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ECS 17: Data, Logic, and Computing
Midterm
February 28, 2024

Notes:

1) Midterm is open book, open notes…
2) You have 50 minutes, no more: I will strictly enforce this.
3) The midterm is graded over 70 points.
4) You can answer directly on these sheets (preferred), or on loose paper.
5) Please write your name at the top right of at least the first page that you turn in!

Part I: logic (2 questions, each 10 points; total 20 points)

Using truth tables, establish for each of the two propositions below if it is a tautology, a contradiction, or neither.

1) $(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow \neg q)$

2) $(p \to (q \wedge r)) \vee ((p \wedge q) \to r)$

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Part II: proofs (5 questions, each 10 points; total 50 points)

1) Prove that if $7n^2+4$ is even then n is even, where n is a natural number.

2) Let a, b, and c be consecutive integers with $a<b<c$. Show that if $a \neq -1$ and $a \neq 3$, then $a^2 + b^2 \neq c^2$

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3) Let a and b be two positive real numbers. Use a **proof by contradiction** to show that if $\frac{a}{b+1} = \frac{b}{a+1}$, then $a = b$

4) Show that $n^2 + n + 9$ is odd for all integer n.

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5) Let *a* and *b* be 2 integers. Use a **direct proof** to show that if $a^2 + 4b^2 - 4ab$ is even, then $a + 2b$ is even.

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Appendix

The ECS17 Prayers

1) Thou shalt not say "there exists *k*" without mentioning the domain of *k*.
2) Thou shalt not say "it is obvious"
3) If *p* and *q* are two propositions, then $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$. This is the basis for the proof by contrapositive.
4) If *p* and *q* are two propositions, then $p \rightarrow q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.
5) An integer *n* is even if and only if there exists and integer *k* such that $n = 2k$. We say also that n is a multiple of 2.
6) An integer *n* is odd if and only if there exists and integer *k* such that $n = 2k + 1$.
7) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let *n* be an integer. Then:
   a) If *n* is even, then $n+1$ and $n-1$ are odd
   b) if *n* is odd, then $n+1$ and $n-1$ are even
2) Let *n* be an integer. Then:
   a) *n* is even, if and only if n^2 is even
   b) *n* is odd, if and only if n^2 is odd
3) $\forall n \in Z, n(n+1)$ is even.
4) $\sqrt{2}$ is irrational.

Identities

Let *a* and *b* be two real numbers:

1) $(a+b)^2 = a^2 + 2ab + b^2$
2) $(a-b)^2 = a^2 - 2ab + b^2$
3) $a^2 - b^2 = (a-b)(a+b)$
4) **Completing the square:** $a^2 + b^2 = (a+b)^2 - 2ab$