

## LIST OF FIGURES

Figure	Page
1.1. Axial and saggital slice of human head (CAT). . . . .	2
2.1. Skull rendered using Levoy's algorithm for CAT-scan data. . . . .	7
2.2. Domain subdivision method for exponential function, $q_x = q_y = q_z = 5, \alpha_x = \alpha_y = \alpha_z = 1.$ . . . . .	12
2.3. Domain subdivision method for exponential function, $q_x = q_y = q_z = 8, \alpha_x = \alpha_y = \alpha_z = 2.$ . . . . .	12
2.4. Domain subdivision method for trigonometric function, unit ball, $q_u = q_v = q_w = 5, \alpha_u = \alpha_v = \alpha_w = 2.$ . . . . .	13
2.5. Domain subdivision and transparency, $n_x = n_y = n_z = 8, t = 0.5.$ . . . . .	15
2.6. Domain subdivision and transparency, $n_x = n_y = n_z = 8, t = 0.95.$ . . . . .	16
2.7. Slicing method, coloring hyperplanes, multiple colors, $k_1 = l_1 = 80, k_2 = l_2 = 130, k_3 = l_3 = 20.$ . . . . .	18
2.8. Slicing method, coloring hyperplanes, single color, $k_1 = l_1 = 80, k_2 = l_2 = 130, k_3 = l_3 = 20.$ . . . . .	18
2.9. Slicing method, bivariate surfaces, Gouraud shaded, $k_1 = k_2 = k_3 = l_1 = l_2 = l_3 = 30.$ . . . . .	20
3.1. Contour triangulation, contour divided into two parts. . . . .	27
3.2. Discontinuous piecewise planar contour approximation. . . . .	29
3.3. Trilinear cell interpolant restricted to a cell face, solution for the ambiguous case. . . . .	35
3.4. Cells containing contour polygons of length six, eight, nine, and twelve. . . . .	37

Figure	Page
3.5. Exact and piecewise linearly approximated contour in a cell, $f(x, y, z) = 2(1-x)(1-y)(1-z) + 1.6x(1-y)(1-z)$ $+ 1.4(1-x)y(1-z) + 1.4(1-x)(1-y)z + .4x(1-y)z$ $+ 2(1-x)yz + 2xyz = 1.5, \quad x, y, z \in [0, 1].$ . . . . .	40
3.6. Triangular approximation of contour level $f(x, y, z) = 60$ for $f(x, y, z) = 1.2 ((x-10)^2 - (y-10)^2 + (z-10)^2),$ $x, y, z \in [0, 20].$ . . . . .	41
3.7. Assigning the part index to a triangle in a contour triangulation. . . . .	46
3.8. The 18 function values and their weights needed for approximating the $x$ -coordinate for a normal using Zucker's operator. . . . .	51
3.9. Human skull obtained from a rectilinear CAT-scan data set, $68 \cdot 64 \cdot 64$ points, $f_{i,j,k} \in [0, 255]$ , approximation for $f(x, y, z) = 12.5.$ . . . . .	53
4.1. Texture map of mean and Gaussian curvature onto a torus, $((2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u)^T, \quad u, v \in [0, 2\pi];$ green/yellow representing negative values, magenta/blue representing positive values. . . . .	61
4.2. Construction of a bivariate polynomial for platelet points in a two-dimensional triangulation. . . . .	72
4.3. Exact curvatures $\kappa_1^{ex}, \kappa_2^{ex}, H^{ex},$ and $K^{ex}$ on the graph of $f(x, y) = .4(x^2 - y^2), \quad x, y \in [-1, 1].$ . . . .	75
4.4. Approximated curvatures $\kappa_1^{app}, \kappa_2^{app}, H^{app},$ and $K^{app}$ on the graph of $f(x, y) = .4(x^2 - y^2), \quad x, y \in [-1, 1].$ . . . .	75
4.5. Exact curvatures $\kappa_1^{ex}, \kappa_2^{ex}, H^{ex},$ and $K^{ex}$ on the graph of $f(x, y) = .15(x^3 + 2x^2y - xy + 2y^2), \quad x, y \in [-1, 1].$ . . . .	76
4.6. Approximated curvatures $\kappa_1^{app}, \kappa_2^{app}, H^{app},$ and $K^{app}$ on the graph of $f(x, y) = .15(x^3 + 2x^2y - xy + 2y^2), \quad x, y \in [-1, 1].$ . . . .	76
4.7. Exact curvatures $\kappa_1^{ex}, \kappa_2^{ex}, H^{ex},$ and $K^{ex}$ on the graph of $f(x, y) = .1(\cos(\pi x) + \cos(\pi y)), \quad x, y \in [-1, 1].$ . . . .	77

Figure	Page
4.8.    Approximated curvatures $\kappa_1^{app}$ , $\kappa_2^{app}$ , $H^{app}$ , and $K^{app}$ on the graph of $f(x, y) = .1 (\cos(\pi x) + \cos(\pi y))$ , $x, y \in [-1, 1]$ . . . . .	77
4.9.    Mean and Gaussian curvature of the graph of $f(x, y, z) = .4 (x^2 + y^2 + z^2)$ , $x, y, z \in [-1, 1]$ . . . . .	80
4.10.   Exact and approximated mean and Gaussian curvatures of the graph of $f(x, y, z) = .4 (x^2 - y^2 - z^2)$ , $x, y, z \in [-1, 1]$ . . . . .	89
4.11.   Exact and approximated mean and Gaussian curvatures of the graph of $f(x, y, z) = .15 (x^3 + 2x^2y - xz^2 + 2y^2)$ , $x, y, z \in [-1, 1]$ . . . . .	90
4.12.   Exact and approximated mean and Gaussian curvatures of the graph of $f(x, y, z) = .1 (\cos(\pi x) + \cos(\pi y) + \cos(\pi z))$ , $x, y, z \in [-1, 1]$ . . . . .	90
5.1.    Triangle platelet with continuous and discontinuous corona and cyclic corona. . . . .	94
5.2.    Triangle platelet with continuous, acyclic corona and boundary vertex set. . . . .	98
5.3.    Boundary vertex set and its projection onto plane $P$ ; triangle $T_i$ passing the half-plane test. . . . .	101
5.4.    Different choices for origin depending on triangle platelet. . . . .	107
5.5.    Removal of triangle $T_i$ and re-triangulation of boundary vertex set and new point. . . . .	111
5.6.    Increasing angle weights of triangles in local re-triangulation (original and improved re-triangulation). . . . .	115
5.7.    Triangle reduction of 50%, 80%, and 90% for the graph of $f(x, y) = .4 (x^2 + y^2)$ , $x, y, \in [-1, 1]$ . . . . .	118
5.8.    Triangle reduction of 50%, 80%, and 90% for the graph of $f(x, y) = .15 (x^3 + 2x^2y - xy + 2y^2)$ , $x, y, \in [-1, 1]$ . . . . .	118

Figure		Page
5.9.	Triangle reduction of 50%, 80%, and 90% for the graph of $f(x, y) = .1 (\cos(\pi x) + \cos(\pi y))$ , $x, y, \in [-1, 1]$ . . . . .	119
5.10.	Triangle reduction of 50%, 80%, and 90% for the torus $((2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u)^T$ , $u, v \in [0, 2\pi]$ . . . . .	120
5.11.	Triangle reduction of 90% for a piecewise triangular approximation of a human skull. . . . .	120
6.1.	Concept of barycentric coordinates for a triangle. . . . .	128
6.2.	Conic in Bézier representation. . . . .	129
6.3.	Degree elevation for a conic in Bézier representation. . . . .	131
6.4.	Generating patch normal along edge $e_1$ . . . . .	134
6.5.	Evaluating first patch building block. . . . .	135
6.6.	Convex and non-convex data configurations defined by end-points and end-tangents in a plane. . . . .	137
6.7.	Triangular surfaces obtained from spherical data using increasing weights (from left to right). . . . .	138
6.8.	Triangular surface for reduced skull triangulation, weights chosen automatically. . . . .	139