

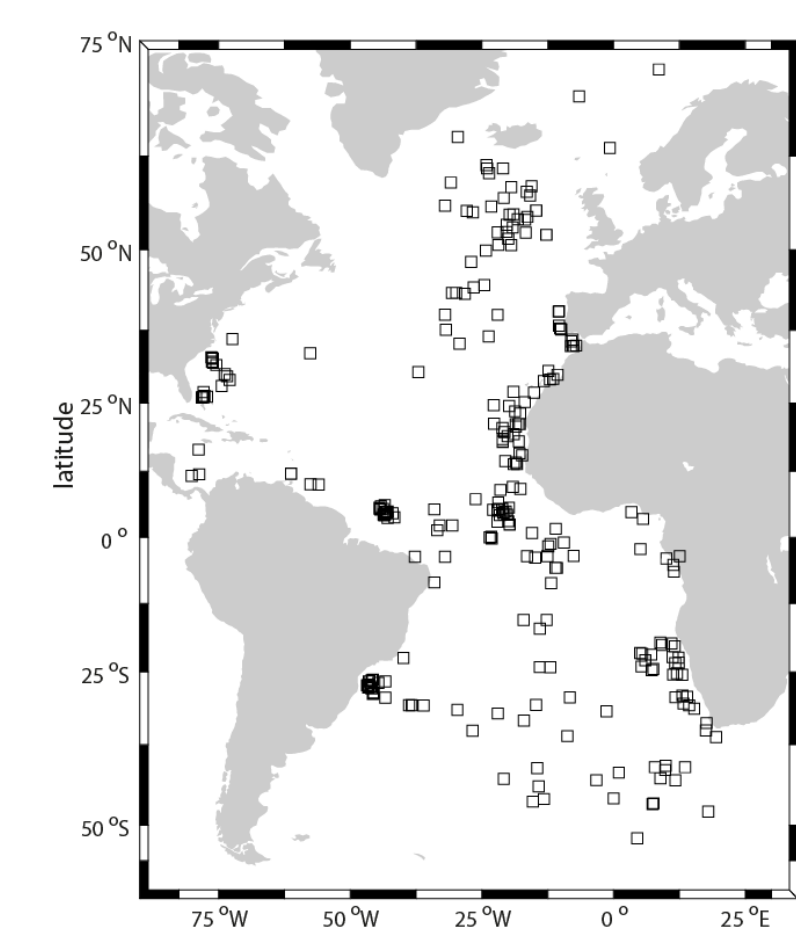
A Comparison of Methods for Ocean Reconstruction from Sparse Observations



Gregory J. Streletz^[1], Markus Kronenberger^[3], Christopher Weber^[1], Geoffrey Gebbie^[2], Hans Hagen^[3], Bernd Hamann^[1], Oliver Kreylos^[1], Louise H. Kellogg^[1], Christoph Garth^[3], and Howard J. Spero^[1]

(1) University of California, Davis, CA 95616 USA (2) Woods Hole Oceanographic Institution, Woods Hole, MA 02543 USA (3) University of Kaiserslautern, Germany

Introduction



- Problem: reconstructions from scattered observations of sediment cores distributed on the ocean floor in a sparse and irregular manner
- Data: measurements from benthic foraminifera in deep sea sediment cores (e.g., the data compiled by Peterson et al. [1])
- Solution: reconstruction methods useful for interpolating or approximating sparse scattered data
- Goal: comparison of the advantages and disadvantages of methods in order to enhance reconstruction quality

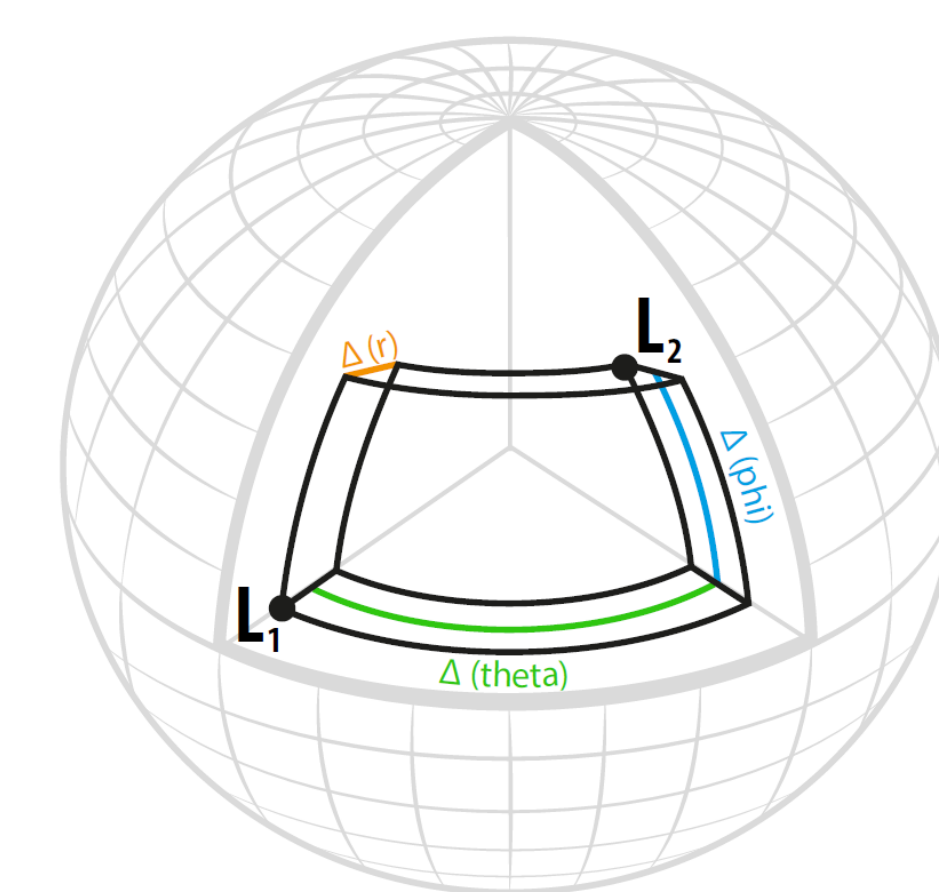
Methods

Reconstruction using a modified Moving Least Squares Approach

- Modified Moving Least Squares (MLS) approach without assuming any additional information (e.g., the flow field)
 - Input: A scalar field of observations \mathbf{f} , precomputed weighting parameters α, β, γ (see Distance Measure section)
 - Computation of matrix \mathbf{B} from quadratic basis functions \mathbf{b}
 - Set up diagonal weight matrix $\mathbf{W}(\rho)$ with weight function ρ (depending on the weighted distance)
 - Evaluate MLS function^[2]

$$MLS(\mathbf{x}) = \mathbf{b}^T(\mathbf{x})(\mathbf{B}^T \mathbf{W}(\rho) \mathbf{B})^{-1} \mathbf{B}^T \mathbf{W}(\rho) \mathbf{f},$$
 with $\rho(L_1, L_2) = 1 / (\text{distance}(L_1, L_2)^2 + \epsilon^2)$ ϵ is a smoothing parameter
 - Output: Reconstructed values

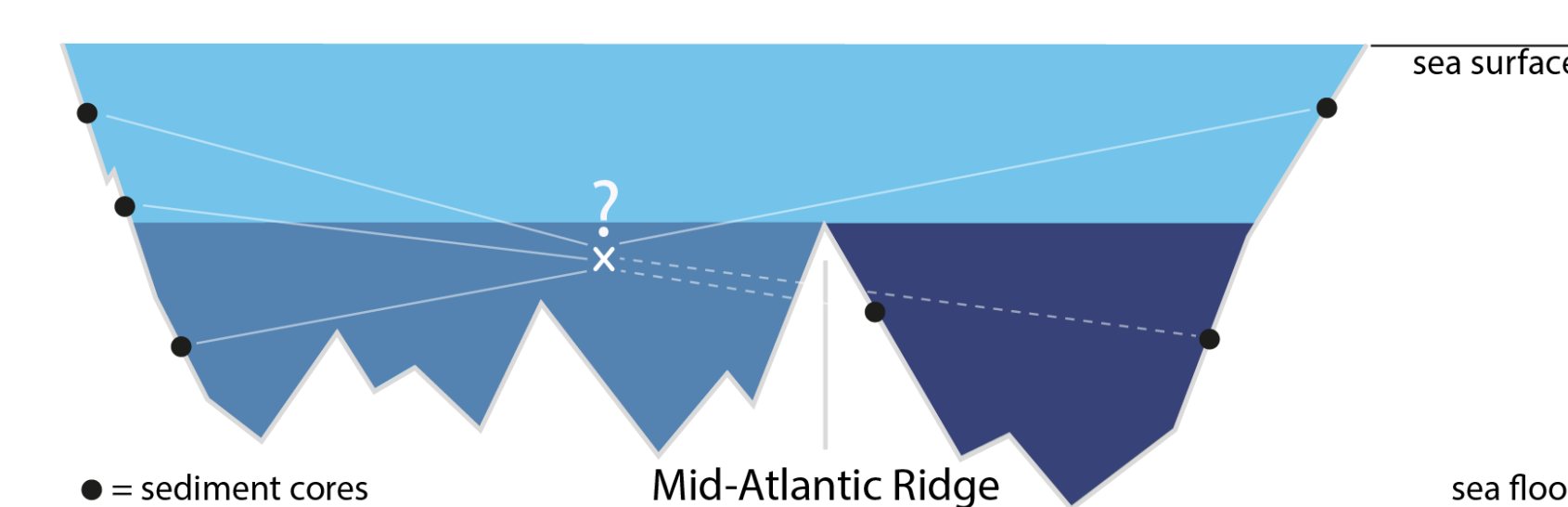
Distance Measure



- Generalized distance measure considering spherical nature of the data set:
 - Input: Two locations \mathbf{L}_1 and \mathbf{L}_2

$$\text{distance}(L_1, L_2) = \alpha \cdot \Delta(\text{phi}) + \beta \cdot \Delta(\text{theta}) + \gamma \cdot \Delta(r)$$
 - α, β, γ are weights
 - $\Delta(\text{phi})$ and $\Delta(\text{theta})$ are geodesic distances between \mathbf{L}_1 and \mathbf{L}_2 (see figure)
 - $\Delta(r)$ is the depth difference between \mathbf{L}_1 and \mathbf{L}_2
- Machine-learning pre-processing step performed to estimate good values for α, β and γ

Physical Boundaries of the Ocean



- Consideration of ocean boundary to improve reconstruction results
- Core samples partitioned into bathymetry-based subsets
- Cores of not directly connected subsets are ignored

Reconstruction using a Flow-Based Approach

- Exploits correlations between the scalar field to be reconstructed and the vector field representing the ocean flow
- Input: A scalar field of observations \mathbf{f} and a vector field representing flow
- Output: Reconstructed values
- Optimal Interpolation used as the underlying reconstruction method
- Modification of underlying method: utilize a non-Euclidean distance measure defined using the input flow field:

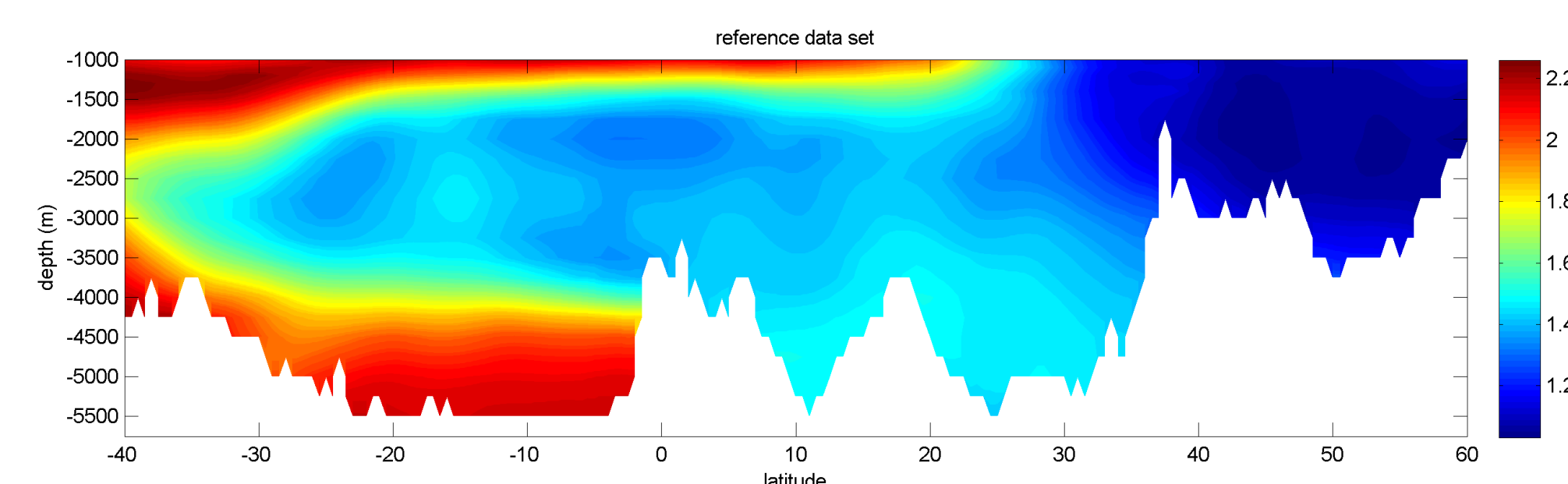
$$\text{distance}(L_1, L_2) = \sqrt{(\alpha \cdot (\text{distance along streamline})^2 + (\text{distance across streamline})^2)}$$
- Streamlines are calculated for the flow field using a fourth-order Runge Kutta method
- Parameters: α , correlation length
- Parameters optimized dynamically using an objective function defined with respect to the RMS error for a leave-one-out cross validation using the given observations

Results

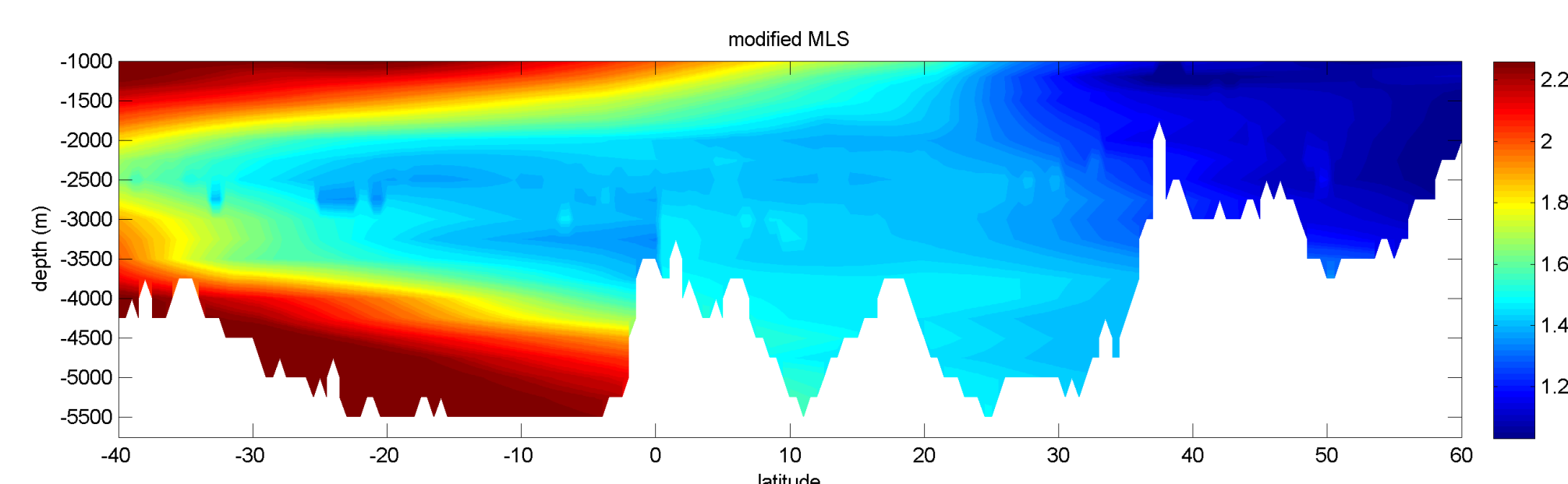
Modern day

- Comparisons of the Reconstructions for the modern-day Atlantic Ocean based on a gridded data product [3] as reference data
- Reconstructions based on a subset of 186 data points corresponding to the distribution of the sediment cores
- Due to lack of a gridded climatology of modern-day $\delta^{13}\text{C}$ data we used phosphate, because of its nearly linear relation to $\delta^{13}\text{C}$

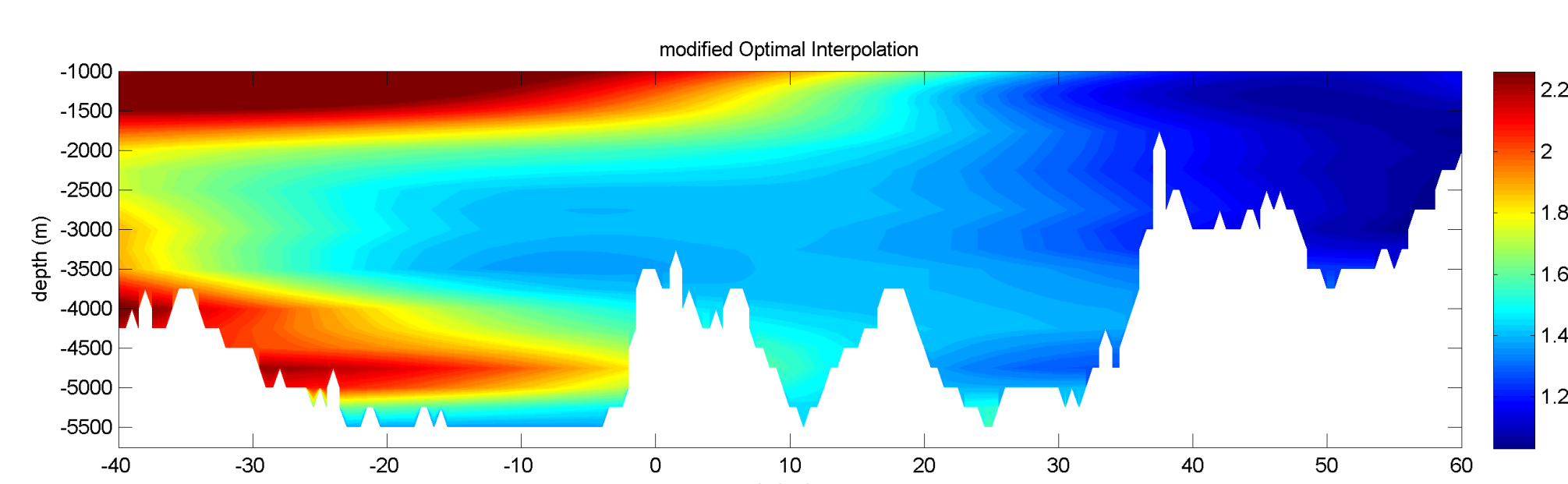
WOCE data set



Modified Moving Least Squares (MLS)

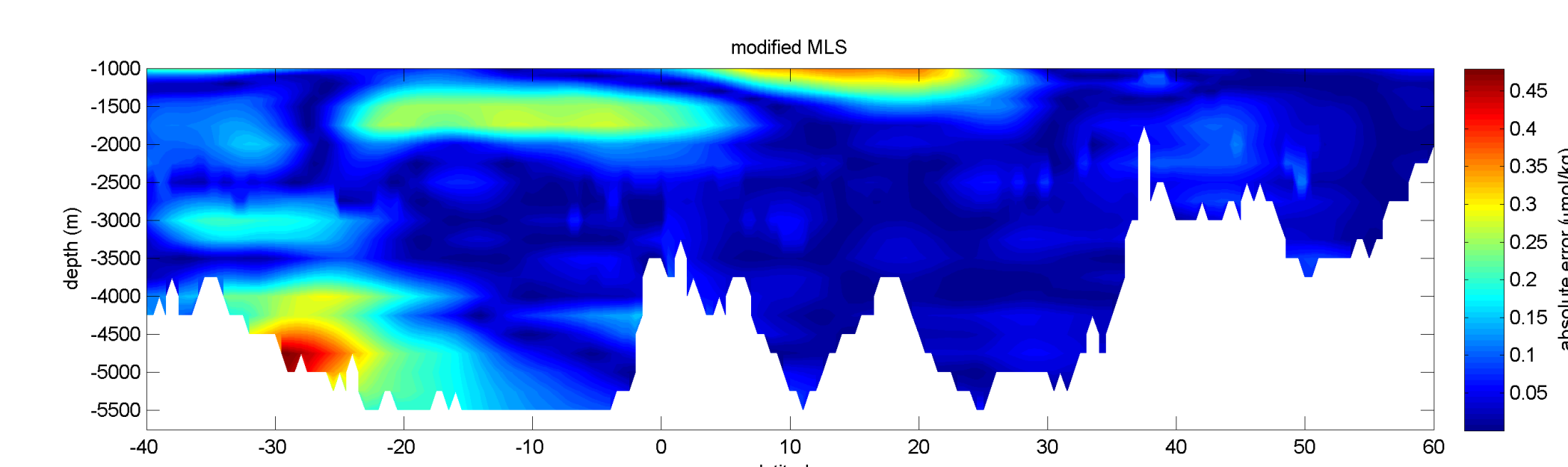


Modified Optimal Interpolation (OI)

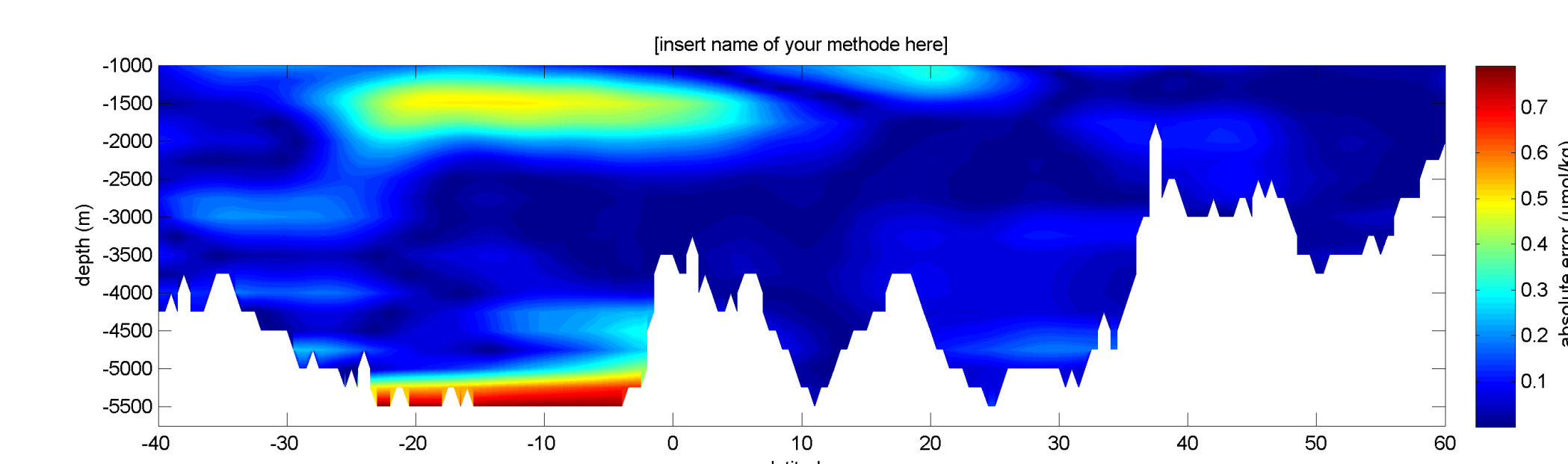


- Difference images between reconstructions and WOCE data set

Modified Moving Least Squares (MLS)

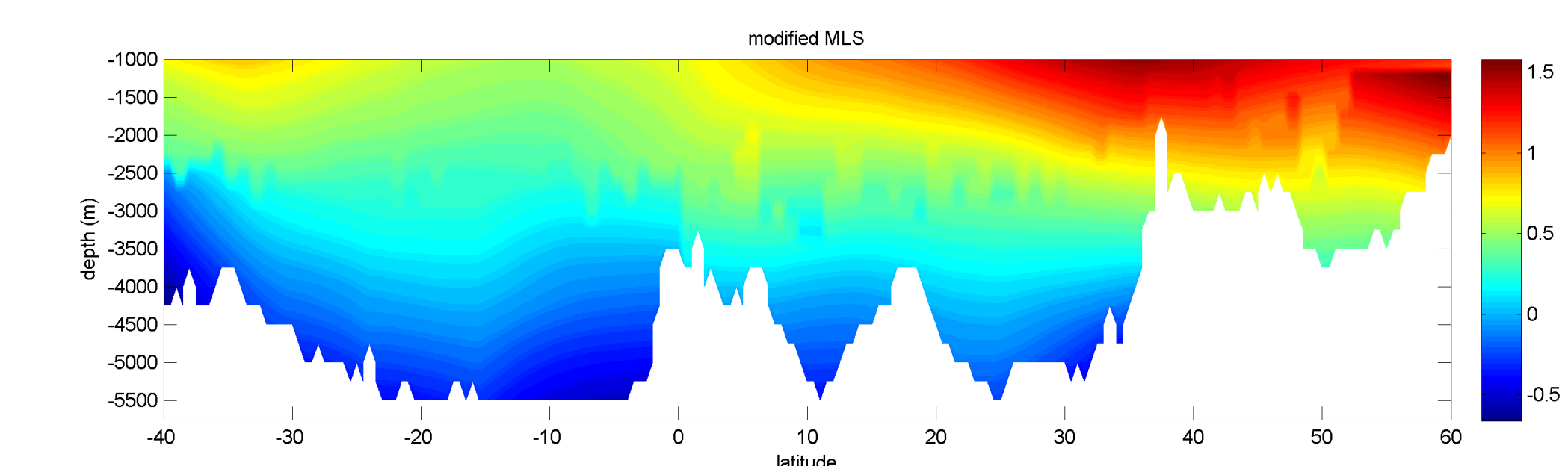


Modified Optimal Interpolation (OI)



LGM

- A reconstruction for the LGM



Summary and Future Work

- Both methods show promising results
- They have advantages and disadvantages in different regions
- In the future a combination of both may merged to a more precise reconstruction
- Further improvements may allow a estimating the glacial changes in multiple seawater properties and guide the analysis of existing and the future collection of sediment cores

References

^[1] Peterson, C., Lisiecki, L., and Stern, J. Deglacial whole-ocean $\delta^{13}\text{C}$ change estimated from 493 benthic foraminiferal records. Submitted to *Paleoceanography*.
^[2] Agranovsky, A., Garth C. and Joy, K., (2011) Extracting Flow Structures Using Sparse Particles. *Vision, Modeling and Visualization Workshop 2011*
^[3] Gouretski, V., and K. Koltermann (2004), WOCE global hydrographic climatology, *Bericht des Bundesamtes für Seeschifffahrt und Hydrogr.*