Abstract

In many real world applications one is concerned with the problem of visualizing and approximating three-dimensional data, commonly referred to as scalar fields (points in three-dimensional space with associated function values). The data themselves can either be physical measurements or be obtained from a mathematical simulation. Typical applications are found in medicine (computerized axial tomography, magnetic resonance imaging), in geology and meteorology (temperature, pressure, radiation), and in the CAD/CAM industry (car body, ship hull, airplane design).

Based on existing methods for visualizing bivariate functions new techniques are presented for rendering three-dimensional data. Assigning transparency properties to the data, and using ray tracing is one possibility being discussed. Slicing the data volume with hyperplanes allows the use of bivariate rendering routines directly. The problem of approximating and modeling contours of scalar fields is specifically emphasized.

The common approach treating scalar valued data in space consists of a two step process. An approximating function to the given data is computed, later typically rendered using contour plots. A different sequence of modeling steps is proposed here. First, a piecewise linear approximation to a contour is constructed from the given data yielding a set of triangulated surfaces. Second, all triangulated surfaces are used for generating smooth contours. Scalar fields can be discontinuous by nature, therefore determining a data set rapidly changing within small distances. The boundaries of internal structures in a volume might be given by a contour level close to a discontinuity. Computerized axial tomography (CAT) is an example for the advantage of the new method. Scanning a bone yields thousands of density values which consequently makes an overall approximation quite expensive. Because it is the shape of the bone which is of interest, the contour corresponding to the bone's boundary should be considered only.

A technique for approximating curvatures for surfaces as well as for trivariate functions are inferred from differential geometry. Curvature approximation is important both as input for surface schemes and as a tool for smoothness analysis. A data reduction algorithm is introduced that iteratively eliminates knots for a general triangulated surface, and a new scheme for producing a tangent-plane-continuous surface is presented.