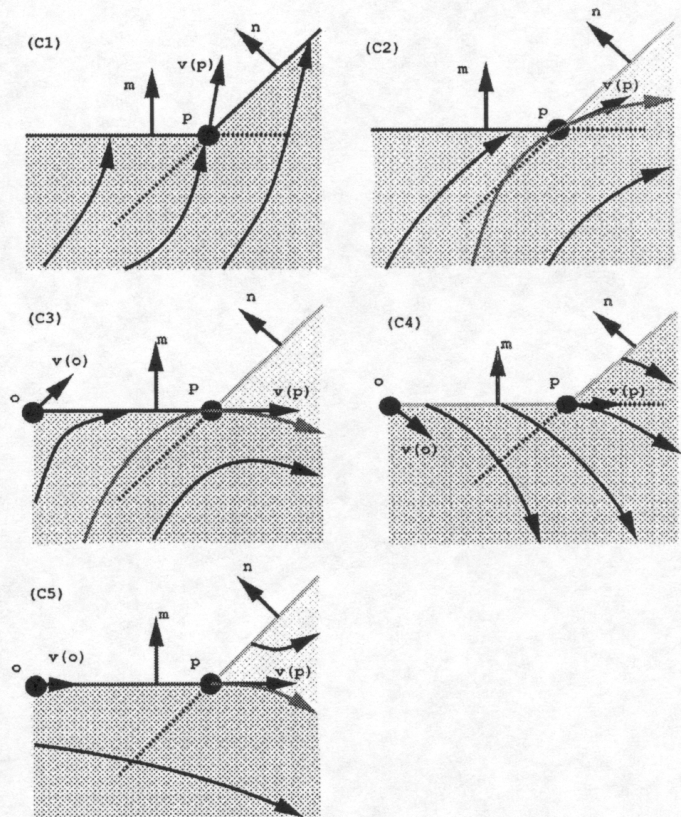


Fig. 7 Concave vertex. Case C1: outflow on both edges; case C2: outflow on one edge and inflow on the other edge; case C3: flow parallel to one edge at vertex p , outflow inside one edge, and inflow inside the other edge; case C4: flow parallel to one edge at vertex p and inflow on both edges; case C5: boundary flow on one edge and inflow on the other edge. Inflow edges and α -basins are colored light gray, outflow edges and ω -basins are colored dark gray, and separatrices and boundary flow are shown in medium gray color.

direction of the tangential component for points x on the two edges, the integration curve through p consists only of p .

The remaining cases, where $v(q) \cdot n = 0$, are equivalent to cases B3 and B5.

Proof: We use figures to explain the arguments. The upper-left corner of Fig. 6 shows the situation in case B1 and the upper-right corner the situation in case B2. These cases are equivalent to the standard case inside an edge in Lemma 4.1. In left of the second row of Fig. 6, it can be seen that in case B3 the integral curves lead into the inner part of D at p , whereas the obvious case B4 is on the right of the second row. Case B5 is shown in the left of the third row in Fig. 6. The integral curve through p ends at p , since there is an outflow region inside the second edge. The right of the third row in Fig. 6 shows the integral curves around p in case B6 with outflow. One can see that the flow can be transformed into topologically uniform outflow. Reversing all vectors yields the inflow case. Case B7 is shown in the lower-left corner of Fig. 6. One integral curve touches p . Thus, three different parts of the flow are visible: One part leaves D , one part enters D , and one part stays inside D . There are two separatrices starting at p , similar to the standard case of a boundary saddle lying in the interior of an

edge. Finally, the lower-right corner of Fig. 6 shows case B8. The flow enters on one side of p and leaves on the other side. Thus, the integral curve through p consists only of p , and there is no separatrix to be calculated. \diamond

The remaining case deals with a concave vertex. Lemma 4.4 summarizes the possible patterns.

Lemma 4.4: Let p be a concave boundary vertex and let m and n be the outward normals of the two edges. Furthermore, let o be the second endpoint of the edge with normal m . There are five cases one can consider.

1. *If $((v(p) \cdot m)(v(p) \cdot n)) > 0$ holds, then p is either an outflow or an inflow point, depending on the common sign of the products.*
2. *If $((v(p) \cdot m)(v(p) \cdot n)) < 0$ holds, then p is a boundary saddle. (The integral curve through p is a separatrix that has to be integrated backward and forward in time.)*
3. *If $v(p) \cdot m = 0$ and $v(o) \cdot m > 0$ holds, then the integral curve through p is a separatrix that has to be integrated backward and forward in time.*
4. *If $v(p) \cdot m = 0$ and $v(o) \cdot m < 0$ holds, then p is an inflow point.*
5. *If $v(p) \cdot m = 0$ and $v(o) \cdot m = 0$ holds, then the integral curve through p stays on one edge and enters the interior of D at p , so one has to calculate one part of the curve. (The orientation of the curve is positive, provided the tangential part of $v(p)$ with respect to m is directed into the interior of D . Otherwise, one has to integrate backwards in time.)*

An analysis of the cases where $v(p) \cdot n = 0$ holds yields the same results.

Proof: The upper-left corner of Fig. 7 shows a typical situation for case C1, with outflow around p . By reversing all vectors and curve orientations, one obtains the inflow for negative scalar products. Case C2 is described in the upper-right corner of Fig. 7. One can see the integral curve touching the boundary at p . This is the situation on the other side of the boundary of case A1. The flow is separated at p into an outflow region along one edge, an inflow region along the other edge, and a region of flow staying inside D below the integral curve through p . Case C3 is shown in the left-middle image of Fig. 7. The topological situation equals case C2, so one obtains the same result. Case C4 is shown in the right-middle image of Fig. 7. The integral curves starting at the boundary around p all enter the interior of D , so one has an inflow point. Finally, in case C5, the integral curve through p stays on one edge and enters the interior of D at p , so one has to calculate the part of this integral curve inside the interior of D . This is shown in the lower left corner of Fig. 7. \diamond

These various case distinctions allow us to establish an algorithm to extract local topology. We determine for all critical points in the domain and classify them into sources, sinks, and saddles; then, we analyze all the edges and vertices on all boundaries identifying boundary saddles and separatrices starting there. Section 5 provides examples.

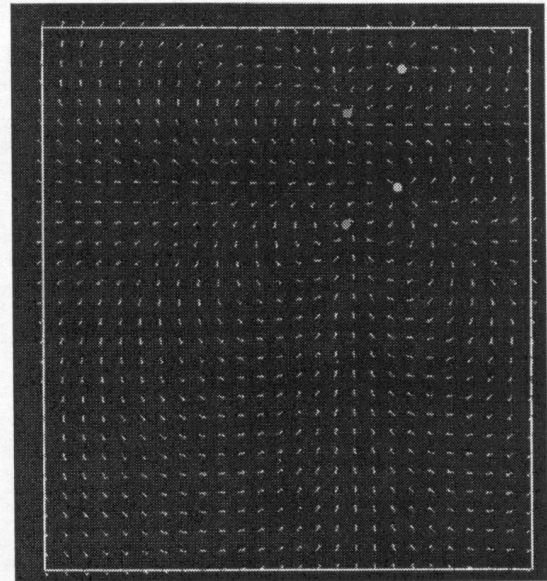


Fig. 8 Vector field containing two sources and two sinks.

5 Examples

The theoretical considerations from the last sections aim at an analysis of vector field topology including the boundary. The first examples provided in this section show the effect of this analysis on the understanding of topological vector field structure. They are based on the study of vector fields given by polynomial equations. The construction of these fields is based on considerations based on *Clifford algebra*, see Refs. 16 and 17. We briefly review the main topological properties discussed in Refs. 16 and 17. Figures 8–13 show unit vectors to indicate the orientation of separatrices and integration curves. Critical points are red, green, or blue. The red color indicates a saddle point; the green color a source; and the blue color a sink. The separatrices are drawn in blue; integral curves are violet; and the boundaries of regions and domains are white.

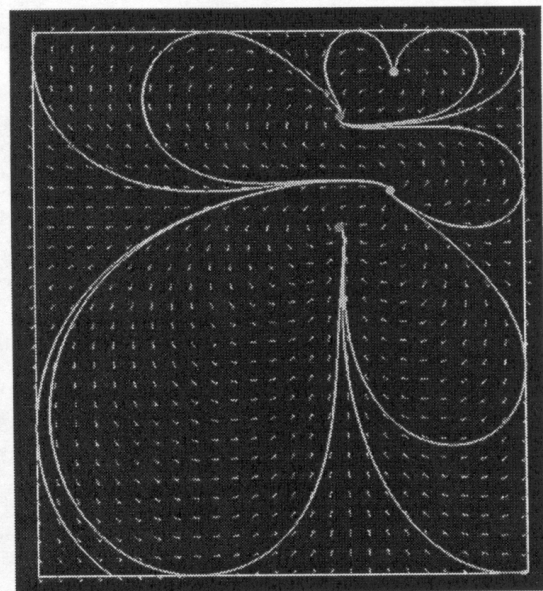


Fig. 9 Local topology showing interaction of sources and sinks.

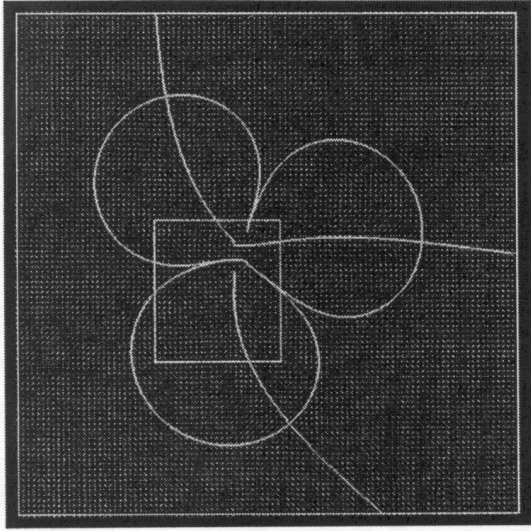


Fig. 10 Global topology derived by considering entire field.

We start with a vector field containing two sinks and two sources in a rectangular area. The conventional analysis, based on the separatrices starting at saddle points alone, will find no separating curves at all, so the user is left with the question of how the two sources and sinks interact. This can be seen in Fig. 8. Since there exist integral curves from one source to both sinks and also to the boundary, not all integration curves belong to the same open basin. We know that, as a result of the piecewise linear interpolation and the analytic structure of the original field, there are no additional critical points or more complicated structures involved in this example. The solution follows as a result of Secs. 3 and 4. There are ten boundary saddles where the flow turns from inflow to outflow. By starting separatrices at these positions it is possible to determine the structure of the flow. The result is shown in Fig. 9. The whole rectangle is now divided into open basins with the same α and ω basin. Every integral curve in one of these basins starts and ends at the same critical point or connected component of the boundary. It is now easy to understand the interaction of the sinks and sources.

As mentioned before, this example was constructed using an analytic field description. The structure of the entire field is shown in Fig. 10. The small white box marks the domain of our example. There are three saddle points

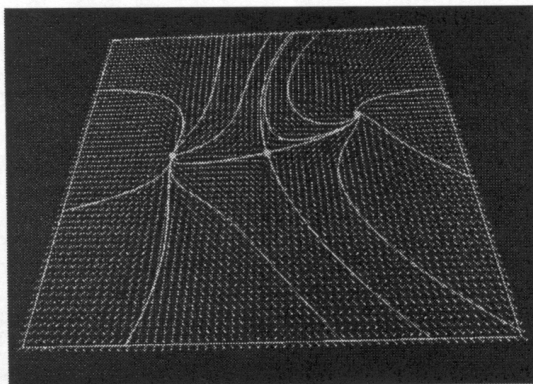


Fig. 11 Analytic vector field with two sources and one saddle.

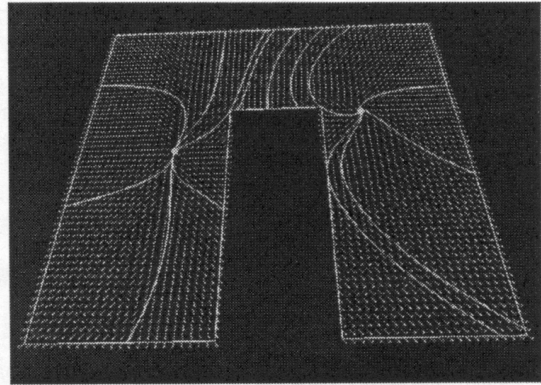


Fig. 12 Local topology analysis—a rectangle containing the saddle was cut out.

where 12 separatrices start. The importance of the saddles for the standard analysis is seen by comparing the result inside the rectangle with the result shown in Fig. 8.

Our next example involves two sources and a saddle point on a plane segment. The simple structure is shown together with some integral curves in Fig. 11. If one cuts out a rectangular area containing the saddle point, one will miss all the separatrices starting at the saddle point. Performing local topology analysis, however, it is possible to analyze the field correctly, see Fig. 12. There are three boundary saddles: One is inside an edge, and the other two are on concave corners, see Sec. 4. These separatrices replace the separatrices starting at the saddle point in the global topology shown in Fig. 11. They assure the separation of the basins of the two sources and the basins of the inflow regions on the boundary. There are obvious differences between the global and local topology. For comparison, we show all separatrices and integral curves together in Fig. 13. It can be seen that the separatrices starting on the boundary are strongly influenced by the placement of the boundary but also by the existence of the saddle point in the overall vector field.

6 Application

We have applied the local topology extraction to a vortex breakdown simulation. Vortex breakdown is a phenomenon observed in a variety of flows ranging from tornadoes to wing tip vortices,¹⁸ pipe flows,^{19–23} and swirling jets.²⁴ The latter flows are important to combustion applications where

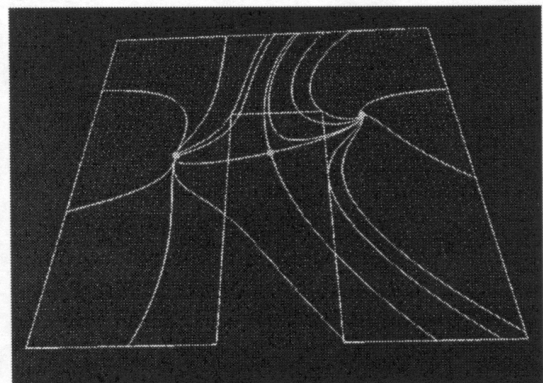


Fig. 13 Comparison of results obtained with local and global topology analysis.

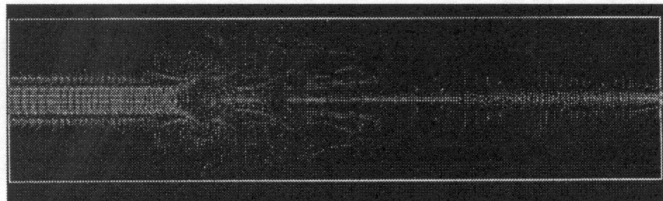


Fig. 14 Turbulent jet data set.

they are able to create recirculation zones with sufficient residence time for the reactions to approach completion. The numerical simulation of this type of flow requires an accurate method for the solution of the initial/boundary value problem of the Navier–Stokes equations. The incompressible Navier–Stokes equations are set up for this purpose in cylindrical domains, which may be bounded or unbounded. Analytic mappings are applied to the radial and axial directions to map the physical domain onto the unit cylinder and to control the distribution of grid points. Cylindrical coordinates are well suited for this geometry since they allow Fourier decomposition with respect to the azimuthal direction. However, the metric coefficients produce singularities at the coordinate axis, which must be carefully analyzed²⁵ to ensure stability and accuracy for the simulation of 3D flows. It can be shown that the azimuthal Fourier modes of scalar, vector and tensor variables possess well defined symmetry properties with respect to the radial coordinate and satisfy radial growth laws near the axis depending on the wave number. A hybrid spectral finite-difference method (Canuto *et al.*²⁶) based on stream functions for the azimuthal Fourier modes is used for the simulations. The solver allows the choice of second to eighth order for first and second derivatives as explicit central difference operators²⁷ or third to ninth order upwind-biased differences.²⁸ High order filters are used to provide the numerical dissipation to stabilize the system if central difference operators are applied to the convective terms.²⁷ Runge–Kutta-type time integration methods with minimal storage requirements,^{27,29} were implemented and the third and fourth order version are currently available in the solver. The Navier–Stokes solver is applied to the vortex breakdown in incompressible jet flows with supercritical swirl numbers²⁴ and the results are used for the topological analysis presented in the previous sections.

The boundary conditions at the nozzle entrance section were determined in accordance with the experiments of Bilant *et al.*²⁴ They measured the axial and azimuthal velocities at $z_0/D=0.6$ ($D=2R$), which show a distinct dependence on the swirl number S . The simulations were performed for various values of the Reynolds and swirl numbers and the experimental profiles at $z/D=0.6$ were used as boundary conditions. The image domain was discretized with $n_r \times n_z = 95 \times 251$ grid points, and the third-order accurate minimal storage Runge–Kutta scheme Eq. (9) of Williamson²⁹ was used for time integration. The Reynolds number was $Re=3000$ and the swirl number $S=1.25$. The initial conditions were generated by extending the entrance profiles throughout the flow field. Disturbances for the axial velocity and azimuthal vorticity of the order of 0.1% were added at the entrance to trigger the Kelvin–

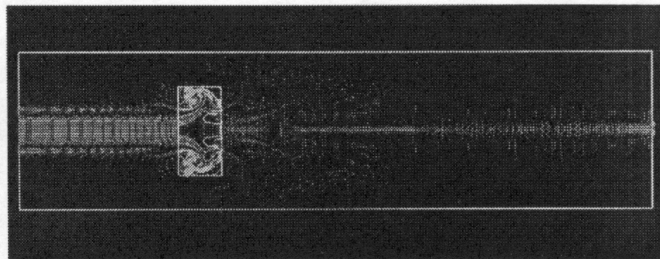


Fig. 15 Rectangular region in jet data set and result of local topology analysis.

Helmholtz instability of the shear layer. The results used for the topological analysis were taken at the dimensionless time $t=40.989$. The vector field is shown in Fig. 14. There are 39 909 data points and 79 000 triangles in this data set. The piecewise linear interpolation contains 703 simple critical points, creating a complex global topology. The jet enters the domain in the middle of the right boundary.

First, we used a rectangle, shown in Fig. 15. The data in this rectangle were used to analyze the local topology. We extract all the critical points and find all the boundary saddles by analyzing the boundary of the rectangle. This resulted in the topological structure shown in Fig. 16. One can see some of the additional separatrices starting at the boundary saddles. They separate regions of flow staying inside the boundary from outflow and inflow parts. No analysis of data outside the rectangle has been used. The required computing time only depends on the size of the region and is (nearly) independent of the size of the overall data set.

It is possible to use multiple regions of interest in the same data set that can be analyzed independently. One may also use arbitrary polygons as boundaries. This is demonstrated in our second analysis of the same jet data set for which we chose three regions, as shown in Fig. 17. One region covers a part of the backstream beside the main inflow jet. The second region shows a part of the rectangle we used before. Neither of these two regions contains critical points, which documents the necessity of boundary analysis. The third region shows the mixing process of the jet and the fluid downstream. Figure 18 shows the first two regions in more detail. One can see clearly forward- and

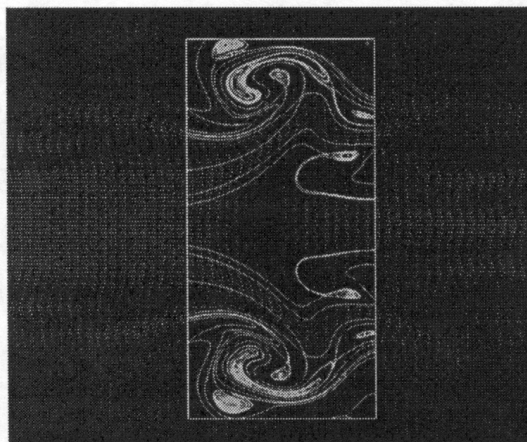


Fig. 16 Magnification of result of local topology analysis shown in Fig. 15.

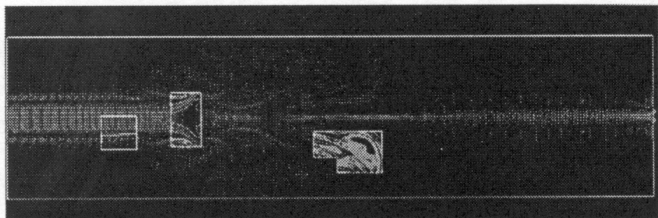


Fig. 17 Three regions in jet data set and respective results of local topology analysis.

backward-facing flow. Without an analysis of the boundary, one obtains no separatrices due to the lack of critical points inside the two regions. The third region is shown in more detail in Fig. 19. Since the analysis is limited to a rather small area, it can be analyzed quickly. In the figure, one can spot several separatrices spiraling around critical points. The critical points inside these areas have Jacobians with complex conjugate eigenvalues, thus there are spirals; the real parts of the eigenvalues may have small absolute values, and a stream line in the neighborhood of the critical points approaches them very slowly. Closed stream lines might be present.

7 Conclusions and Future Work

We have presented a method to analyze the local topology of arbitrary regions in 2D vector fields. Our idea is based on an extraction of the critical points in the domain and an examination of the region's boundary. By determining the inflow, outflow, and boundary flow segments one can separate the domain into regions of topologically uniform flow. A detailed analysis of the possible cases in a piecewise linear flow allows local structure extraction for analytical and gridded data sets. We have outlined the differences to the global topology algorithm in theory and applications, demonstrating the relevance of our localized approach when applied to regions with complicated flow patterns on the boundary. This case is typical for most interesting regions inside a larger data set. Another important situation that we studied is the absence of critical points in a region that provides interesting structure, like backward facing flow. Our algorithm detects these areas and separates them from other parts of the flow leading to better visualizations of local flow structure. Since the local topology analysis

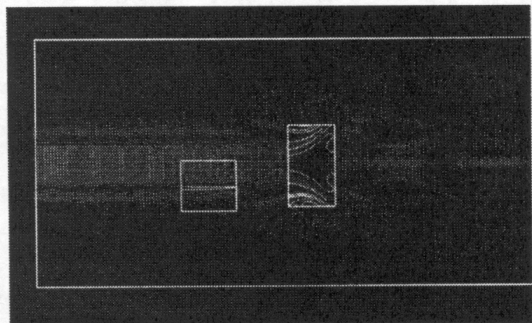


Fig. 18 Local topology analysis inside two regions without critical point (jet data set).

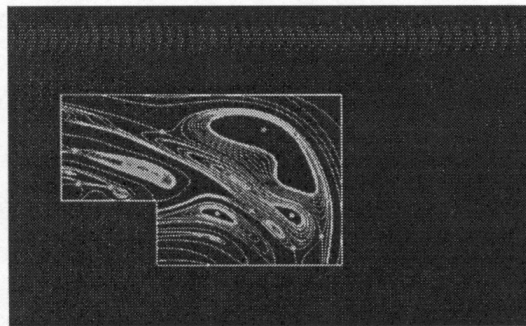


Fig. 19 Local topology analysis result of highly complicated region (jet data set).

does not use any information outside a region of interest, it is very attractive when analyzing large data sets locally and thus reducing computing time significantly.

We plan to extend our ideas to vector fields defined over 3D domains. We will also apply our algorithm to other data sets to gain a deeper understanding of the relation between local and global topology. Besides 3D domains, an analysis of time-dependent simulations is one final goal. The analysis in this paper is limited so far to steady flow, since pathlines will cross each other, especially in a turbulent flow like our application. New ideas are needed to accomplish an analysis over time, but we believe that topology provides a framework to find new ways for analysis and visualization.

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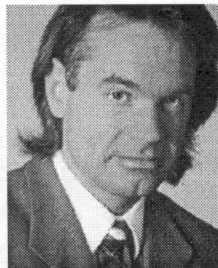
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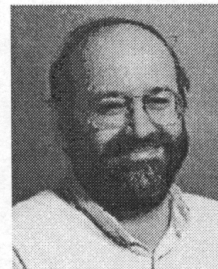


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